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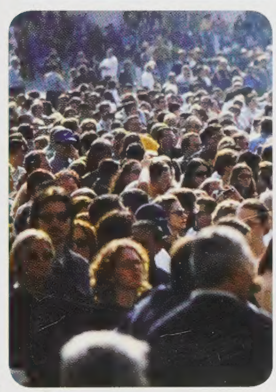
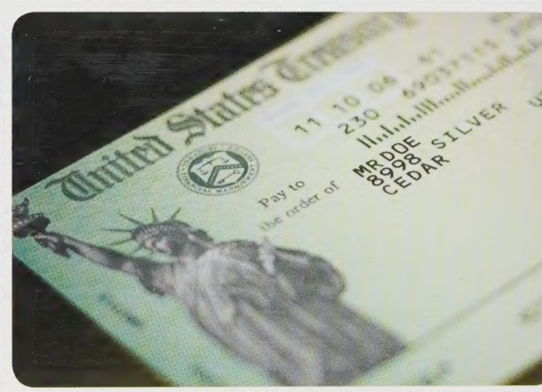
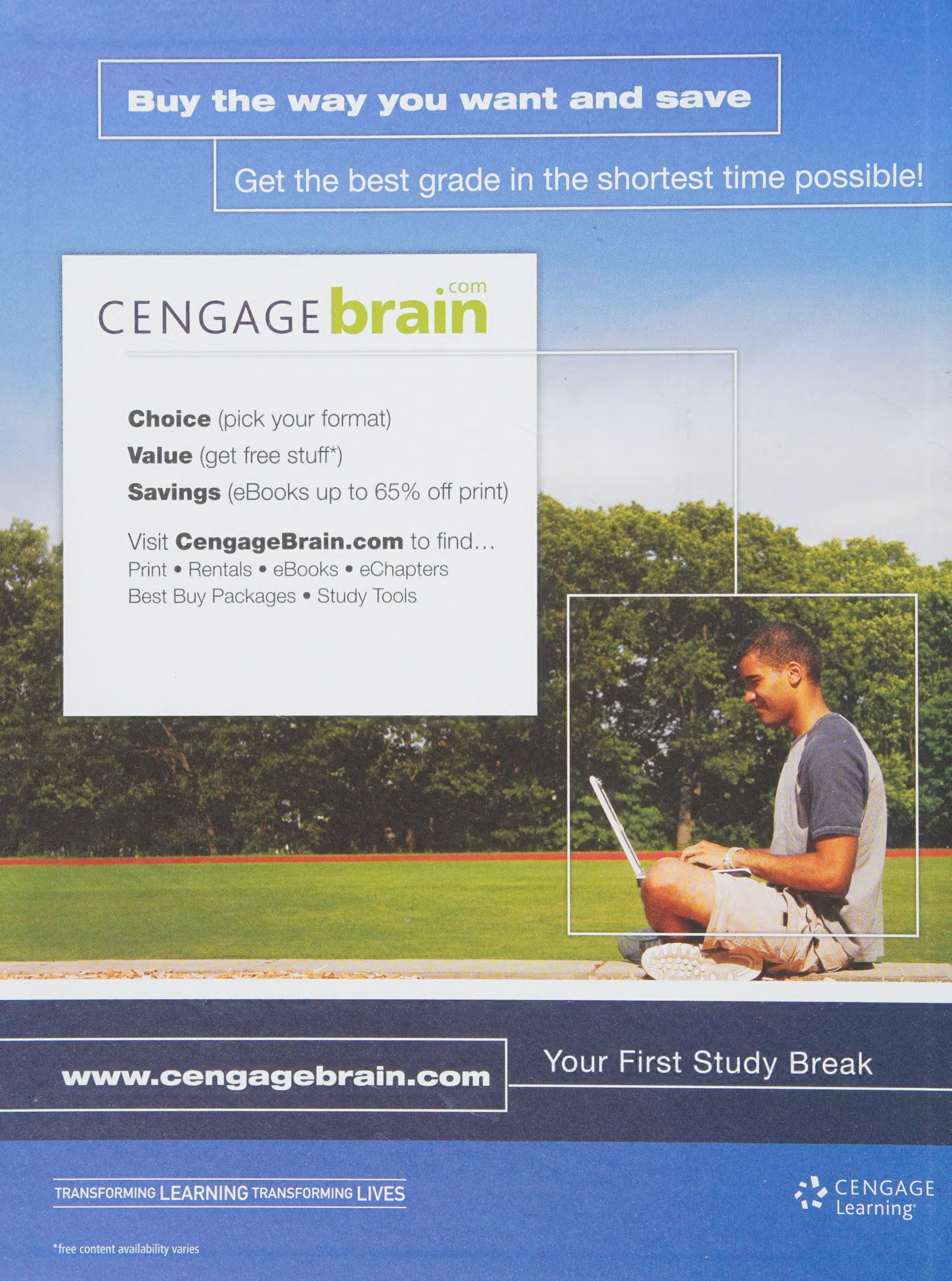
Applied Calculus

For the Managerial, Life, and Social Sciences

Ninth Edition



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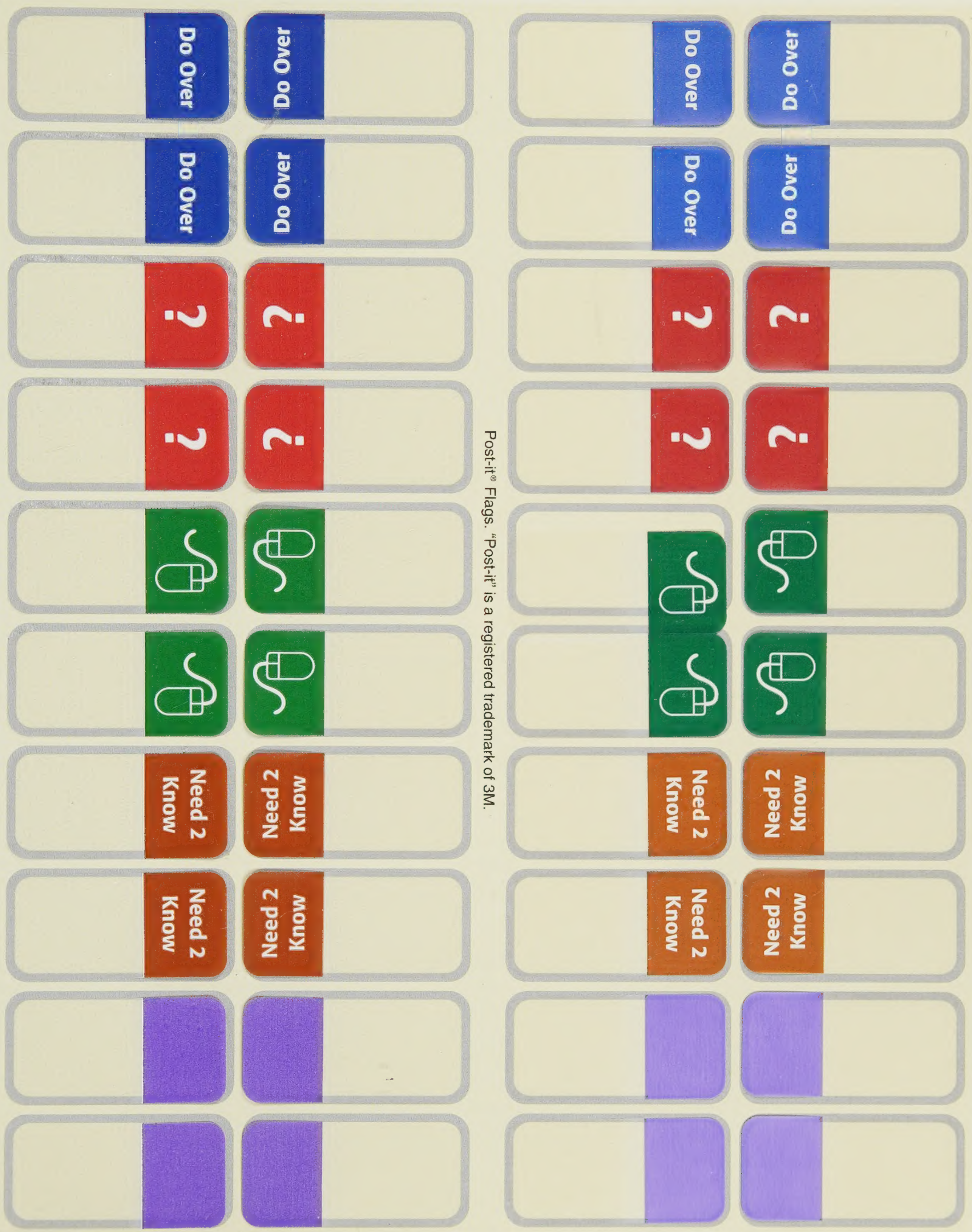

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Tab it. Do it. Ace it.

Do Over

Do you need to review something? Try again? Work it out on your own after class? Tab it.

?

Got a question for office hours? Do you need to review an example on your own to get a full understanding? Do you need to look something up before moving on? Tab it.

Do you need to see the video? Check out an online source? Complete your online homework? Tab it.

Need 2 Know

Is this going to be on the test? Need to mark a key formula? Do you need to memorize these steps? Tab it.

Do you have your own study system? Do you need to make a note? Do you want to express yourself? Tab it.

Tab it. Do it. Ace it.

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EDITION 9

APPLIED CALCULUS

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

SOO T. TAN

STONEHILL COLLEGE

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TO PAT, BILL, AND MICHAEL

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PREFACE

Applied Calculus for the Managerial, Life, and Social Sciences is intended for use in a two-semester or three-quarter introductory calculus course for students in the managerial, life, and social sciences. In preparing the Ninth Edition, I have kept in mind two longstanding goals: (1) to write an applied text that motivates students and (2) to make the book a useful teaching tool for instructors. Underlying this is my belief that math is an integral part of everyone's daily life. One of the most important lessons I have learned from many years of teaching undergraduate mathematics courses is that most students—mathematics majors and non-mathematics majors alike—respond well when introduced to mathematical concepts and results using real-life illustrations.

I also learned from my experience teaching courses in the managerial, life, and social sciences that many students come to these courses with some degree of apprehension. This awareness led to the intuitive approach I have adopted in all of my texts. As you will see, I try to introduce each abstract mathematical concept through an example drawn from a common real-life experience. Once the idea has been conveyed, I then proceed to make it precise, thereby assuring that no mathematical rigor is lost in this intuitive treatment of the subject.

Another lesson I learned from my students is that they have a much greater appreciation of the material if the applications are drawn from their fields of interest and from situations that occur in the real world. This is one reason you will see so many exercises in my texts that are modeled on data gathered from newspapers, magazines, journals, and other media. Whether it be the market for cholesterol-reducing drugs, financing a home, bidding for cable rights, broadband Internet households, or Starbucks' annual sales, I weave topics of current interest into my examples and exercises to keep the book relevant to all of my readers.

THE APPROACH

Level of Presentation

My approach is intuitive, and I state the results informally. However, I have taken special care to ensure that this approach does not compromise the mathematical content and accuracy.

Problem-Solving Approach

A problem-solving approach is stressed throughout the book. Numerous examples and applications illustrate each new concept and result. Special emphasis is placed on helping students formulate, solve, and interpret the results of the problems involving applications. Because students often have difficulty setting up and solving word problems, I have taken extra care to help students master these skills:

- Very early on in the text, students are given practice in setting up word problems (see Section 2.3).
- Guidelines are given to help formulate and solve related-rates problems in Section 3.6.

- In Chapter 4, optimization problems are covered in two sections. First, the techniques of calculus are used to solve problems in which the function to be optimized is given (Section 4.4); second, optimization problems that require the additional step of formulating the problem are treated (Section 4.5).
- In Chapter 9, "Differential Equations," students are once again encouraged to set up problems involving applications (see Section 9.1) before being exposed to the methods of solution for these problems in Sections 9.2–9.4.

Intuitive Introduction to Concepts

Mathematical concepts are introduced with concrete, real-life examples wherever appropriate. Following are some of the topics introduced in this manner:

- **Limits:** The Motion of a Maglev
- **The algebra of functions:** The U.S. Budget Deficit
- **The Chain Rule:** The Population of Americans Aged 55 Years and Older
- **Differentials:** Calculating Mortgage Payments
- **Increasing and decreasing functions:** The Fuel Economy of a Car
- **Concavity:** U.S. and World Population Growth
- **Inflection points:** The Point of Diminishing Returns
- **Curve sketching:** The Dow Jones Industrial Average on "Black Monday"
- **Exponential functions:** Income Distribution of American Families
- **Area between two curves:** Petroleum Saved with Conservation Measures
- **Approximating definite integrals:** The Cardiac Output of a Heart

Connections

One example (the maglev) is used as a common thread throughout the development of calculus—from limits through integration. The goal here is to show students the connections between the concepts presented: limits, continuity, rates of change, the derivative, the definite integral, and so on.

Motivation

Illustrating the practical value of mathematics in applied areas is an objective of my approach. Many of the applications are based on mathematical models (functions) that I have constructed using data drawn from various sources, including current newspapers, magazines, and the Internet. Sources are given in the text for these applied problems.

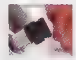
Modeling

I believe that one of the important skills that a student should acquire is the ability to translate a real problem into a mathematical model that can provide insight into the problem. In Section 2.3, the modeling process is discussed, and students are asked to use models (functions) constructed from real-life data to answer questions. Students get hands-on experience constructing these models in the Using Technology sections.

NEW TO THIS EDITION

Motivating Real-World Applications

Among the many new and updated applied examples and exercises are problems involving global warming, the solvency of the U.S. Social Security trust fund, the Case-Shiller Home Price Index, smartphone sales, bounced-check charges, solar panel production, Mexico's hedging tactic, the Gini Index, Facebook users, e-book audience, the growth of credit unions, the waiting time for a *"Fountains of Bellagio"* show, and mobile phone users in China.



APPLIED EXAMPLE 2 Global Warming The increase in carbon dioxide (CO_2) in the atmosphere is a major cause of global warming. The Keeling curve, named after Charles David Keeling, a professor at Scripps Institution of Oceanography, gives the average amount of CO_2, measured in parts per million volume (ppmv), in the atmosphere from 1958 through 2010. Even though data were available for every year in this time interval, we'll construct the curve based only on the following randomly selected data points.

Year	1958	1970	1974	1978	1985	1991	1998	2003	2007	2010
Amount	315	325	330	335	345	355	365	375	380	390

The **scatter plot** associated with these data is shown in Figure 18a. A mathematical model giving the approximate amount of CO_2 in the atmosphere during this period is given by

$$A(t) = 0.012313t^2 + 0.7545t + 313.9 \qquad (1 \le t \le 53)$$

Modeling with Data

As in the previous edition, Modeling with Data exercises are found in many of the Using Technology sections throughout the text. Here, students can actually see how some of the functions found in the exercises are constructed. Many of these applications have been updated, and some new exercises have been added. (See Age at First Marriage, Exercise 92, page 281, and the corresponding exercise in which the model is derived in Exercise 12, page 265.)

92. Age at First Marriage According to a study conducted by Rutgers University, the median age of women in the U.S. at first marriage is approximated by the function

$$f(t) = -0.083t^3 + 0.6t^2 + 0.18t + 20.1 \qquad (0 \le t \le 5)$$

where t is measured in d
the beginning of 1960.

a. What was the median
the beginning of 196
2000?

b. When was the median
changing most rapidly
sideration?

Source: The National Marriag

12. Modeling With Data The following data gives the median age of women in the United States at first marriage from 1960 through 2000.

Decade	1960	1970	1980	1990	2000
Median Age	20.1	21.0	22.0	24.0	25.1

a. Use **CubicReg** to find a third-degree polynomial regression model for these data. Let t be measured in decades, with $t = 0$ corresponding to 1960.

b. Plot the graph of the polynomial function f found in part (a) using the viewing window $[0, 4] \times [18, 28]$.

c. Where is f increasing? What does this tell us?

d. Verify the result of part (c) analytically.

Source: The National Marriage Project, Rutgers University

Making Connections with Technology

All of the art in the Using Technology sections has been redone. The graphing calculator screens now show numbered scales for both axes, making it easier for students to use and understand these graphs. Many of the applications in the examples and exercises of the Using Technology sections have been updated.

7. Bank Failure The Haven Trust Bank of Duluth, Ga., founded in 2000, quickly increased its risky commercial real estate portfolio, despite many red flags from regulators. The bank failed in December 2008. The amount of construction loans of the bank as a percentage of its capital is approximated by the function

$$f(t) = -5.92t^4 + 58.89t^3 - 165.75t^2 + 56.21t + 629 \qquad (0 \le t \le 5)$$

where $t = 0$ corresponds to the beginning of 2003.

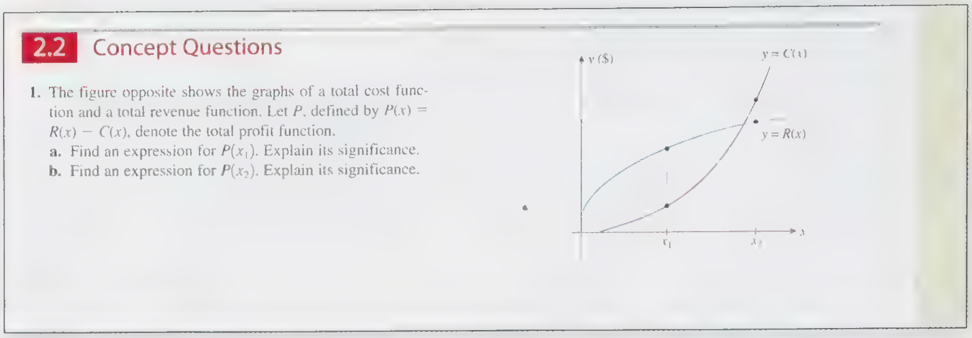
a. Plot the graph of f using the viewing window $[0, 5] \times [0, 650]$.

b. Show that at no time during the period from the beginning of 2003 through the beginning of 2008 did the amount of construction loans of the bank as a percentage of its capital fall below 415%. Note: The maximum percentage recommended by regulators in 2008 was 100%.

Source: FDIC Office of Inspector General

Variety of Problem Types

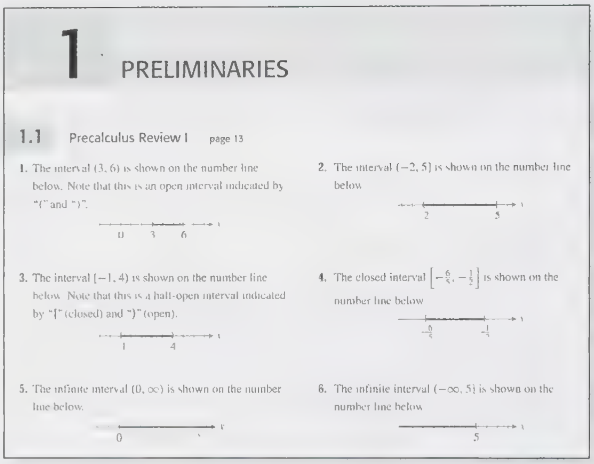
Additional rote questions, true-or-false questions, and concept questions have been added throughout the text to enhance the exercise sets. (See, for example, Concept Questions 2.2, Exercise 1, page 71.)


2.2 Concept Questions

1. The figure opposite shows the graphs of a total cost function and a total revenue function. Let P, defined by $P(x) = R(x) - C(x)$, denote the total profit function.
   a. Find an expression for $P(x_1)$. Explain its significance.
   b. Find an expression for $P(x_2)$. Explain its significance.

Carefully Crafted Solutions

The *Complete Solutions Manual* has been completely revamped. All new art has been created for the manual, and the solutions have been copyedited and streamlined for ease of use. As in previous editions, the solutions to all of the exercises have been written by the author.


1 PRELIMINARIES

1.1 Precalculus Review I page 13

1. The interval $(3, 6)$ is shown on the number line below. Note that this is an open interval indicated by "(" and ")".

2. The interval $(-2, 5]$ is shown on the number line below.

3. The interval $[-1, 4)$ is shown on the number line below. Note that this is a half-open interval indicated by "[" (closed) and ")" (open).

4. The closed interval $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ is shown on the number line below.

5. The infinite interval $(0, \infty)$ is shown on the number line below.

6. The infinite interval $(-\infty, 5]$ is shown on the number line below.

Enhanced Graphics

The three-dimensional artwork in Section 7.5 and Chapter 8 has been redone with an emphasis on making it easier for students to see the concepts being described in 3-space. For example, Figure 7 in Section 8.1 now shows the trace of the graph of $z = f(x, y)$ and the plane $z = k$ and its projection onto the xy-plane (Figure 7a) and the corresponding level curve (Figure 7b).

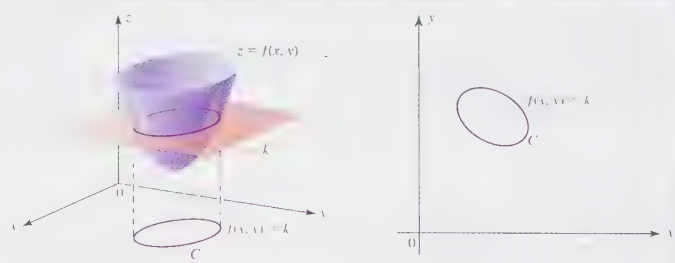


FIGURE 7

(a) The level curve C with equation $f(x, y) = k$ is the projection of the trace of f in the plane $z = k$ onto the xy-plane.

(b) The level curve C

Specific Content Changes

- Examples 4, 7b, and 10c were added to Section 1.1.
- Applied Example 4 in the Using Technology subsection for Section 2.2 has been redone. The budget-deficit graphs that are used as motivation to introduce Section 2.2, "The Algebra of Functions," have been redone to reflect the current deficit figures. A new graphical concept exercise has been added to Section 2.2. In Section 2.3, "Functions and Mathematical Models," new models for the first four applied examples—Bounced Check Charges, Global Warming, Social Security Trust Fund Assets, and Driving Costs—have been constructed.
- Applied Examples 8 in Sections 3.1 and 3.3 have been modified. In Section 3.4, the subsection on Elasticity of Demand has been rewritten, and the discussion has been simplified. In Section 3.6, "Implicit Differentiation and Related Rates," two new

examples have been added. Example 5 illustrates the process of finding the second derivative of a function implicitly, and Applied Example 6 is an economic application that is used to introduce the marginal rate of technical substitution (MRTS).

- In Chapter 4, the budget curves used to motivate relative extrema were updated to reflect the current deficit. Seven new graphical exercises were added to Exercise Set 4.2, including Rumors of a Run on a Bank and the Case-Shiller Home Price Index. The Average Age of Cars application that is used to motivate the concept of absolute extrema in Section 4.4 was updated. Applied Example 7 in that section has also been modified.
- In Chapters 5 through 7, a number of new and unique applications have been added to the exercise sets. Among these are Pharmaceutical Theft, Federal Lobbying, Total Knee Replacement Procedures, Mexico's Hedging Tactics, the British Deficit, Consumer Spending on Entertainment, and Medical Costs for Veterans. In Section 5.3, the examples and exercises have been updated to reflect the current lower interest rates. Section 5.4 is now introduced by a new model for the distribution of income in the United States in 2010. Two new applications on the Gini index in the United States have been added to exercise set 7.2 for numerical integration.
- In Chapter 8, more than 30 new applications have been added to the exercise sets. In Sections 9.2 and 9.3, two new examples have been added. In Chapter 10, two new examples have been added to Section 10.1 and 11 applications have been added to the exercise sets. In Chapter 11, a new Using Technology section has been added for Newton's method. In Chapter 12, new applications and rote problems were added to the exercise sets.
- The three-dimensional artwork in Section 7.5 and Chapter 8 has been redone. The new graphics make it easier for students to see the concepts being described in 3-space.

TRUSTED FEATURES

In addition to the new features, we have retained many of the hallmarks that made this series so usable and well-received in past editions:

- Review material to reinforce prerequisite algebra skills
- Section exercises to help students understand and apply concepts
- Optional technology sections to explore mathematical ideas and solve problems
- End-of-chapter review sections to assess understanding and problem-solving skills
- Features to motivate further exploration

Algebra Review Gives Students a Plan of Action

A Diagnostic Test precedes the precalculus review. Each question is referenced by the section and example in the text where the relevant topic can be reviewed. Students can use this test to diagnose their weaknesses and review the material on an as-needed basis.

Diagnostic Test

1. a. Evaluate the expression:

   (i) $\left(\frac{16}{9}\right)^{3/2}$ (ii) $\sqrt[3]{\frac{27}{125}}$

   b. Rewrite the expression using positive exponents only: $(x^{-2}y^{-1})^3$

   (Exponents and radicals, Examples 1 and 2, pages 6–7)

2. Rationalize the numerator: $\sqrt[3]{\frac{x^2}{yz^3}}$

   (Rationalization, Example 5, page 7)

Algebra Review Where Students Need It Most

Well-placed algebra review notes, keyed to the review chapter, appear where students need them most throughout the text. These are indicated by the (x^2) icon. See this feature in action on pages 105 and 535.

EXAMPLE 6 Evaluate:

$$\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h}$$

Solution Letting h approach zero, we obtain the indeterminate form 0/0. Next, we rationalize the numerator of the quotient by multiplying both the numerator and the denominator by the expression $(\sqrt{1+h}+1)$, obtaining

$$\begin{aligned}\frac{\sqrt{1+h}-1}{h} &= \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)} \quad (x^2)\text{ See page 19.}\\ &= \frac{1+h-1}{h(\sqrt{1+h}+1)} \quad (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b\\ &= \frac{h}{h(\sqrt{1+h}+1)}\\ &= \frac{1}{\sqrt{1+h}+1}\end{aligned}$$

Therefore,

$$\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h \to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Self-Check Exercises

Offering students immediate feedback on key concepts, the Self-Check Exercises begin each end-of-section exercise set. Fully worked-out solutions to these exercises can be found at the end of each exercise section.

7.1 Self-Check Exercises

1. Evaluate $\int x^2 \ln x\,dx$.
2. Since the inauguration of Ryan's Express at the beginning of 2009, the number of passengers (in millions) flying on this commuter airline has been growing at the rate of
$$R(t) = 0.1 + 0.2te^{-0.4t}$$
passengers/year ($t = 0$ corresponds to the beginning of 2009). Assuming that this trend continues through 2013, determine how many passengers will have flown on Ryan's Express by that time.

Solutions to Self-Check Exercises 7.1 can be found on page 492.

Concept Questions

Designed to test students' understanding of the basic concepts discussed in the section, the Concept Questions encourage students to explain learned concepts in their own words.

7.1 Concept Questions

1. Write the formula for integration by parts.
2. Explain how you would choose u and dv when using the integration by parts formula. Illustrate your answer with $\int x^2 e^{-x}\,dx$. What happens if you reverse your choices of u and dv?

Exercises

Each exercise section contains an ample set of problems of a routine computational nature followed by an extensive set of application-oriented problems.

7.1 Exercises

In Exercises 1–26, find each indefinite integral.

1. $\int xe^{2x}\,dx$
2. $\int xe^{-x}\,dx$
3. $\int \frac{1}{2}xe^{x/4}\,dx$
4. $\int 6xe^{3x}\,dx$
5. $\int (e^x - x)^2\,dx$
6. $\int (e^{-x} + x)^2\,dx$
7. $\int (x+1)e^x\,dx$
8. $\int (x-3)e^{3x}\,dx$
9. $\int x(x+1)^{-3/2}\,dx$
10. $\int x(x+4)^{-2}\,dx$
11. $\int x\sqrt{x-5}\,dx$
12. $\int \frac{3x}{\sqrt{2x+3}}\,dx$
13. $\int x \ln 2x\,dx$
14. $\int x^2 \ln 2x\,dx$
15. $\int x^3 \ln x\,dx$
16. $\int \sqrt{x} \ln x\,dx$
17. $\int \sqrt{x} \ln \sqrt{x}\,dx$
18. $\int \frac{\ln x}{\sqrt{x}}\,dx$
19. $\int \frac{\ln x}{x^2}\,dx$
20. $\int \frac{\ln x}{x^3}\,dx$
21. $\int \ln x\,dx$
Hint: Let $u = \ln x$ and $dv = dx$.
22. $\int \ln(x+1)\,dx$
23. $\int x^2 e^{-x}\,dx$
Hint: Integrate by parts twice.
24. $\int e^{-\sqrt{x}}\,dx$
Hint: First, make the substitution $u = \sqrt{x}$; then integrate by parts.
25. $\int x(\ln x)^2\,dx$
Hint: Integrate by parts twice

Using Technology

The optional Using Technology features appear after the section exercises. They can be used in the classroom if desired or as material for self-study by the student. Here, the graphing calculator is used as a tool to solve problems. These sections are written in the traditional example-exercise format with answers given at the back of the book. Illustrations showing graphing calculator screens are used extensively. In keeping with the theme of motivation through real-life examples, many sourced applications are included. Students can construct their own models using real-life data in many of the Using Technology sections. These include models for the growth of the Indian gaming industry, health-care spending, TiVo owners, nicotine content of cigarettes, computer security, and online gaming, among others.

A *How-To Technology Index* is included at the back of the book for easy reference.

USING TECHNOLOGY

Although the proof is outside the scope of this book, it can be proved that an exponential function of the form $f(x) = b^x$, where $b > 1$, will ultimately grow faster than the power function $g(x) = x^n$ for *any* positive real number n. To give a visual demonstration of this result for the special case of the exponential function $f(x) = e^x$, we can use a graphing utility to plot the graphs of both f and g (for selected values of n) on the same set of axes in an appropriate viewing window and observe that the graph of f ultimately lies above that of g.

EXAMPLE 1 Use a graphing utility to plot the graphs of (a) $f(x) = e^x$ and $g(x) = x^3$ on the same set of axes in the viewing window $[0, 6] \times [0, 250]$ and (b) $f(x) = e^x$ and $g(x) = x^5$ in the viewing window $[0, 20] \times [0, 1{,}000{,}000]$.

Solution

a. The graphs of $f(x) = e^x$ and $g(x) = x^3$ in the viewing window $[0, 6] \times [0, 250]$ are shown in Figure T1a.

b. The graphs of $f(x) = e^x$ and $g(x) = x^5$ in the viewing window $[0, 20] \times [0, 1{,}000{,}000]$ are shown in Figure T1b.

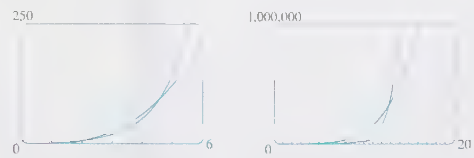


FIGURE **T1**

(a) The graphs of $f(x) = e^x$ and $g(x) = x^3$ in the viewing window $[0, 6] \times [0, 250]$

(b) The graphs of $f(x) = e^x$ and $g(x) = x^5$ in the viewing window $[0, 20] \times [0, 1{,}000{,}000]$

In the exercises that follow, you are asked to use a graphing utility to reveal the properties of exponential functions.

Exploring with Technology

Designed to explore mathematical concepts and to shed further light on examples in the text, the optional Exploring with Technology questions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Often the solution of an example in the text is augmented with a graphical or numerical solution. Solutions to these exercises are given in Solution Builder and the *Complete Solutions Manual.*

Exploring with TECHNOLOGY

To obtain a visual confirmation of the fact that the expression $(1 + 1/m)^m$ approaches the number $e = 2.71828\ldots$ as m increases without bound, plot the graph of $f(x) = (1 + 1/x)^x$ in a suitable viewing window and observe that $f(x)$ approaches $2.71828\ldots$ as x increases without bound. Use ZOOM and TRACE to find the value of $f(x)$ for large values of x.

Summary of Principal Formulas and Terms

Each review section begins with the Summary highlighting important equations and terms, with page numbers given for quick review.

CHAPTER 5 Summary of Principal Formulas and Terms

FORMULAS

1. Exponential function with base b	$y = b^x$
2. The number e	$e = \lim_{m \to \infty} \left(1 + \frac{1}{m}\right)^m = 2.71828\ldots$
3. Exponential function with base e	$y = e^x$
4. Logarithmic function with base b	$y = \log_b x$
5. Logarithmic function with base e	$y = \ln x$
6. Inverse properties of $\ln x$ and e^x	$\ln e^x = x$ and $e^{\ln x} = x$
7. Compound interest (accumulated amount)	$A = P\left(1 + \frac{r}{m}\right)^{mt}$
8. Effective rate of interest	$r_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1$
9. Compound interest (present value)	$P = A\left(1 + \frac{r}{m}\right)^{-mt}$
10. Continuous compound interest	$A = Pe^{rt}$
11. Derivative of the exponential function	$\frac{d}{dx}(e^x) = e^x$
12. Chain Rule for Exponential Functions	$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$
13. Derivative of the logarithmic function	$\frac{d}{dx} \ln x = \frac{1}{x}$
14. Chain Rule for Logarithmic Functions	$\frac{d}{dx} \ln \lvert u \rvert = \frac{1}{u}\frac{du}{dx}$

TERMS

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natural logarithm (338)
compound interest (346)
logarithmic differentiation (374)
exponential growth (381)
growth constant (381)
exponential decay (382)
decay constant (382)
half-life of a radioactive substance (383)
logistic growth function (386)

Concept Review Questions

The Concept Review Questions give students a chance to check their knowledge of the basic definitions and concepts given in each chapter.

CHAPTER 5 Concept Review Questions

Fill in the blanks.

1. The function $f(x) = x^b$ (b, a real number) is called a/an _____ function, whereas the function $g(x) = b^x$, where $b >$ _____ and $b \neq$ _____, is called a/an _____ function.

2. a. The domain of the function $y = 3^x$ is _____, and its range is _____.
b. The graph of the function $y = 0.3^x$ passes through the point _____ and is decreasing on _____.

3. a. If $b > 0$ and $b \neq 1$, then the logarithmic function $y = \log_b x$ has domain _____ and range _____; its graph passes through the point _____.
b. The graph of $y = \log_b x$ is decreasing if b _____ and increasing if b _____.

4. a. If $x > 0$, then $e^{\ln x} =$ _____.
b. If x is any real number, then $\ln e^x =$ _____.

5. In the compound interest formula $A = P(1 + \frac{r}{m})^{mt}$, A stands for the _____ _____, P stands for the _____, r stands for the _____ _____ _____ per year, m stands for the _____ _____ _____ per year, and t stands for the _____.

6. The effective rate r_{eff} is related to the nominal interest rate r per year and the number of conversion periods per year by $r_{\text{eff}} =$ _____.

7. If interest earned at the rate of r per year is compounded continuously over t years, then a principal of P dollars will have an accumulated value of $A =$ _____ dollars.

8. a. If $g(x) = e^{f(x)}$, where f is a differentiable function, then $g'(x) =$ _____.
b. If $g(x) = \ln f(x)$, where $f(x) > 0$ is differentiable, then $g'(x) =$ _____.

Review Exercises

Offering a solid review of the chapter material, the Review Exercises contain routine computational exercises followed by applied problems.

CHAPTER 5 Review Exercises

1. Sketch on the same set of coordinate axes the graphs of the exponential functions defined by the equations.
a. $y = 2^{-x}$ **b.** $y = \left(\frac{1}{2}\right)^x$

In Exercises 2 and 3, express each equation in logarithmic form.

2. $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$ **3.** $16^{-3/4} = 0.125$

In Exercises 4 and 5, solve each equation for x.

4. $\log_4(2x + 1) = 2$

5. $\ln(x - 1) + \ln 4 = \ln(2x + 4) - \ln 2$

In Exercises 6–8, given that $\ln 2 = x$, $\ln 3 = y$, and $\ln 5 = z$, express each of the given logarithmic values in terms of x, y, and z.

23. $f(x) = x^2e^x + e^x$ **24.** $g(t) = t \ln t$

25. $f(x) = \ln(e^{x^2} + 1)$ **26.** $f(x) = \frac{x}{\ln x}$

27. $f(x) = \frac{\ln x}{x + 1}$ **28.** $y = (x + 1)e^x$

29. $y = \ln(e^{4x} + 3)$ **30.** $f(r) = \frac{re^r}{1 + r^2}$

31. $f(x) = \frac{\ln x}{1 + e^x}$ **32.** $g(x) = \frac{e^{x^2}}{1 + \ln x}$

33. Find the second derivative of the function $y = \ln(3x + 1)$.

34. Find the second derivative of the function $y = x \ln x$.

Before Moving On . . .

Found at the end of each chapter review, the Before Moving On exercises give students a chance to find out whether they have mastered the basic computational skills developed in that chapter.

CHAPTER 5 Before Moving On . . .

1. Solve the equation $\frac{100}{1 + 2e^{0.3t}} = 40$ for t.
2. Find the accumulated amount after 4 years if \$3000 is invested at 8%/year compounded weekly.
3. Find the slope of the tangent line to the graph of $f(x) = e^{\sqrt{x}}$.
4. Find the rate at which $y = x \ln(x^2 + 1)$ is changing at $x = 1$.
5. Find the second derivative of $y = e^{2x} \ln 3x$.
6. The temperature of a cup of coffee at time t (in minutes) is

$$T(t) = 70 + ce^{-kt}$$

Initially, the temperature of the coffee was 200°F. Three minutes later, it was 180°. When will the temperature of the coffee be 150°F?

Explore & Discuss

The optional Explore & Discuss questions can be used in class or assigned as homework. These questions generally require more thought and effort than the usual exercises. They may also be used to add a writing component to the class or as team projects. Solutions to these exercises are given in Solution Builder and the *Complete Solutions Manual.*

Explore & Discuss

The average price of gasoline at the pump over a 3-month period, during which there was a temporary shortage of oil, is described by the function f defined on the interval [0, 3]. During the first month, the price was increasing at an increasing rate. Starting with the second month, the good news was that the rate of increase was slowing down, although the price of gas was still increasing. This pattern continued until the end of the second month. The price of gas peaked at $t = 2$ and began to fall at an increasing rate until $t = 3$.

1. Describe the signs of $f'(t)$ and $f''(t)$ over each of the intervals (0, 1), (1, 2), and (2, 3).
2. Make a sketch showing a plausible graph of f over [0, 3].

Portfolios

The real-life experiences of a variety of professionals who use mathematics in the workplace are related in the Portfolio interviews. Among those interviewed are Gary Li, an associate at JPMorgan Chase, and Todd Kodet, the Senior Vice-President of Supply at Earthbound Farm.

PORTFOLIO **Gary Li**

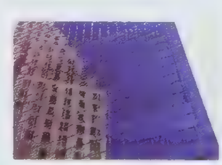
TITLE Associate
INSTITUTION JPMorgan Chase

As one of the leading financial institutions in the world, JPMorgan Chase & Co. depends on a wide range of mathematical disciplines from statistics to linear programming to calculus. In assessing the creditworthiness of a borrower, recommending portfolio investments, or pricing an exotic derivative, quantitative understanding is a critical tool in serving the financial needs of clients.

I work in the Fixed-Income Derivatives Strategy group. A derivative in finance is an instrument whose value depends on the price of some other underlying instrument. A simple type of derivative is the forward contract, in which two parties agree to a future trade at a specified price. In agriculture, for instance, farmers will often pledge their crops for sale to buyers at an agreed price before even planting the seed. Depending on the weather, demand, and other factors, the market price may turn out higher or lower. Either the buyer or seller of the forward contract benefits accordingly. The value of the contract changes one-for-one with the market price. In derivatives lingo, we borrow from calculus and say that forward contracts have a delta of 1.

Nowadays, the bulk of derivatives deal with interest-rate risk rather than agricultural risk. The value of any asset with fixed payments over time varies with interest rates. With trillions of dollars in this form, especially government bonds and mortgages, fixed-income derivatives are vital to the economy. The job of our strategy group is to track and anticipate key drivers and developments in the market using, in significant part, quantitative analysis. Some of the derivatives that we look at are the forward kind such as interest-rate swaps, in which over time, you receive fixed-rate payments in exchange for paying a floating rate or vice versa. A whole other class of derivatives in which statistics and calculus are especially relevant are options.

Whereas forward contracts bind both parties to a future trade, options give the holder the right but not the obligation to trade at a specified time and price. Similar to an insurance policy, the holder of the option pays an upfront premium in exchange for potential gain. Solving this pricing problem requires statistics, stochastic calculus, and enough insight to win a Nobel Prize. Fortunately for us, this was taken care of by Fischer Black, Myron Scholes, and Robert Merton in the early 1970s (the 1997 Nobel Prize in Economics went to Scholes and Merton). The Black–Scholes differential equation was the first accurate options pricing model, making possible the rapid growth of the derivatives market. Black–Scholes and the many derivatives of it continue to be used to this day.



Cengage Learning (inset) © AA World Travel Library/Alamy

Example Videos

Available on the companion Website and through Enhanced WebAssign, these videos offer hours of instruction from award-winning teacher Deborah Upton of Molloy College. Watch as she walks students through key examples from the text, step by step, giving them a foundation in the skills that they need to have. Each example that is available online is identified by the video icon located in the margin.

VIDEO

APPLIED EXAMPLE 8 Consumer Price Index An economy's consumer price index (CPI) is described by the function

$$I(t) = -0.2t^3 + 3t^2 + 100 \qquad (0 \le t \le 10)$$

where $t = 0$ corresponds to the beginning of 2003. Find the point of inflection of the function I, and discuss its significance.

Action-Oriented Study Tabs

Convenient color-coded study tabs, similar to Post-it® flags, make it easy for students to tab pages that they want to return to later, whether for additional review, exam preparation, online exploration, or identifying a topic to be discussed with the instructor.

TEACHING AIDS

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COMPLETE SOLUTIONS MANUAL (ISBN 978-1-285-08802-0) by Soo T. Tan
The *Complete Solutions Manual* contains fully worked-out solutions to all section exercises in the text, including Technology Exercises in Using Technology, Explore & Discuss activities, and Exploring with Technology activities.

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LEARNING AIDS

STUDENT SOLUTIONS MANUAL (ISBN 978-1-133-96092-8) by Soo T. Tan
Giving you more in-depth explanations, this insightful resource includes fully worked-out solutions for the answers to select exercises included at the back of the textbook, as well as problem-solving strategies, additional algebra steps, and review for selected problems.

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S. T. Tan

ABOUT THE AUTHOR

SOO T. TAN received his S.B. degree from Massachusetts Institute of Technology, his M.S. degree from the University of Wisconsin–Madison, and his Ph.D. from the University of California at Los Angeles. He has published numerous papers in optimal control theory, numerical analysis, and mathematics of finance. He is also the author of a series of calculus textbooks.

1

PRELIMINARIES

How much money is needed to purchase at least 100,000 shares of the Starr Communications Company? Corbyco, a giant conglomerate, wishes to purchase a minimum of 100,000 shares of the company. In Example 11, page 21, you will see how Corbyco's management determines how much money they will need for the acquisition.

THE FIRST TWO sections of this chapter contain a brief review of algebra. We then introduce the Cartesian coordinate system, which allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. This chapter also covers straight lines. The slope of a straight line plays an important role in the study of calculus.

Use this test to diagnose any weaknesses that you might have in the algebra that you will need for the calculus material that follows. The review section and examples that will help you brush up on the skills necessary to work the problem are indicated after each exercise. The answers follow the test.

Diagnostic Test

1. **a.** Evaluate the expression:

(i) $\left(\dfrac{16}{9}\right)^{3/2}$ (ii) $\sqrt[3]{\dfrac{27}{125}}$

b. Rewrite the expression using positive exponents only: $(x^{-2}y^{-1})^3$

(Exponents and radicals, Examples 1 and 2, pages 6–7)

2. Rationalize the numerator: $\sqrt[3]{\dfrac{x^2}{yz^3}}$

(Rationalization, Example 5, page 7)

3. Simplify the following expressions:

a. $(3x^4 + 10x^3 + 6x^2 + 10x + 3) + (2x^4 + 10x^3 + 6x^2 + 4x)$

b. $(3x - 4)(3x^2 - 2x + 3)$

(Operations with algebraic expressions, Examples 6 and 7, pages 8–9)

4. Factor completely:

a. $6a^4b^4c - 3a^3b^2c - 9a^2b^2$ **b.** $6x^2 - xy - y^2$

(Factoring, Examples 8–10, pages 9–11)

5. Use the quadratic formula to solve the following equation: $9x^2 - 12x = 4$

(The quadratic formula, Example 11, pages 12–13)

6. Simplify the following expressions:

a. $\dfrac{2x^2 + 3x - 2}{2x^2 + 5x - 3}$ **b.** $\dfrac{(t^2 + 4)(2t - 4) - (t^2 - 4t + 4)(2t)}{(t^2 + 4)^2}$

(Rational expressions, Example 1, page 16)

7. Perform the indicated operations and simplify:

a. $\dfrac{2x - 6}{x + 3} \cdot \dfrac{x^2 + 6x + 9}{x^2 - 9}$ **b.** $\dfrac{3x}{x^2 + 2} + \dfrac{3x^2}{x^3 + 1}$

(Rational expressions, Examples 2 and 3, pages 16–18)

8. Perform the indicated operations and simplify:

a. $\dfrac{1 + \dfrac{1}{x + 2}}{x - \dfrac{9}{x}}$ **b.** $\dfrac{x(3x^2 + 1)}{x - 1} \cdot \dfrac{3x^3 - 5x^2 + x}{x(x - 1)(3x^2 + 1)^{1/2}}$

(Rational expressions, Examples 4 and 5, pages 18–19)

9. Rationalize the denominator: $\dfrac{3}{1 + 2\sqrt{x}}$

(Rationalizing algebraic fractions, Example 6, page 19)

10. Solve the inequalities:

a. $x^2 + x - 12 \le 0$

(Inequalities, Example 9, page 20)

b. $|3x - 4| \le 2$

(Absolute value, Example 14, page 22)

ANSWERS:

1. a. (i) $\frac{64}{27}$ **(ii)** $\frac{3}{5}$ **b.** $\frac{1}{x^6y^3}$ **2.** $\frac{x}{z\sqrt[3]{xy}}$

3. a. $5x^4 + 20x^3 + 12x^2 + 14x + 3$ **b.** $9x^3 - 18x^2 + 17x - 12$

4. a. $3a^2b^2(2a^2b^2c - ac - 3)$ **b.** $(2x - y)(3x + y)$

5. $\frac{2}{3}(1 - \sqrt{2}); \frac{2}{3}(1 + \sqrt{2})$ **6. a.** $\frac{x+2}{x+3}$ **b.** $\frac{4(t^2-4)}{(t^2+4)^2}$

7. a. 2 **b.** $\frac{3x(2x^3 + 2x + 1)}{(x^2 + 2)(x^3 + 1)}$

8. a. $\frac{x}{(x+2)(x-3)}$ **b.** $\frac{x\sqrt{1+3x^2}(3x^2 - 5x + 1)}{(x-1)^2}$

9. $\frac{3(1 - 2\sqrt{x})}{1 - 4x}$ **10. a.** $[-4, 3]$ **b.** $\left[\frac{2}{3}, 2\right]$

1.1 Precalculus Review I

Sections 1.1 and 1.2 review some basic concepts and techniques of algebra that are essential in the study of calculus. The material in this review will help you work through the examples and exercises in this book. You can read through this material now and do the exercises in areas where you feel a little "rusty," or you can review the material on an as-needed basis as you study the text. The self-diagnostic test that precedes this section will help you pinpoint the areas where you might have any weaknesses.

The Real Number Line

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division.

We can represent real numbers geometrically by points on a **real number,** or **coordinate, line.** This line can be constructed as follows. Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin.** If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number 1. This determines the scale for the number line. Each positive real number lies at an appropriate distance to the right of the origin, and each negative real number lies at an appropriate distance to the left of the origin (Figure 1).

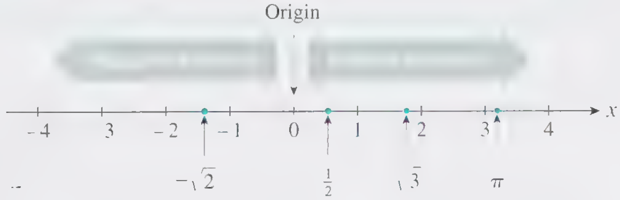


FIGURE 1
The real number line

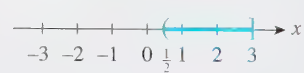
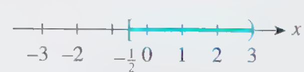
A *one-to-one correspondence* is set up between the set of all real numbers and the set of points on the number line; that is, exactly one point on the line is associated with each real number. Conversely, exactly one real number is associated with each point on the line. The real number that is associated with a point on the real number line is called the **coordinate** of that point.

Intervals

Throughout this book, we will often restrict our attention to subsets of the set of real numbers. For example, if x denotes the number of cars rolling off a plant assembly line each day, then x must be nonnegative—that is, $x \geq 0$. Further, suppose management decides that the daily production must not exceed 200 cars. Then, x must satisfy the inequality $0 \leq x \leq 200$.

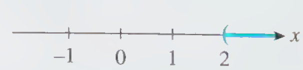
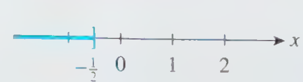
More generally, we will be interested in the following subsets of real numbers: open intervals, closed intervals, and half-open intervals. The set of all real numbers that lie *strictly* between two fixed numbers a and b is called an **open interval** (a, b). It consists of all real numbers x that satisfy the inequalities $a < x < b$, and it is called "open" because neither of its endpoints is included in the interval. A **closed interval** contains *both* of its endpoints. Thus, the set of all real numbers x that satisfy the inequalities $a \leq x \leq b$ is the closed interval $[a, b]$. Notice that square brackets are used to indicate that the endpoints are included in this interval. **Half-open intervals** contain only *one* of their endpoints. Thus, the interval $[a, b)$ is the set of all real numbers x that satisfy $a \leq x < b$, whereas the interval $(a, b]$ is described by the inequalities $a < x \leq b$. Examples of these **finite intervals** are illustrated in Table 1.

TABLE 1
Finite Intervals

Interval	Graph		Example
Open: (a, b)		$(-2, 1)$	
Closed: $[a, b]$		$[-1, 2]$	
Half-open: $(a, b]$		$(\frac{1}{2}, 3]$	
Half-open: $[a, b)$		$[-\frac{1}{2}, 3)$	

In addition to finite intervals, we will encounter **infinite intervals.** Examples of infinite intervals are the half-lines (a, ∞), $[a, \infty)$, $(-\infty, a)$, and $(-\infty, a]$ defined by the set of all real numbers that satisfy $x > a$, $x \geq a$, $x < a$, and $x \leq a$, respectively. The symbol ∞, called *infinity*, is not a real number. It is used here only for notational purposes. The notation $(-\infty, \infty)$ is used for the set of all real numbers x, since by definition, the inequalities $-\infty < x < \infty$ hold for any real number x. Infinite intervals are illustrated in Table 2.

TABLE 2
Infinite Intervals

Interval	Graph		Example
(a, ∞)		$(2, \infty)$	
$[a, \infty)$		$[-1, \infty)$	
$(-\infty, a)$		$(-\infty, 1)$	
$(-\infty, a]$		$(-\infty, -\frac{1}{2}]$	

Exponents and Radicals

Recall that if b is any real number and n is a positive integer, then the expression b^n (read "b to the power n") is defined as the number

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}$$

The number b is called the **base,** and the superscript n is called the **power** of the exponential expression b^n. For example,

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \qquad \text{and} \qquad \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$$

If $b \neq 0$, we define

$$b^0 = 1$$

For example, $2^0 = 1$ and $(-\pi)^0 = 1$, but the expression 0^0 is undefined.

Next, recall that if n is a positive integer, then the expression $b^{1/n}$ is defined to be the number that, when raised to the nth power, is equal to b. Thus,

$$(b^{1/n})^n = b$$

Such a number, if it exists, is called the ***n*th root of *b*,** also written $\sqrt[n]{b}$.

If n is even, the nth root of a negative number is not defined. For example, the square root of -2 ($n = 2$) is not defined because there is no real number b such that $b^2 = -2$. Also, given a number b, more than one number might satisfy our definition of the nth root. For example, both 3 and -3 squared equal 9, and each is a square root of 9. So to avoid ambiguity, we define $b^{1/n}$ to be the positive nth root of b whenever it exists. Thus, $\sqrt{9} = 9^{1/2} = 3$. That's why your calculator will give the answer 3 when you use it to evaluate $\sqrt{9}$.

Next, recall that if p/q (where p and q are positive integers and $q \neq 0$) is a rational number in lowest terms, then the expression $b^{p/q}$ is defined as the number $(b^{1/q})^p$ or, equivalently, $\sqrt[q]{b^p}$, whenever it exists. For example,

$$2^{3/2} = (2^{1/2})^3 \approx (1.4142)^3 \approx 2.8283$$

Expressions involving negative rational exponents are taken care of by the definition

$$b^{-p/q} = \frac{1}{b^{p/q}}$$

Thus,

$$4^{-5/2} = \frac{1}{4^{5/2}} = \frac{1}{(4^{1/2})^5} = \frac{1}{2^5} = \frac{1}{32}$$

The rules defining the exponential expression a^n, where $a > 0$, for all rational values of n are given in Table 3.

The first three definitions in Table 3 are also valid for negative values of a. The fourth definition holds for all values of a if n is odd but only for nonnegative values of a if n is even. Thus,

$$(-8)^{1/3} = \sqrt[3]{-8} = -2 \qquad n \text{ is odd.}$$

$$(-8)^{1/2} \text{ has no real value} \qquad n \text{ is even.}$$

Finally, it can be shown that a^n has meaning for *all* real numbers n. For example, using a calculator with a $\boxed{y^x}$ key, we see that $2^{\sqrt{2}} \approx 2.665144$.

TABLE 3

Rules for Defining a^n

Definition of a^n $(a > 0)$	Example	Definition of a^n $(a > 0)$	Example
Integer exponent: If n is a positive integer, then $a^n = a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors of a)	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ (5 factors) $= 32$	**Fractional exponent:** **a.** If n is a positive integer, then $a^{1/n}$ or $\sqrt[n]{a}$ denotes the nth root of a.	$16^{1/2} = \sqrt{16} = 4$
Zero exponent: If n is equal to zero, then $a^0 = 1$ (0^0 is not defined.)	$7^0 = 1$	**b.** If m and n are positive integers, then $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$8^{2/3} = (\sqrt[3]{8})^2 = 4$
Negative exponent: If n is a positive integer, then $a^{-n} = \dfrac{1}{a^n}$ $(a \neq 0)$	$6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$	**c.** If m and n are positive integers, then $a^{-m/n} = \dfrac{1}{a^{m/n}}$ $(a \neq 0)$	$9^{-3/2} = \dfrac{1}{9^{3/2}} = \dfrac{1}{27}$

The five laws of exponents are listed in Table 4.

TABLE 4

Laws of Exponents

Law	Example
1. $a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^3 = x^{2+3} = x^5$
2. $\dfrac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3. $(a^m)^n = a^{m \cdot n}$	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$
4. $(ab)^n = a^n \cdot b^n$	$(2x)^4 = 2^4 \cdot x^4 = 16x^4$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $(b \neq 0)$	$\left(\dfrac{x}{2}\right)^3 = \dfrac{x^3}{2^3} = \dfrac{x^3}{8}$

These laws are valid for any real numbers a, b, m, and n whenever the quantities are defined.

Remember, $(x^2)^3 \neq x^5$. The correct equation is $(x^2)^3 = x^{2 \cdot 3} = x^6$.

The next several examples illustrate the use of the laws of exponents.

VIDEO **EXAMPLE 1** Simplify the expressions:

a. $(3x^2)(4x^3)$ **b.** $\dfrac{16^{5/4}}{16^{1/2}}$ **c.** $(6^{2/3})^3$ **d.** $(x^3y^{-2})^{-2}$ **e.** $\left(\dfrac{y^{3/2}}{x^{1/4}}\right)^2$

Solution

a. $(3x^2)(4x^3) = 12x^{2+3} = 12x^5$ Law 1

b. $\dfrac{16^{5/4}}{16^{1/2}} = 16^{5/4-1/2} = 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$ Law 2

c. $(6^{2/3})^3 = 6^{(2/3)(3)} = 6^2 = 36$ Law 3

d. $(x^3y^{-2})^{-2} = (x^3)^{-2}(y^{-2})^{-2} = x^{(3)(-2)}y^{(-2)(-2)} = x^{-6}y^4 = \dfrac{y^4}{x^6}$ Law 4

e. $\left(\dfrac{y^{3/2}}{x^{1/4}}\right)^{-2} = \dfrac{y^{(3/2)(-2)}}{x^{(1/4)(-2)}} = \dfrac{y^{-3}}{x^{-1/2}} = \dfrac{x^{1/2}}{y^3}$ Law 5

We can also use the laws of exponents to simplify expressions involving radicals, as illustrated in the next example.

EXAMPLE 2 Simplify the expressions. (Assume that x, y, m, and n are positive.)

a. $\sqrt[4]{16x^4y^8}$ **b.** $\sqrt{12m^3n} \cdot \sqrt{3m^5n}$ **c.** $\dfrac{\sqrt[3]{-27x^6}}{\sqrt[3]{8y^3}}$

Solution

a. $\sqrt[4]{16x^4y^8} = (16x^4y^8)^{1/4} = 16^{1/4} \cdot x^{4/4}y^{8/4} = 2xy^2$

b. $\sqrt{12m^3n} \cdot \sqrt{3m^5n} = \sqrt{36m^8n^2} = (36m^8n^2)^{1/2} = 36^{1/2} \cdot m^4n = 6m^4n$

c. $\dfrac{\sqrt[3]{-27x^6}}{\sqrt[3]{8y^3}} = \dfrac{(-27x^6)^{1/3}}{(8y^3)^{1/3}} = \dfrac{-27^{1/3}x^2}{8^{1/3}y} = -\dfrac{3x^2}{2y}$

If a radical appears in the numerator or denominator of an algebraic expression, we often try to simplify the expression by eliminating the radical from the numerator or denominator. This process, called **rationalization,** is illustrated in the next two examples.

EXAMPLE 3 Rationalize the denominator of the expression $\dfrac{3x}{2\sqrt{x}}$.

Solution

$$\frac{3x}{2\sqrt{x}} = \frac{3x}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3x\sqrt{x}}{2\sqrt{x^2}} = \frac{3x\sqrt{x}}{2x} = \frac{3}{2}\sqrt{x}$$

EXAMPLE 4 Express $\dfrac{1}{2}x^{-1/2}$ as a radical and rationalize the denominator of the expression that you obtain.

Solution

$$\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

EXAMPLE 5 Rationalize the numerator of the expression $\dfrac{3\sqrt{x}}{2x}$.

Solution

$$\frac{3\sqrt{x}}{2x} = \frac{3\sqrt{x}}{2x} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3\sqrt{x^2}}{2x\sqrt{x}} = \frac{3x}{2x\sqrt{x}} = \frac{3}{2\sqrt{x}}$$

Operations with Algebraic Expressions

In calculus, we often work with algebraic expressions such as

$$2x^{4/3} - x^{1/3} + 1 \qquad 2x^2 - x - \frac{2}{\sqrt{x}} \qquad \frac{3xy + 2}{x + 1} \qquad 2x^3 + 2x + 1$$

An algebraic expression of the form $ax^m y^n$, where the coefficient a is a real number and m and n are nonnegative integers, is called a **monomial,** meaning it consists of one term. For example, $7x^2$ is a monomial. A **polynomial** is a monomial or the sum of two or more monomials. For example,

$$x^2 + 4x + 4 \qquad x^3 + 5 \qquad x^4 + 3x^2 + 3 \qquad x^2y + xy + y$$

are all polynomials. The degree of a polynomial is the highest power $(m + n)$ of the variables that appears in the polynomial.

Constant terms and terms containing the same variable factor are called **like,** or **similar, terms.** Like terms may be combined by adding or subtracting their numerical coefficients. For example,

$$3x + 7x = 10x \quad \text{and} \quad \frac{1}{2}xy + 3xy = \frac{7}{2}xy$$

The distributive property of the real number system,

$$ab + ac = a(b + c)$$

is used to justify this procedure.

To add or subtract two or more algebraic expressions, first remove the parentheses and then combine like terms. The resulting expression is written in order of nonincreasing degree from left to right.

EXAMPLE 6

a. $(2x^4 + 3x^3 + 4x + 6) - (3x^4 + 9x^3 + 3x^2)$

$= 2x^4 + 3x^3 + 4x + 6 - 3x^4 - 9x^3 - 3x^2$ Remove parentheses.

$= 2x^4 - 3x^4 + 3x^3 - 9x^3 - 3x^2 + 4x + 6$

$= -x^4 - 6x^3 - 3x^2 + 4x + 6$ Combine like terms.

b. $2t^3 - \{t^2 - [t - (2t - 1)] + 4\}$

$= 2t^3 - \{t^2 - [t - 2t + 1] + 4\}$

$= 2t^3 - \{t^2 - [-t + 1] + 4\}$ Remove parentheses and combine like terms within brackets.

$= 2t^3 - \{t^2 + t - 1 + 4\}$ Remove brackets.

$= 2t^3 - \{t^2 + t + 3\}$ Combine like terms within braces.

$= 2t^3 - t^2 - t - 3$ Remove braces.

Observe that when the algebraic expression in Example 6b was simplified, the innermost grouping symbols were removed first; that is, the parentheses ( ) were removed first, the brackets [ ] second, and the braces { } third.

When algebraic expressions are multiplied, each term of one algebraic expression is multiplied by each term of the other. The resulting algebraic expression is then simplified.

EXAMPLE 7 Perform the indicated operations:

a. $(x^2 + 1)(3x^2 + 10x + 3)$ **b.** $x\left(300 - \frac{1}{4}x - \frac{1}{8}y\right) + y\left(240 - \frac{1}{8}x - \frac{3}{8}y\right)$

c. $(e^t + e^{-t})e^t - e^t(e^t - e^{-t})$

Solution

a. $$(x^2+1)(3x^2+10x+3) = x^2(3x^2+10x+3) + 1(3x^2+10x+3)$$
$$= 3x^4 + 10x^3 + 3x^2 + 3x^2 + 10x + 3$$
$$= 3x^4 + 10x^3 + 6x^2 + 10x + 3$$

b. $$x\left(300 - \frac{1}{4}x - \frac{1}{8}y\right) + y\left(240 - \frac{1}{8}x - \frac{3}{8}y\right)$$
$$= 300x - \frac{1}{4}x^2 - \frac{1}{8}xy + 240y - \frac{1}{8}xy - \frac{3}{8}y^2$$
$$= -\frac{1}{4}x^2 - \frac{3}{8}y^2 - \frac{1}{4}xy + 300x + 240y$$

c. $$(e^t + e^{-t})e^t - e^t(e^t - e^{-t}) = e^{2t} + e^0 - e^{2t} + e^0$$
$$= e^{2t} - e^{2t} + e^0 + e^0$$
$$= 1 + 1 \qquad \text{Recall that } e^0 = 1.$$
$$= 2$$

Certain product formulas that are frequently used in algebraic computations are given in Table 5.

TABLE 5
Some Useful Product Formulas

Formula	Example
$(a+b)^2 = a^2 + 2ab + b^2$	$(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$ $= 4x^2 + 12xy + 9y^2$
$(a-b)^2 = a^2 - 2ab + b^2$	$(4x-2y)^2 = (4x)^2 - 2(4x)(2y) + (2y)^2$ $= 16x^2 - 16xy + 4y^2$
$(a+b)(a-b) = a^2 - b^2$	$(2x+y)(2x-y) = (2x)^2 - (y)^2$ $= 4x^2 - y^2$

Factoring

Factoring is the process of expressing an algebraic expression as a product of other algebraic expressions. For example, by applying the distributive property, we may write

$$3x^2 - x = x(3x - 1)$$

To factor an algebraic expression, first check to see whether any of its terms have common factors. If they do, then factor out the greatest common factor. For example, the common factor of the algebraic expression $2a^2x + 4ax + 6a$ is $2a$ because

$$2a^2x + 4ax + 6a = 2a \cdot ax + 2a \cdot 2x + 2a \cdot 3 = 2a(ax + 2x + 3)$$

EXAMPLE 8 Factor out the greatest common factor in each expression:

a. $-3t^2 + 3t$ **b.** $2x^{3/2} - 3x^{1/2}$ **c.** $2ye^{xy^2} + 2xy^3e^{xy^2}$

d. $4x(x+1)^{1/2} - 2x^2\left(\frac{1}{2}\right)(x+1)^{-1/2}$

Solution

a. $-3t^2 + 3t = -3t(t - 1)$
b. $2x^{3/2} - 3x^{1/2} = x^{1/2}(2x - 3)$
c. $2ye^{xy^2} + 2xy^3e^{xy^2} = 2ye^{xy^2}(1 + xy^2)$

d. $4x(x+1)^{1/2} - 2x^2\left(\frac{1}{2}\right)(x+1)^{-1/2} = 4x(x+1)^{1/2} - x^2(x+1)^{-1/2}$

$$= x(x+1)^{-1/2}[4(x+1)^{1/2}(x+1)^{1/2} - x]$$
$$= x(x+1)^{-1/2}[4(x+1) - x]$$
$$= x(x+1)^{-1/2}(4x+4-x) = x(x+1)^{-1/2}(3x+4)$$

Here we select $(x+1)^{-1/2}$ as the greatest common factor because it is the highest power of $(x+1)$ in each algebraic term. In particular, observe that

$$(x+1)^{-1/2}(x+1)^{1/2}(x+1)^{1/2} = (x+1)^{-1/2+1/2+1/2} = (x+1)^{1/2}$$

Sometimes an algebraic expression may be factored by regrouping and rearranging its terms so that a common term can be factored out. This technique is illustrated in Example 9.

EXAMPLE 9 Factor:

a. $2ax + 2ay + bx + by$ **b.** $3x\sqrt{y} - 4 - 2\sqrt{y} + 6x$

Solution

a. First, factor the common term $2a$ from the first two terms and the common term b from the last two terms. Thus,

$$2ax + 2ay + bx + by = 2a(x+y) + b(x+y)$$

Since $(x+y)$ is common to both terms of the polynomial, we may factor it out. Hence

$$2a(x+y) + b(x+y) = (2a+b)(x+y)$$

b. $3x\sqrt{y} - 4 - 2\sqrt{y} + 6x = 3x\sqrt{y} - 2\sqrt{y} + 6x - 4$ Rearrange terms.

$$= \sqrt{y}(3x-2) + 2(3x-2) \quad \text{Factor out common factors.}$$
$$= (3x-2)(\sqrt{y}+2)$$

As we have seen, the first step in factoring a polynomial is to find the common factors. The next step is to express the polynomial as the product of a constant and/or one or more prime polynomials.

Certain product formulas that are useful in factoring binomials and trinomials are listed in Table 6.

TABLE 6

Product Formulas Used in Factoring

Formula	Example
Difference of two squares:	
$x^2 - y^2 = (x+y)(x-y)$	$x^2 - 36 = (x+6)(x-6)$
	$8x^2 - 2y^2 = 2(4x^2 - y^2)$
	$= 2(2x+y)(2x-y)$
	$9 - a^6 = (3+a^3)(3-a^3)$
Perfect-square trinomial:	
$x^2 + 2xy + y^2 = (x+y)^2$	$x^2 + 8x + 16 = (x+4)^2$
$x^2 - 2xy + y^2 = (x-y)^2$	$4x^2 - 4xy + y^2 = (2x-y)^2$
Sum of two cubes:	
$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$	$z^3 + 27 = z^3 + (3)^3$
	$= (z+3)(z^2 - 3z + 9)$
Difference of two cubes:	
$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$	$8x^3 - y^6 = (2x)^3 - (y^2)^3$
	$= (2x - y^2)(4x^2 + 2xy^2 + y^4)$

The factors of the second-degree polynomial with integral coefficients

$$px^2 + qx + r$$

are $(ax + b)(cx + d)$, where $ac = p$, $ad + bc = q$, and $bd = r$. Since only a limited number of choices are possible, we can use a trial-and-error method to factor polynomials having this form.

For example, to factor $x^2 - 2x - 3$, we first observe that the only possible first-degree terms are

$$(x \quad)(x \quad) \qquad \text{Since the coefficient of } x^2 \text{ is 1}$$

Next, we observe that the product of the constant terms is (-3). This gives us the following possible factors:

$$(x - 1)(x + 3)$$
$$(x + 1)(x - 3)$$

Looking once again at the polynomial $x^2 - 2x - 3$, we see that the coefficient of x is -2. Checking to see which set of factors yields -2 for the coefficient of x, we find that

Coefficients of inner terms

Coefficients of outer terms

$$(-1)(1) + (1)(3) = 2$$

Factors

Outer terms

$$(x - 1)(x + 3)$$

Inner terms

Coefficients of inner terms

Coefficients of outer terms

$$(1)(1) + (1)(-3) = -2$$

Outer terms

$$(x + 1)(x - 3)$$

Inner terms

and we conclude that the correct factorization is

$$x^2 - 2x - 3 = (x + 1)(x - 3)$$

With practice, you will soon find that you can perform many of these steps mentally and the need to write out each step will be eliminated.

EXAMPLE 10 Factor:

a. $3x^2 + 4x - 4$ **b.** $3x^2 - 6x - 24$ **c.** $-3t^2 + 192t + 195$

Solution

a. Using trial and error, we find that the correct factorization is

$$3x^2 + 4x - 4 = (3x - 2)(x + 2)$$

b. Since each term has the common factor 3, we have

$$3x^2 - 6x - 24 = 3(x^2 - 2x - 8)$$

Using the trial-and-error method of factorization, we find that

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

Thus, we have

$$3x^2 - 6x - 24 = 3(x - 4)(x + 2)$$

c. Since each term has the common factor -3, we have

$$-3t^2 + 192t + 195 = -3(t^2 - 64t - 65)$$

Using the trial-and-error method of factorization, we find that

$$(t^2 - 64t - 65) = (t - 65)(t + 1)$$

Therefore,

$$-3t^2 + 192t + 195 = -3(t - 65)(t + 1)$$

Roots of Polynomial Equations

A polynomial equation of degree n in the variable x is an equation of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$$

where n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$. For example, the equation

$$-2x^5 + 8x^3 - 6x^2 + 3x + 1 = 0$$

is a polynomial equation of degree 5 in x.

The **roots of a polynomial equation** are precisely the values of x that satisfy the given equation.* One way to find the roots of a polynomial equation is to factor the polynomial and then solve the resulting equation. For example, the polynomial equation

$$x^3 - 3x^2 + 2x = 0$$

may be rewritten in the form

$$x(x^2 - 3x + 2) = 0 \quad \text{or} \quad x(x-1)(x-2) = 0$$

Since the product of two real numbers can be equal to zero if and only if one (or both) of the factors is equal to zero, we have

$$x = 0 \qquad x - 1 = 0 \qquad \text{or} \qquad x - 2 = 0$$

from which we see that the desired roots are $x = 0$, 1, and 2.

The Quadratic Formula

In general, the problem of finding the roots of a polynomial equation is a difficult one. But the roots of a quadratic equation (a polynomial equation of degree 2) are easily found either by factoring or by using the following quadratic formula.

Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note If you use the quadratic formula to solve a quadratic equation, first make sure that the equation is in the *standard form* $ax^2 + bx + c = 0$.

EXAMPLE 11 Solve each of the following quadratic equations:

a. $2x^2 + 5x - 12 = 0$ **b.** $x^2 = -3x + 8$

Solution

a. The equation is in standard form, with $a = 2$, $b = 5$, and $c = -12$. Using the quadratic formula, we find

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)} \\ &= \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} \\ &= -4 \quad \text{or} \quad \frac{3}{2} \end{aligned}$$

*In this book, we are interested only in the *real* roots of an equation.

This equation can also be solved by factoring. Thus,

$$2x^2 + 5x - 12 = (2x - 3)(x + 4) = 0$$

from which we see that the desired roots are $x = \frac{3}{2}$ or $x = -4$, as obtained earlier.

b. We first rewrite the given equation in the standard form $x^2 + 3x - 8 = 0$, from which we see that $a = 1$, $b = 3$, and $c = -8$. Using the quadratic formula, we find

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(1)(-8)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{41}}{2}$$

That is, the solutions are

$$\frac{-3 + \sqrt{41}}{2} \approx 1.7 \quad \text{and} \quad \frac{-3 - \sqrt{41}}{2} \approx -4.7$$

In this case, the quadratic formula proves quite handy!

1.1 Exercises

In Exercises 1–6, show the interval on a number line.

1. $(3, 6)$ **2.** $(-2, 5]$ **3.** $[-1, 4)$

4. $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ **5.** $(0, \infty)$ **6.** $(-\infty, 5]$

In Exercises 7–22, evaluate the expression.

7. $27^{2/3}$ **8.** $8^{-4/3}$

9. $\left(\frac{1}{\sqrt{5}}\right)^0$ **10.** $(7^{1/2})^6$

11. $\left[\left(\frac{1}{8}\right)^{1/3}\right]^{-2}$ **12.** $\left[\left(-\frac{1}{3}\right)^2\right]^{-3}$

13. $\left(\frac{8^{-5} \cdot 8^2}{8^{-2}}\right)^{-1}$ **14.** $\left(\frac{9}{16}\right)^{-1/2}$

15. $(125^{2/3})^{-1/2}$ **16.** $\sqrt[3]{2^6}$

17. $\frac{\sqrt{72}}{\sqrt{18}}$ **18.** $\sqrt[3]{\frac{-8}{27}}$

19. $\frac{16^{5/8}16^{1/2}}{16^{7/8}}$ **20.** $\left(\frac{9^{-3.5} \cdot 9^{2.5}}{9^{-2}}\right)^{-0.5}$

21. $16^{1/4} \cdot 8^{-1/3}$ **22.** $\frac{6^{2.5} \cdot 6^{-1.9}}{6^{-1.4}}$

In Exercises 23–32, determine whether the statement is true or false. Give a reason for your choice.

23. $x^4 + 2x^4 = 3x^4$ **24.** $3^2 \cdot 2^2 = 6^2$

25. $x^3 \cdot 2x^2 = 2x^6$ **26.** $3^3 + 3^2 = 3^5$

27. $\frac{2^{4x}}{1^{3x}} = 2^{4x-3x}$ **28.** $(2^2 \cdot 3^2)^2 = 6^4$

29. $\frac{1}{4^{-3}} = \frac{1}{64}$ **30.** $\frac{4^{3/2}}{2^4} = \frac{1}{2}$

31. $(1.2^{1/2})^{-1/2} = 1$ **32.** $5^{2/3} \cdot (25)^{2/3} = 25$

In Exercises 33–38, rewrite the expression using positive exponents only.

33. $(xy)^{-2}$ **34.** $3s^{1/3} \cdot 2s^{-7/3}$

35. $\frac{3x^{-1/3}}{x^{1/2}}$ **36.** $\sqrt{4x^{-1}} \cdot \sqrt{9x^{-3}}$

37. $12^0(s + t)^{-3}$ **38.** $(x - y)(x^{-1} + y^{-1})$

In Exercises 39–54, simplify the expression. (Assume that x, y, r, s, and t are positive.)

39. $\frac{x^{7/3}}{x^{-2}}$ **40.** $(49x^{-2})^{-1/2}$

41. $(x^2y^{-3})(x^{-5}y^3)$ **42.** $\frac{5x^{5/2}y^{3/2}}{2x^{3/2}y^{7/4}}$

43. $\frac{x^{3/4}}{x^{-1/4}}$ **44.** $\left(\frac{x^3y^2}{z^2}\right)^2$

45. $\left(\frac{x^3}{-27y^{-6}}\right)^{-2/3}$ **46.** $\left(16\frac{e^x}{e^{x-2}}\right)^{-1/2}$

47. $\left(\frac{x^{-3}}{y^{-2}}\right)^2\left(\frac{y}{x}\right)^4$ **48.** $\frac{8^{2/3}(r^n)^4}{2^4r^{5-2n}}$

49. $\sqrt[3]{x^{-2}} \cdot \sqrt{4x^5}$ **50.** $\sqrt{121x^6y^{-4}}$

51. $-\sqrt[4]{16x^4y^8}$ **52.** $\sqrt[3]{x^{3a+b}}$

53. $\sqrt[6]{64x^8y^3}$ **54.** $\sqrt[3]{27r^6} \cdot \sqrt{9s^2t^4}$

In Exercises 55–58, use the fact that $2^{1/2} \approx 1.414$ and $3^{1/2} \approx 1.732$ to evaluate the expression without using a calculator.

55. $2^{3/2}$ **56.** $8^{1/2}$ **57.** $9^{3/4}$ **58.** $6^{1/2}$

In Exercises 59–62, use the fact that $10^{1/2} \approx 3.162$ and $10^{1/3} \approx 2.154$ to evaluate the expression without using a calculator.

59. $10^{3/2}$ **60.** $1000^{3/2}$

61. $10^{2.5}$ **62.** $(0.0001)^{-1/3}$

In Exercises 63–68, rationalize the denominator.

63. $\dfrac{3}{2\sqrt{x}}$ **64.** $\dfrac{5}{2\sqrt{xy}}$ **65.** $\dfrac{2y}{\sqrt{3y}}$

66. $\dfrac{5x^2}{\sqrt{5x}}$ **67.** $\dfrac{1}{\sqrt[3]{x}}$ **68.** $\sqrt{\dfrac{2x}{y}}$

In Exercises 69–74, rationalize the numerator.

69. $\dfrac{2\sqrt{x}}{3}$ **70.** $\dfrac{\sqrt[3]{8x}}{24}$ **71.** $\sqrt{\dfrac{2y}{3x}}$

72. $\sqrt[3]{\dfrac{2x}{3y}}$ **73.** $\dfrac{\sqrt[3]{x^2z}}{y}$ **74.** $\dfrac{\sqrt[3]{x^2y}}{2x}$

In Exercises 75–98, perform the indicated operations and/or simplify each expression.

75. $(7x^2 - 3x + 5) + (2x^2 + 5x - 4)$

76. $(3x^2 + 5xy + 2y) + (4 - 3xy - 2x^2)$

77. $(5y^2 - 2y + 1) - (y^2 - 3y - 7)$

78. $3(2a - b) - 4(b - 2a)$

79. $x - \{2x - [-x - (1 - x)]\}$

80. $3x^2 - \{x^2 + 1 - x[x - (2x - 1)]\} + 2$

81. $\left(\dfrac{1}{3} - 1 + e\right) - \left(-\dfrac{1}{3} - 1 + e^{-1}\right)$

82. $-\dfrac{3}{4}y - \dfrac{1}{4}x + 100 + \dfrac{1}{2}x + \dfrac{1}{4}y - 120$

83. $3\sqrt{8} + 8 - 2\sqrt{y} + \dfrac{1}{2}\sqrt{x} - \dfrac{3}{4}\sqrt{y}$

84. $\dfrac{8}{9}x^2 + \dfrac{2}{3}x + \dfrac{16}{3}x^2 - \dfrac{16}{3}x - 2x + 2$

85. $(3x + 8)(x - 2)$ **86.** $(5x + 2)(3x - 4)$

87. $(a + 5)^2$ **88.** $(3a - 4b)^2$

89. $(x + 2y)^2$ **90.** $(6y - 3x)^2$

91. $(2x + y)(2x - y)$ **92.** $(3x + 2)(2 - 3x)$

93. $(2x^2 - 1)(3x^2) + (x^2 + 3)(4x)$

94. $(x^2 - 1)(2x) - x^2(2x)$

95. $6x\left(\dfrac{1}{2}\right)(2x^2 + 3)^{-1/2}(4x) + 6(2x^2 + 3)^{1/2}$

96. $(x^{1/2} + 1)\left(\dfrac{1}{2}x^{-1/2}\right) - (x^{1/2} - 1)\left(\dfrac{1}{2}x^{-1/2}\right)$

97. $100(-10te^{-0.1t} - 100e^{-0.1t})$

98. $2(t + \sqrt{t})^2 - 2t^2$

In Exercises 99–106, factor out the greatest common factor from each expression.

99. $4x^5 - 12x^4 - 6x^3$

100. $4x^2y^2z - 2x^5y^2 + 6x^3y^2z^2$

101. $7a^4 - 42a^2b^2 + 49a^3b$

102. $3x^{2/3} - 2x^{1/3}$

103. $3e^{-x} - 9xe^{-x}$

104. $2ye^{xy^2} + 2xy^3e^{xy^2}$

105. $2x^{-5/2}y^2 - \dfrac{3}{2}x^{-3/2}y^4$

106. $\dfrac{1}{2}\left(\dfrac{2}{3}u^{5/2} - 2u^{3/2}\right)$

In Exercises 107–120, factor each expression completely.

107. $6ac + 3bc - 4ad - 2bd$

108. $3x^3 - x^2 + 3x - 1$

109. $4a^2 - 9b^2$ **110.** $12x^2 - 3y^2$

111. $10 - 14x - 12x^2$ **112.** $6x^2 - 7x - 20$

113. $3x^2 - 6x - 24$ **114.** $15x^2 - 4x - 4$

115. $12x^2 - 2x - 30$ **116.** $(x + y)^2 - 1$

117. $9x^2 - 16y^2$ **118.** $8a^2 - 2ab - 6b^2$

119. $8x^6 + 125$ **120.** $27x^3 - 64$

In Exercises 121–128, perform the indicated operations and simplify each expression.

121. $(x^2 + y^2)x - xy(2y)$

122. $2kr(R - r) - kr^2$

123. $2(x - 1)(2x + 2)^3[4(x - 1) + (2x + 2)]$

124. $5x^2(3x^2 + 1)^4(6x) + (3x^2 + 1)^5(2x)$

125. $4(x - 1)^2(2x + 2)^3(2) + (2x + 2)^4(2)(x - 1)$

126. $(x^2 + 1)(4x^3 - 3x^2 + 2x) - (x^4 - x^3 + x^2)(2x)$

127. $(x^2 + 2)^2[5(x^2 + 2)^2 - 3](2x)$

128. $(x^2 - 4)(x^2 + 4)(2x + 8) - (x^2 + 8x - 4)(4x^3)$

In Exercises 129–134, find the real roots of each equation by factoring.

129. $x^2 + x - 12 = 0$

130. $3x^2 - x - 4 = 0$

131. $4t^2 + 2t - 2 = 0$

132. $-6x^2 + x + 12 = 0$

133. $\frac{1}{4}x^2 - x + 1 = 0$

134. $\frac{1}{2}a^2 + a - 12 = 0$

In Exercises 135–140, solve the equation by using the quadratic formula.

135. $4x^2 + 5x - 6 = 0$

136. $3x^2 - 4x + 1 = 0$

137. $8x^2 - 8x - 3 = 0$

138. $x^2 - 6x + 6 = 0$

139. $2x^2 + 4x - 3 = 0$

140. $2x^2 + 7x - 15 = 0$

141. Distribution of Incomes The distribution of income in a certain city can be described by the mathematical model $y = (5.6 \cdot 10^{11})(x)^{-1.5}$, where y is the number of families with an income of x or more dollars.

a. How many families in this city have an income of \$30,000 or more?

b. How many families have an income of \$60,000 or more?

c. How many families have an income of \$150,000 or more?

In Exercises 142–144, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

142. If $b^2 - 4ac > 0$, then $ax^2 + bx + c = 0$ $(a \neq 0)$ has two real roots.

143. If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ $(a \neq 0)$ has no real roots.

144. $\sqrt{(a + b)(b - a)} = \sqrt{b^2 - a^2}$ for all real numbers a and b.

1.2 Precalculus Review II

Rational Expressions

Quotients of polynomials are called **rational expressions.** Examples of rational expressions are

$$\frac{6x - 1}{2x + 3} \qquad \frac{3x^2y^3 - 2xy}{4x} \qquad \frac{2}{5ab}$$

Since rational expressions are quotients in which the variables represent real numbers, the properties of real numbers apply to rational expressions as well, and operations with rational fractions are performed in the same manner as operations with arithmetic fractions. For example, using the properties of the real number system, we may write

$$\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

where a, b, and c are any real numbers and b and c are not zero.

Similarly, using the same properties of real numbers, we may write

$$\frac{(x + 2)(x - 3)}{(x - 2)(x - 3)} = \frac{x + 2}{x - 2} \qquad (x \neq 2, 3)$$

after "canceling" the common factors.

An example of incorrect cancellation is

$$\frac{\cancel{3} + 4x}{\cancel{3}} \neq 1 + 4x$$

because 3 is not a factor of the numerator. Instead, we need to write

$$\frac{3 + 4x}{3} = \frac{3}{3} + \frac{4x}{3} = 1 + \frac{4x}{3}$$

An algebraic fraction is simplified, or in lowest terms, when the numerator and denominator have no common factors other than 1 and -1 and the fraction contains no negative exponents.

EXAMPLE 1 Simplify the following expressions:

a. $\dfrac{x^2 + 2x - 3}{x^2 + 4x + 3}$ **b.** $\dfrac{(x^2 + 1)^2(-2) + (2x)(2)(x^2 + 1)(2x)}{(x^2 + 1)^4}$

Solution

a. $$\frac{x^2 + 2x - 3}{x^2 + 4x + 3} = \frac{(x + 3)(x - 1)}{(x + 3)(x + 1)} = \frac{x - 1}{x + 1}$$

b. $$\frac{(x^2 + 1)^2(-2) + (2x)(2)(x^2 + 1)(2x)}{(x^2 + 1)^4}$$

$$= \frac{(x^2 + 1)[(x^2 + 1)(-2) + (2x)(2)(2x)]}{(x^2 + 1)^4}$$ Factor out $(x^2 + 1)$.

$$= \frac{(x^2 + 1)(-2x^2 - 2 + 8x^2)}{(x^2 + 1)^4}$$ Carry out indicated multiplication.

$$= \frac{(x^2 + 1)(6x^2 - 2)}{(x^2 + 1)^4}$$ Combine like terms.

$$= \frac{(6x^2 - 2)}{(x^2 + 1)^3}$$ Cancel the common factors.

$$= \frac{2(3x^2 - 1)}{(x^2 + 1)^3}$$ Factor out 2 from the numerator.

The operations of multiplication and division are performed with algebraic fractions in the same manner as with arithmetic fractions (Table 7).

TABLE 7

Rules of Multiplication and Division: Algebraic Fractions

Operation	Example
If P, Q, R, and S are polynomials, then	
Multiplication:	
$\dfrac{P}{Q} \cdot \dfrac{R}{S} = \dfrac{PR}{QS}$ $\quad (Q, S \neq 0)$	$\dfrac{2x}{y} \cdot \dfrac{(x + 1)}{(y - 1)} = \dfrac{2x(x + 1)}{y(y - 1)} = \dfrac{2x^2 + 2x}{y^2 - y}$
Division:	
$\dfrac{P}{Q} \div \dfrac{R}{S} = \dfrac{P}{Q} \cdot \dfrac{S}{R} = \dfrac{PS}{QR}$ $\quad (Q, R, S \neq 0)$	$\dfrac{x^2 + 3}{y} \div \dfrac{y^2 + 1}{x} = \dfrac{x^2 + 3}{y} \cdot \dfrac{x}{y^2 + 1} = \dfrac{x^3 + 3x}{y^3 + y}$

When rational expressions are multiplied and divided, the resulting expressions should be simplified if possible.

VIDEO **EXAMPLE 2** Perform the indicated operations and simplify:

$$\frac{2x - 8}{x + 2} \cdot \frac{x^2 + 4x + 4}{x^2 - 16}$$

Solution

$$\begin{aligned}\frac{2x-8}{x+2}\cdot\frac{x^2+4x+4}{x^2-16} &= \frac{2(x-4)}{x+2}\cdot\frac{(x+2)^2}{(x+4)(x-4)}\\ &= \frac{2(x-4)(x+2)(x+2)}{(x+2)(x+4)(x-4)}\\ &= \frac{2(x+2)}{x+4} && \text{Cancel the common factors } (x+2)(x-4).\end{aligned}$$

For rational expressions, the operations of addition and subtraction are performed by finding a common denominator of the fractions and then adding or subtracting the numerators. Table 8 shows the rules for fractions with equal denominators.

TABLE 8

Rules of Addition and Subtraction: Fractions with Equal Denominators

Operation	Example
If P, Q, and R are polynomials, then	
Addition:	
$\dfrac{P}{R}+\dfrac{Q}{R}=\dfrac{P+Q}{R}\quad (R\neq 0)$	$\dfrac{2x}{x+2}+\dfrac{6x}{x+2}=\dfrac{2x+6x}{x+2}=\dfrac{8x}{x+2}$
Subtraction:	
$\dfrac{P}{R}-\dfrac{Q}{R}=\dfrac{P-Q}{R}\quad (R\neq 0)$	$\dfrac{3y}{y-x}-\dfrac{y}{y-x}=\dfrac{3y-y}{y-x}=\dfrac{2y}{y-x}$

$$\frac{x}{2+y}\neq\frac{x}{2}+\frac{x}{y}$$

To add or subtract fractions that have different denominators, first find a common denominator, preferably the least common denominator (LCD). Then carry out the indicated operations following the procedure described in Table 8.

To find the LCD of two or more rational expressions:

1. *Find the prime factors of each denominator.*
2. *Form the product of the different prime factors that occur in the denominators.* Each prime factor in this product should be raised to the highest power of that factor appearing in the denominators.

EXAMPLE 3 Simplify:

a. $\dfrac{2x}{x^2+1}+\dfrac{6(3x^2)}{x^3+2}$ **b.** $\dfrac{1}{x+h}-\dfrac{1}{x}$

Solution

a.
$$\begin{aligned}\frac{2x}{x^2+1}+\frac{6(3x^2)}{x^3+2} &= \frac{2x(x^3+2)+6(3x^2)(x^2+1)}{(x^2+1)(x^3+2)} && \text{LCD} = (x^2+1)(x^3+2)\\ &= \frac{2x^4+4x+18x^4+18x^2}{(x^2+1)(x^3+2)} && \text{Carry out the indicated multiplication.}\\ &= \frac{20x^4+18x^2+4x}{(x^2+1)(x^3+2)} && \text{Combine like terms.}\\ &= \frac{2x(10x^3+9x+2)}{(x^2+1)(x^3+2)} && \text{Factor.}\end{aligned}$$

b. $$\frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)} \qquad \text{LCD} = x(x+h)$$

$$= \frac{x-x-h}{x(x+h)} \qquad \text{Remove parentheses.}$$

$$= \frac{-h}{x(x+h)} \qquad \text{Combine like terms.}$$

Other Algebraic Fractions

The techniques used to simplify rational expressions may also be used to simplify algebraic fractions in which the numerator and denominator are not polynomials, as illustrated in Example 4.

EXAMPLE 4 Simplify:

a. $$\frac{1+\dfrac{1}{x+1}}{x-\dfrac{4}{x}}$$ **b.** $$\frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$$

Solution

a. $$\frac{1+\dfrac{1}{x+1}}{x-\dfrac{4}{x}} = \frac{\dfrac{x+1+1}{x+1}}{\dfrac{x^2-4}{x}} \qquad \text{LCD for numerator is } x+1 \text{ and LCD for denominator is } x.$$

$$= \frac{x+2}{x+1}\cdot\frac{x}{x^2-4} = \frac{x+2}{x+1}\cdot\frac{x}{(x+2)(x-2)}$$

$$= \frac{x}{(x+1)(x-2)}$$

b. $$\frac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}} = \frac{\dfrac{1}{x}+\dfrac{1}{y}}{\dfrac{1}{x^2}-\dfrac{1}{y^2}} = \frac{\dfrac{y+x}{xy}}{\dfrac{y^2-x^2}{x^2y^2}} \qquad x^{-n} = \frac{1}{x^n}$$

$$= \frac{y+x}{xy}\cdot\frac{x^2y^2}{y^2-x^2} = \frac{y+x}{xy}\cdot\frac{x^2y^2}{(y+x)(y-x)}$$

$$= \frac{xy}{y-x}$$

EXAMPLE 5 Perform the given operations and simplify:

a. $$\frac{x^2(2x^2+1)^{1/2}}{x-1}\cdot\frac{4x^3-6x^2+x-2}{x(x-1)(2x^2+1)}$$ **b.** $$\frac{12x^2}{\sqrt{2x^2+3}} + 6\sqrt{2x^2+3}$$

Solution

a. $$\frac{x^2(2x^2+1)^{1/2}}{x-1}\cdot\frac{4x^3-6x^2+x-2}{x(x-1)(2x^2+1)} = \frac{x(4x^3-6x^2+x-2)}{(x-1)^2(2x^2+1)^{1-1/2}}$$

$$= \frac{x(4x^3-6x^2+x-2)}{(x-1)^2(2x^2+1)^{1/2}}$$

b. $$\frac{12x^2}{\sqrt{2x^2+3}} + 6\sqrt{2x^2+3} = \frac{12x^2}{(2x^2+3)^{1/2}} + 6(2x^2+3)^{1/2}$$

Write radicals in exponential form

$$= \frac{12x^2 + 6(2x^2+3)^{1/2}(2x^2+3)^{1/2}}{(2x^2+3)^{1/2}}$$

LCD is $(2x^2+3)^{1/2}$

$$= \frac{12x^2 + 6(2x^2+3)}{(2x^2+3)^{1/2}}$$

$$= \frac{24x^2+18}{(2x^2+3)^{1/2}} = \frac{6(4x^2+3)}{\sqrt{2x^2+3}}$$

Rationalizing Algebraic Fractions

When the denominator of an algebraic fraction contains sums or differences involving radicals, we may **rationalize the denominator**—that is, transform the fraction into an equivalent one with a denominator that does not contain radicals. In doing so, we make use of the fact that

$$\begin{aligned}(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) &= (\sqrt{a})^2 - (\sqrt{b})^2 \\ &= a - b\end{aligned}$$

This procedure is illustrated in Example 6.

EXAMPLE 6 Rationalize the denominator: $\dfrac{1}{1+\sqrt{x}}$.

Solution Upon multiplying the numerator and the denominator by $(1-\sqrt{x})$, we obtain

$$\begin{aligned}\frac{1}{1+\sqrt{x}} &= \frac{1}{1+\sqrt{x}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}} \\ &= \frac{1-\sqrt{x}}{1-(\sqrt{x})^2} \\ &= \frac{1-\sqrt{x}}{1-x}\end{aligned}$$

$\dfrac{1-\sqrt{x}}{1-\sqrt{x}} = 1$

In other situations, it may be necessary to rationalize the numerator of an algebraic expression. In calculus, for example, one encounters the following problem.

VIDEO **EXAMPLE 7** Rationalize the numerator: $\dfrac{\sqrt{1+h}-1}{h}$.

Solution

$$\begin{aligned}\frac{\sqrt{1+h}-1}{h} &= \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} \\ &= \frac{(\sqrt{1+h})^2 - (1)^2}{h(\sqrt{1+h}+1)} \\ &= \frac{1+h-1}{h(\sqrt{1+h}+1)} \\ &= \frac{h}{h(\sqrt{1+h}+1)} \\ &= \frac{1}{\sqrt{1+h}+1}\end{aligned}$$

$$\begin{aligned}(\sqrt{1+h})^2 &= \sqrt{1+h}\cdot\sqrt{1+h} \\ &= 1+h\end{aligned}$$

Inequalities

The following properties may be used to solve one or more inequalities involving a variable.

Properties of Inequalities

If a, b, and c are any real numbers, then

		Example
Property 1	If $a < b$ and $b < c$, then $a < c$.	$2 < 3$ and $3 < 8$, so $2 < 8$.
Property 2	If $a < b$, then $a + c < b + c$.	$-5 < -3$, so $-5 + 2 < -3 + 2$; that is, $-3 < -1$.
Property 3	If $a < b$ and $c > 0$, then $ac < bc$.	$-5 < -3$, and since $2 > 0$, we have $(-5)(2) < (-3)(2)$; that is, $-10 < -6$.
Property 4	If $a < b$ and $c < 0$, then $ac > bc$.	$-2 < 4$, and since $-3 < 0$, we have $(-2)(-3) > (4)(-3)$; that is, $6 > -12$.

Similar properties hold if each inequality sign, $<$, between a and b and between b and c is replaced by $\geq$, $>$, or $\leq$. Note that Property 4 says that an inequality sign is reversed if the inequality is multiplied by a negative number.

A real number is a *solution of an inequality* involving a variable if a true statement is obtained when the variable is replaced by that number. The set of all real numbers satisfying the inequality is called the *solution set*. We often use interval notation to describe the solution set.

VIDEO **EXAMPLE 8** Find the set of real numbers that satisfy $-1 \leq 2x - 5 < 7$.

Solution Add 5 to each member of the given double inequality, obtaining

$$4 \leq 2x < 12$$

Next, multiply each member of the resulting double inequality by $\frac{1}{2}$, yielding

$$2 \leq x < 6$$

Thus, the solution is the set of all values of x lying in the interval $[2, 6)$.

EXAMPLE 9 Solve the inequality $x^2 + 2x - 8 < 0$.

Solution Observe that $x^2 + 2x - 8 = (x + 4)(x - 2)$, so the given inequality is equivalent to the inequality $(x + 4)(x - 2) < 0$. Since the product of two real numbers is negative if and only if the two numbers have opposite signs, we solve the inequality $(x + 4)(x - 2) < 0$ by studying the signs of the two factors $x + 4$ and $x - 2$. Now, $x + 4 > 0$ when $x > -4$, and $x + 4 < 0$ when $x < -4$. Similarly, $x - 2 > 0$ when $x > 2$, and $x - 2 < 0$ when $x < 2$. These results are summarized graphically in Figure 2.

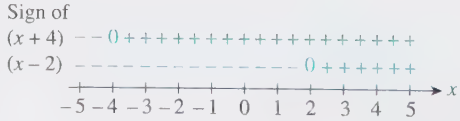


FIGURE 2
Sign diagram for $(x + 4)(x - 2)$

From Figure 2, we see that the two factors $x + 4$ and $x - 2$ have opposite signs when and only when x lies strictly between -4 and 2. Therefore, the required solution is the interval $(-4, 2)$.

EXAMPLE 10 Solve the inequality $\dfrac{x + 1}{x - 1} \geq 0$.

Solution The quotient $(x + 1)/(x - 1)$ is strictly positive if and only if both the numerator and the denominator have the same sign. The signs of $x + 1$ and $x - 1$ are shown in Figure 3.

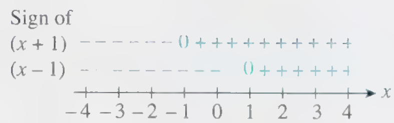


FIGURE 3
Sign diagram for $\dfrac{x + 1}{x - 1}$

From Figure 3, we see that $x + 1$ and $x - 1$ have the same sign if $x < -1$ or $x > 1$. The quotient $(x + 1)/(x - 1)$ is equal to zero if $x = -1$. Therefore, the required solution is the set of all x in the intervals $(-\infty, -1]$ and $(1, \infty)$.

APPLIED EXAMPLE 11 Stock Purchase The management of Corbyco, a giant conglomerate, has estimated that x thousand dollars is needed to purchase

$$100{,}000(-1 + \sqrt{1 + 0.001x})$$

shares of common stock of Starr Communications. Determine how much money Corbyco needs to purchase at least 100,000 shares of Starr's stock.

Solution The amount of money Corbyco needs to purchase at least 100,000 shares is found by solving the inequality

$$100{,}000(-1 + \sqrt{1 + 0.001x}) \geq 100{,}000$$

Proceeding, we find

$$\begin{aligned} -1 + \sqrt{1 + 0.001x} &\geq 1 \\ \sqrt{1 + 0.001x} &\geq 2 \\ 1 + 0.001x &\geq 4 \qquad \text{Square both sides.} \\ 0.001x &\geq 3 \\ x &\geq 3000 \end{aligned}$$

so Corbyco needs at least \$3,000,000. (Recall that x is measured in thousands of dollars.)

Absolute Value

Absolute Value

The **absolute value** of a number a is denoted by $|a|$ and is defined by

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$