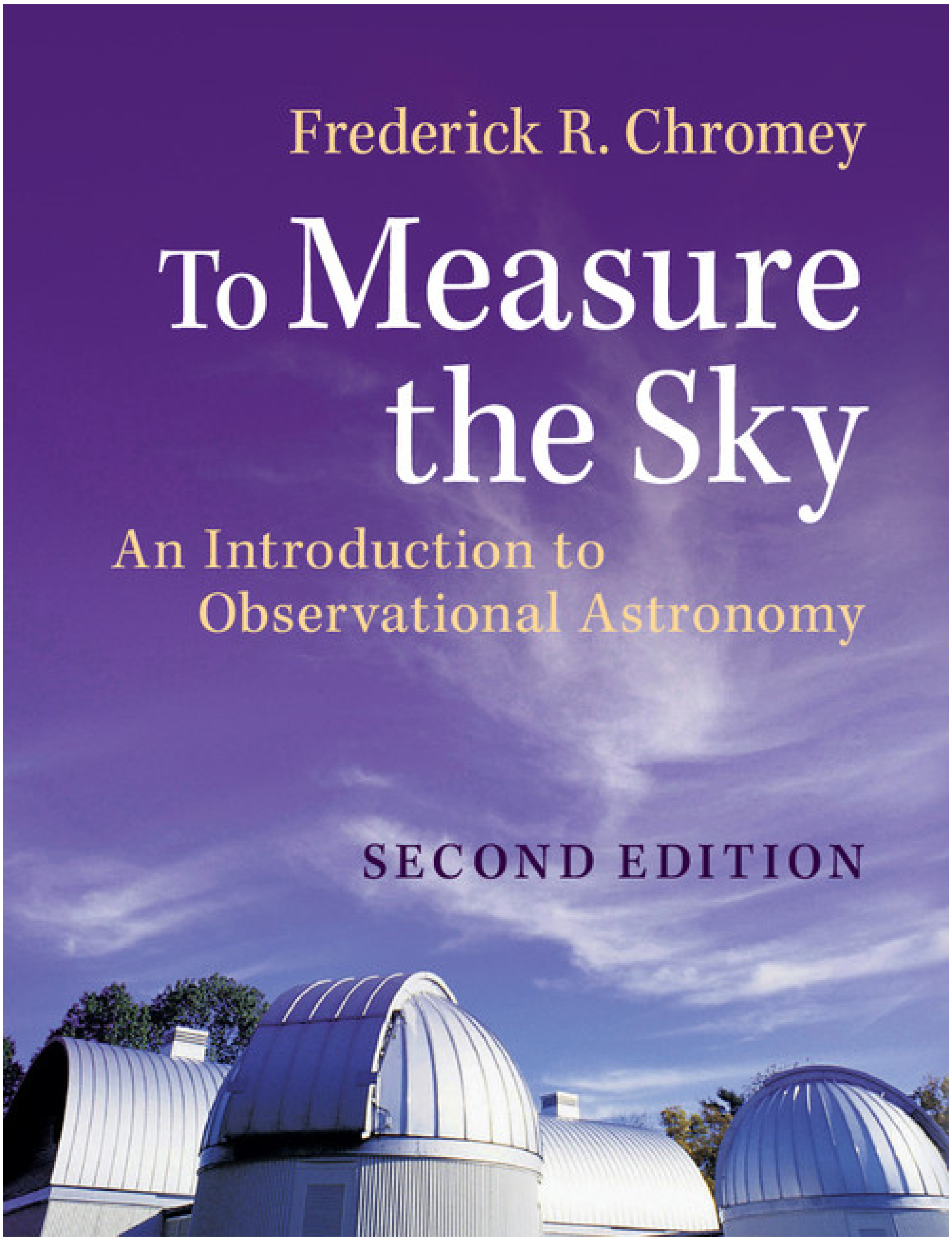
Frederick R. Chromey
To Measure the Sky
An Introduction to Observational Astronomy
SECOND EDITION

To Measure the Sky

An Introduction to Observational Astronomy

Second Edition

The second edition of this popular text provides undergraduates with a quantitative yet accessible introduction to the physical principles underlying the collection and analysis of observational data in contemporary optical and infrared astronomy. The text clearly links recent developments in ground- and space-based telescopes, observatory and instrument design, adaptive optics, and detector technologies to the more modest telescopes and detectors students may use themselves. Beginning with reviews of the most relevant physical concepts and an introduction to elementary statistics, students are given the firm theoretical foundation they need. New topics, including an expanded treatment of spectroscopy, Gaia, the Large Synoptic Survey Telescope, and photometry at large redshifts bring the text up to date. Historical development of topics and quotations emphasize that astronomy is both a scientific and a human endeavor, while extensive end-of-chapter exercises facilitate the students' practical learning experience.

Frederick R. Chromey is Professor of Astronomy on the Matthew Vassar, Jr. Chair at Vassar College, and Director of the Vassar College Observatory. He has almost 45 years' experience in observational astronomy research in the optical, radio, and near infrared on stars, gaseous nebulae, and galaxies, and has taught astronomy to undergraduates at Brooklyn College and Vassar.

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To Molly

Contents

Preface

There is an old joke: a lawyer, a priest, and an observational astronomer walk into a bar. The bartender turns out to be a visiting extraterrestrial who presents the trio with a complicated-looking black box. The alien first demonstrates that when a bucketful of garbage is fed into the entrance chute of the box, a small bag of high-quality diamonds and a gallon of pure water appear at its output. Then, assuring the three that the machine is his gift to them, the bartender vanishes.

The lawyer says, "Boys, we're rich! It's the goose that lays the golden egg! We need to form a limited partnership so we can keep this thing secret and share the profits."

The priest says, "No, no, my brothers, we need to take this to the United Nations, so it can benefit all humanity."

"We can decide all that later," the observational astronomer says. "Get me a screwdriver. I need to take this thing apart and see how it works."

The first edition of this text grew out of 16 years of teaching observational astronomy to undergraduates, and this second edition benefited from six years of using that edition in my classes and from hearing from colleagues who had done the same. In both editions, my intent has been partly to satisfy – but mainly to cultivate – my students' need to look inside black boxes. The text introduces the primary tools for making astronomical observations at visible and infrared wavelengths: telescopes, detectors, cameras, and spectrometers, as well as the methods for securing and understanding the quantitative measurements they make. I hope that after this introduction, none of these tools will remain a completely black box, and that the reader will be ready to use them to pry into other boxes. The second edition has brought the discussion a bit more up to date with current technology and practices, and has added a few recent examples of discoveries that relied on careful application of fundamental practices.

The book, then, aims at an audience similar to my students: nominally second- or third-year science majors, but with a sizable minority containing advanced first-year students, non-science students, and adult amateur astronomers. About three-quarters of those in my classes are *not* bound for graduate school in astronomy or physics, and the text has that set of backgrounds in mind.

I assume my students have little or no preparation in astronomy, but do presume that each has had one year of college-level physics and an introduction to integral and differential calculus. A course in modern physics, although very helpful, is not essential. I make the same assumptions about readers of this book. Since readers' mastery of physics varies, I include reviews of the most relevant physical concepts: optics, atomic structure, and solid-state physics. I also include a brief introduction to elementary statistics. I have written qualitative chapter summaries, but the problems posed at the end of each chapter are all quantitative exercises meant to strengthen and further develop student understanding.

My approach is to be rather thorough on fundamental topics in astronomy, in the belief that individual instructors will supply enrichment in specialized areas as they see fit. The table of contents indicates that my choice of topics is blatantly selective and slanted toward the kind of observations students might make themselves at a campus observatory.

The text lends itself to either a one- or two-semester course. I personally use the book for a two-semester sequence, where, in addition to most of the text and a selection of its end-of-chapter problems, I incorporate a number of at-the-telescope projects both for individuals and for "research teams" of students. I try to vary the large team projects: these have included a photometric time series of a variable object (in different years an eclipsing exoplanetary system, a Cepheid, and a blazar), an H–R diagram, and spectroscopy of the atmosphere of a Jovian planet. I am mindful that astronomers who teach with this text will have their own special interests in particular objects or techniques, and will have their own limitations and capabilities for student access to telescopes and equipment. My very firm belief, though, is that this book will be most effective if the instructor can devise appropriate exercises that require students to put their hands on actual hardware to measure actual photons from the sky.

To use the text for a one-semester course, the instructor will have to judiciously skip many topics. Certainly, if students are well prepared in physics and mathematics, one can dispense with much of Chapters 5 and 6 (geometrical optics and telescopes), Chapter 7 (atomic and solid-state physics), and possibly all detectors (Chapter 8) except the CCD. One would still need to choose between a more thorough treatment of photometry (skipping Chapter 11, on spectrometers) and the inclusion of spectrometry with exclusion of some photometric topics (compressing the early sections of both Chapters 9 and 10).

Compared with other texts, this book has strengths and counterbalancing weaknesses. I have taken some care with the physical and mathematical treatment of basic topics, like detection, uncertainty, telescope design, astronomical seeing, and array processing, but at the cost of a more descriptive or encyclopedic survey of specialized areas of concern to observers (e.g. little treatment of the details of astrometry or of variable star observing). I believe the book is an excellent fit for courses in which students will do their own optical/infrared observing. Because I confine myself to that wavelength region, I can develop ideas more systematically, beginning with those that arise from fundamental astronomical questions like position, brightness, and spectrum. But that narrowness makes the book less suitable for a more general survey that includes radio or X-ray techniques.

The sheer number of people and institutions contributing to the production of both editions of this book makes an adequate acknowledgment of all those to whom I am indebted impossible. Inadequate thanks are better than none, and I am deeply grateful to all who helped along the way.

A book requires an audience. The audience I had uppermost in mind was filled with those students brave enough to enroll in my Astronomy 240–340 courses at Vassar College. Over the years, more than a hundred of these students have challenged and rewarded me. All made contributions that found their way into this text, but I especially thank those who asked the hardest questions (so sorry this list is incomplete): Liz Blanton, Megan Vogelaar, Claire Webb, Deep Anand, Sherri Stephan, David Hasselbacher, Trent Adams, Leslie Sherman, Kate Eberwein, Olivia Johnson, Iulia Deneva, Laura Ruocco, Ben Knowles, Aaron Warren, Jessica Warren, Gabe Lubell, Scott Fleming, Alex Burke, Colin Wilson, Charles Wisotzkey, Peter Robinson, Tom Ferguson, David Vollbach, Krista Romita, Ximena Fernandez, Max Fagin, Jenna Lemonias, Max Marcus, Rachel Wagner-Kaiser, Tim Taber, Max Fagin, Roni Teich, Zeeve Rogozinski, Alex Shvonski, Nico Mongillo, Lauren Bearden, Angelica Rivera, Megan Lewis, Sean Sellers, Alex Trunnell, Caitlin Rose, and Liz McGrath.

I owe particular thanks to Jay Pasachoff, without whose constant encouragement and timely assistance this book would probably not exist. Likewise, Tom Balonek, who introduced me to CCD astronomy, has shared ideas, data, students, and friendship over many years. I am grateful as well to my astronomical colleagues in the Keck Northeast Astronomical Consortium; all provided crucial discussions on how to thrive as an astronomer at a small college, and many, like Tom and Jay, have read or used portions of the manuscript and the completed first edition in their observational courses. The entire text, and especially the second edition, have benefited from their feedback. I thank every Keckie, but especially Frank Winkler, Eric Jensen, Lee Hawkins, Karen Kwitter, Steve Sousa, Ed Moran, Bill Herbst, Kim McLeod, and Allyson Sheffield.

The anonymous reviewers of my proposal for the second edition made many very helpful suggestions, as did Colette Salyk and Gautham Narayan. I appreciate all the readers of the first edition who have alerted me to errors, and the many excellent conversations with Zosia Krusberg about how to teach science.

Debra Elmegreen, my colleague at Vassar, collaborated with me on multiple research projects and on the notable enterprise of building a campus observatory. Much of our joint experience found its way into this volume. Vassar College, financially and communally, has been a superb environment for both my teaching and my practice of astronomy, and deserves my gratitude. My editors at Cambridge University Press have been uniformly helpful and skilled.

My family and friends have had to bear some of the burden of this writing. Clara Bargellini and Gabriel Camera opened their home to me and my laptop during extended visits, as did my sisters, Nancy and Tina Chromey. Ann Congelton supplied useful quotations and spirited discussions. I thank my children, Kate and Anthony, who gently remind me that what is best in life is not in a book.

Finally, I thank my wife, Molly Shanley, for just about everything.

Chapter 1
Light

> Always the laws of light are the same, but the modes and degrees of seeing vary.
>
> – Henry David Thoreau, *A Week on the Concord and Merrimack Rivers*, 1849

Astronomy is not for the faint of heart. Almost everything it cares for is forbiddingly remote, tantalizingly untouchable, and invisible in the daytime, when most sensible people do their work. Nevertheless, many – including you, brave reader – have enough curiosity and courage to collect the flimsy evidence that trickles in from the universe outside our atmosphere and hope it may hold a message.

In this chapter we introduce you to astronomical evidence. Some is in the form of material, like meteorites, but most is in the form of light from faraway objects. Accordingly, we begin with three familiar theories describing the behavior of light: light as a wave, light as a quantum entity called a photon, and light as a geometrical ray. The ray picture is simplest, and we use it to introduce some basic ideas about measuring the brightness of a source. Most information in astronomy, however, comes from analyzing how brightness varies with wavelength, so we next introduce the important idea of spectroscopy. We end with a discussion of the astronomical magnitude system. We begin, however, with a few thoughts on the nature of astronomy as an intellectual enterprise.

1.1 The story

> . . . as I say, the world itself has changed. . .. For this is the great secret, which was known by all educated men in our day: that by what men think, we create the world around us, daily new.
>
> – Marion Zimmer Bradley, *The Mists of Avalon*, 1982

Astronomers are storytellers. They spin tales of the universe and of its important parts. Sometimes they envision landscapes of another place, like the roiling liquid-metal core of the planet Jupiter. Sometimes they describe another time, like the era before Earth when dense buds of gas first flowered into stars, and a darkening universe filled with the sudden blooms of galaxies. Often the stories solve mysteries or illuminate something commonplace or account for something monstrous: How is it that stars shine, age, or explode? Some of the best stories tread the same ground as myth: What threw up the mountains of the Moon? How did the skin of our Earth come to teem with life? Sometimes there are fantasies: What would happen if a comet hit the Earth? Sometimes there are prophecies: How will the universe end?

Like all stories, creation of astronomical tales demands imagination. Like all storytellers, astronomers are restricted in their creations by many conventions of language as well as by the characters and plots already in the literature. Astronomers are no less a product of their upbringing, heritage, and society than any other crafts people. Astronomers, however, think their stories are special, that they hold a larger dose of "truth" about the universe than any others. Clearly, the subject matter of astronomy – the universe and its important parts – does not belong only to astronomers. Many others speak with authority about just these things: theologians, philosophers, and poets, for example. Is there some characteristic of astronomers, besides arrogance, that sets them apart from these others? Which story about the origin of the Moon, for example, is the truer: the astronomical story about a collision 4500 million years ago between the proto-Earth and a somewhat smaller proto-planet, or the mythological story about the birth of the Sumerian/Babylonian deity Nanna-Sin (a rather formidable fellow who had a beard of lapis lazuli and rode a winged bull)?

This question of which is the "truer" story is not an idle one. Over the centuries, people have discovered (by being proved wrong) that it is very difficult to have a commonsense understanding of what the whole universe and its most important parts are like. Common sense just isn't up to the task. For that reason, as Morgan le Fay tells us in *The Mists of Avalon*, created stories about the universe themselves actually *create* the universe the listener lives in. The real universe (like most scientists, you and I behave as if there is one) is not silent, but whispers very softly to us humans. Many whispers go unheard, and the real universe is probably very different from the one you read about today in any book that claims to tell its story. People, nevertheless, must act. Most recognize that the bases for their actions are fallible stories, and they must therefore select the most trustworthy stories that they can find.

Most of you won't have to be convinced that it is better to talk about colliding planets than about Nanna-Sin if your aim is to understand the Moon or perhaps plan a visit. Still, it is useful to ask the question: what is it, if anything, that makes astronomical stories a more reliable basis for action, and in that sense more truthful or factual than any others? Only one thing, I think: *discipline*. Astronomers feel an obligation to tell their story with great care, following a rather strict, scientific, discipline.

Scientists, philosophers, and sociologists have written about what it is that makes science different from other human endeavors. There is much discussion and disagreement about the necessity of making scientific stories "broad and deep and simple," about the centrality of paradigms, the importance of predictions, the strength or relevance of motivations, and the inevitability of conformity to social norms and professional hierarchies.

But most agree on the perhaps obvious point that a scientist, in creating a story (scientists usually call them "theories" or "models") of, say, the Moon, must pay a great deal of attention to all the relevant evidence. A scientist, unlike a science-fiction writer, may only fashion a theory that never, ever, violates that evidence.

This is a book about how to collect and interpret relevant evidence in astronomy. Most of that evidence is in the form of light arriving from far, far away.

1.2 Models for the behavior of light

Some (not astronomers!) regard astronomy as applied physics. There is some justification for this, since astronomers, to help tell some astronomical story, persistently drag out theories proposed by physicists. Physics and astronomy differ partly because astronomers are interested in telling the story of an object, whereas physicists are interested in uncovering the most fundamental rules of the natural world. Astronomers tend to find physics useful but sterile; physicists tend to find astronomy messy and mired in detail. We now ponder the question: how does light behave? More specifically, what properties of light are important in making meaningful astronomical observations and predictions? Physics has the answers.

1.2.1 Electromagnetic waves

> . . . we may be allowed to infer, that homogeneous light, at certain equal distances in the direction of its motion, is possessed of opposite qualities, capable of neutralizing or destroying each other, and extinguishing the light, where they happen to be united; . . .
>
> – Thomas Young, *Philosophical Transactions, The Bakerian Lecture*, 1804

Electromagnetic waves are a model for the behavior of light. We know this model is incorrect (*incomplete* is perhaps a better term). Nevertheless, since the wave theory precisely describes so much of light's behavior, we need to review its claims. Christian Huygens,[1] in his 1678 book, *Traité de la Lumière*, made the first serious argument that visible light is best regarded as a *wave* phenomenon.

[1] Huygens (1629–95), a Dutch natural philosopher and major figure in seventeenth-century science, had an early interest in lens grinding. He discovered the rings of Saturn and its large satellite, Titan, in 1655–56, with a refracting telescope of his manufacture. At about the same time, he invented the pendulum clock and formulated a theory of elastic bodies. He developed his wave theory of light later in his career, after he moved from The Hague to the more cosmopolitan environment of Paris. Near the end of his life, he wrote a treatise on the possibility of extraterrestrial life.

A ***wave*** is a disturbance that propagates through space. If some property of the environment (say, the level of the water in your bathtub) is disturbed at one place (perhaps by a splash), a wave is present if that disturbance moves continuously from place to place in the environment (ripples from one end of your bathtub to the other, for example). Material particles, like bullets or ping-pong balls, also propagate from place to place. Waves and particles share many characteristic behaviors – both can ***reflect*** (change directions at an interface), ***refract*** (change speed or direction in response to a change in the transmitting medium), and can carry energy from place to place.

However, waves exhibit two characteristic behaviors not shared by particles. ***Diffraction*** is the ability to bend around obstacles. A water wave entering a narrow opening, for example, will travel not only in the "shadow" of the opening but will spread in all directions on the far side. ***Interference*** is the ability to combine with other waves in predictable ways. Two water waves can, for example, destructively interfere if they combine so that the troughs of one always coincide with the peaks of the other – the same phenomenon that permits noise-cancelling earphones.

Although Huygens knew that light exhibited the properties of diffraction and interference, he unfortunately did not discuss them in his book. Isaac Newton, his younger contemporary, opposed Huygens' wave hypothesis and argued that light was composed of tiny solid particles. Newton's reputation was such that his view prevailed until the early part of the nineteenth century, when Thomas Young and Augustin Fresnel drew attention to diffraction and interference in light. Soon the evidence for "light waves" proved irresistible.

Well-behaved waves exhibit certain measurable qualities: amplitude, wavelength, frequency, and wave speed. Physicists in the generation following Fresnel were able to measure these quantities for visible light waves. Since light was a wave, and since waves are disturbances that propagate, it was natural to ask: "What 'stuff' does a light wave disturb?" In one of the major triumphs of nineteenth-century physics, James Clerk Maxwell proposed an answer in 1873.

Maxwell (1831–79), a Scot, is a major figure in the history of physics, comparable to Newton and Einstein. His doctoral thesis demonstrated that the rings of Saturn (discovered by Huygens) must be made of many small solid particles in order to be gravitationally stable. He conceived the kinetic theory of gases in 1866 (Ludwig Boltzmann did similar work independently) and transformed thermodynamics into a science based on statistics rather than determinism. His most important achievement was the mathematical formulation of the laws of electricity and magnetism in the form of four partial differential equations. Published in 1873, ***Maxwell's equations*** completely accounted for separate electric and magnetic phenomena and also demonstrated the connection between the two forces. Maxwell's work is the culmination of classical physics, and its limits led to both the theory of relativity and the theory of quantum mechanics.

Maxwell proposed that light is a propagating ***electric and magnetic*** disturbance. The following example illustrates his idea.

Consider a single, motionless electron, electron A, attached to the rest of an atom by means of a spring. (The spring is just a mechanical model for the electrostatic attraction that holds the electron to the nucleus.) This pair of charges, the negative electron and the positive ion, constitute a dipole. A second electron, electron B, is also attached to the rest of its atom by a spring, but this second dipole is at some distance from A. Electron A repels B, and B's stationary position in its atom is in part determined by the location of A. The two atoms are sketched in Figure 1.1. Now to make a wave: Set electron A vibrating on its spring. Electron B must respond to this vibration, since the force it feels is changing direction. It moves in a way that will echo the motion of A. The lower part of Figure 1.1 shows the changing electric force on B as A moves through a cycle of its vibration.

The disturbance of dipole A has propagated to B in a way that suggests a wave is operating. Electron B behaves like an object floating in your bathtub that moves in response to the rising and falling level of a water wave.

In trying to imagine the actual thing that a vibrating dipole disturbs, you might envision the water in a bathtub. Now imagine some stuff that fills space around the electrons, the way a fluid would, so a disturbance caused by moving one electron can propagate from place to place. The physicist Michael Faraday[2] supplied the very useful idea of a ***field*** – an abstract *entity* (not a material fluid at all) created by any charged particle. The field permeates space and gives other charged particles instructions about what force they should experience. In this conception, electron B consults the local field in order to decide how to move. You are probably familiar with understanding magnetic and gravitational forces as also arising from their corresponding fields. Shaking (accelerating) the electron at A distorts the field in its vicinity, and this distortion propagates to vast distances, just like the ripples from a rock dropped into a calm and infinite ocean.

The details of propagating a field disturbance turned out to be a little complicated. Hans Christian Oersted and André Marie Ampère in 1820 had shown experimentally that a changing electric field, such as the one generated by an accelerated electron, produces a magnetic field. Acting on his intuition of an underlying unity in physical forces, Faraday experimentally confirmed his guess

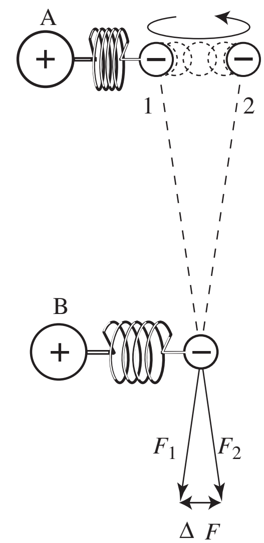


Fig. 1.1 Acceleration of an electron produces a wave. The electrons in initially undisturbed atoms are in stationary positions. Each electron is attached to the rest of the atom (the heavy, positively charged ion) by some force, which we represent as a spring. If the electron in the source atom (A) is disturbed so that it oscillates between positions (1) and (2), then the electron in the receiver (B) experiences a force that changes from F_1 to F_2 in the course of A's oscillation. The difference ΔF, sets the amplitude of the changing part of the electric force seen by B.

[2] Michael Faraday (1791–1867), considered by many the greatest experimentalist in history, began his career as a bookbinder with minimal formal education. His amateur interest in chemistry led to a position in the laboratory of the renowned chemist, Sir Humphrey Davy, at the Royal Institution in London. Faraday continued work as a chemist for most of his productive life, but conducted an impressive series of experiments in electromagnetism in the period 1834–55. His ideas, although largely rejected by physicists on the Continent, eventually formed the empirical basis for Maxwell's theory of electromagnetism.

that a changing magnetic field must in turn generate an electric field. Maxwell had the genius to realize that his equations implied that the electric and magnetic field changes in a vibrating dipole would support one another and produce a wavelike self-propagating disturbance. Change the electric field and you thereby create a magnetic field, which then creates a different electric field, which creates a magnetic field, and so on, forever. Thus, it is proper to speak of the waves produced by an accelerated charged particle as ***electromagnetic***. Figure 1.2 shows a schematic version of an electromagnetic wave. The changes in the two fields, electric and magnetic, vary at right angles to one another and the direction of propagation is at right angles to both (a ***transverse wave***).

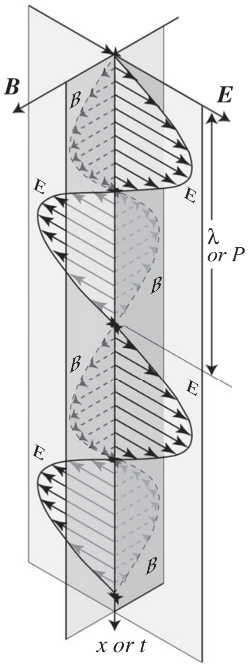


Fig. 1.2 A plane-polarized electromagnetic wave. The electric and magnetic field strengths are drawn as vectors that vary in both space and time. The illustrated waves are said to be plane-polarized because all electric vectors are confined to the *x*–*y* plane.

Thus, a disturbance in the electric field does indeed seem to produce a wave. Is this *electromagnetic* wave the same thing as the *light* wave we see with our eyes?

From his four equations – the laws of electric and magnetic force – Maxwell derived the speed of any electromagnetic wave, which, in a vacuum, turned out to depend only on constants

$$c = \sqrt{\varepsilon\mu} \tag{1.1}$$

Here ε and μ are well-known constants that describe the strengths of the electric and magnetic forces. (They are, respectively, the electric permittivity and magnetic permeability of the vacuum.) When he entered the experimental values for ε and μ in the above equation, Maxwell computed the electromagnetic wave speed, which turned out to be numerically identical to the speed of light, a quantity that had been experimentally measured with improving precision over the preceding century. This equality of predicted and experimentally measured speeds was a quite convincing argument that light waves and electromagnetic waves were the same thing. Maxwell had shown that three different entities, electricity, magnetism, and light, were really tightly related.

Other predictions based on Maxwell's theory further strengthened this view of the nature of light. For one thing, one can note that for any well-behaved wave the speed of the wave is the product of its frequency and wavelength:

$$c = \lambda\upsilon \tag{1.2}$$

There is only one speed that electromagnetic waves can have in a vacuum; therefore, there should be a one-dimensional classification of electromagnetic waves (the ***electromagnetic spectrum***). In this spectrum, each wave is characterized only by its particular wavelength (or frequency). A single light wave of a particular wavelength is usually represented as the harmonic function

$$E(x,t) = E_0 \sin\left\{\frac{2\pi}{\lambda}(x - ct)\right\} = E_0 \sin\{\phi\} \tag{1.3}$$

where E_0 and ϕ are, respectively, the ***amplitude*** and the ***phase*** of the wave. Table 1.1 gives the modern names for various portions or ***bands*** of the

Table 1.1 *The electromagnetic spectrum. Region boundaries are not well defined, so there is some overlap. Subdivisions are based in part on distinct detection methods.*

Band	Wavelength range	Frequency range	Subdivisions (long λ – short λ)
Radio	$>$ 1 mm	$<$ 300 GHz	VLF–AM–VHF–UHF
Microwave	0.1 mm–3 cm	100 MHz–3000 GHz	Millimeter–Submillimeter
Infrared	700 nm–1 mm	3×10^{11}–4×10^{14} Hz	Far–Middle–Near
Visible	300 nm–800 nm	4×10^{14}–1×10^{15} Hz	Red–Blue
Ultraviolet	10 nm–400 nm	7×10^{14}–3×10^{16} Hz	Near–Extreme
X-rays	0.001 nm–10 nm	3×10^{16}–3×10^{20} Hz	Soft–Hard
Gamma ray	$<$ 0.1 nm	$>3\times10^{18}$ Hz	Soft–Hard

electromagnetic spectrum. William Herschel and Johann Wilhelm Ritter had already discovered infrared and ultraviolet "light," respectively, in 1800–01 – well before Maxwell's theory. In 1888, Heinrich Hertz demonstrated the production of radio waves based on Maxwell's principles. These experimental confirmations convinced physicists that Maxwell had discovered the secret of light. Humanity had made a tremendous leap in understanding reality. This leap to new heights, however, soon revealed that Maxwell had discovered only a part of the secret.

The wave theory of light very accurately describes the way light behaves in most macroscopic situations. In summary, the theory says:

1. Light exhibits all the properties of classical, well-behaved waves, namely: reflection at interfaces, refraction upon changes in the medium, diffraction around edges, interference with other light waves, and polarization in a particular direction (plane of vibration of the electric vector).
2. A light wave can have any positive wavelength. The range of possible wavelengths constitutes the electromagnetic spectrum. Frequency and wavelength are related by Equation (1.2).
3. In a vacuum, light waves travel in a straight line at speed c. Travel in other media is slower and subject to refraction and absorption.
4. A light wave carries energy whose magnitude depends on the squares of the amplitudes of the electric and magnetic waves.

1.2.2 Quantum mechanics and light

> It is very important to know that light behaves like particles, especially for those of you who have gone to school, where you were probably told something about light behaving like waves. I'm telling you the way it does behave – like particles.
>
> – Richard Feynman: *Q.E.D.*, 1985

Toward the end of the nineteenth century, physicists realized that electromagnetic theory could not account for certain behaviors of light. The theory that eventually replaced it, ***quantum mechanics***, postulates that light possesses the properties of a particle as well as the wavelike properties described by Maxwell's theory. Quantum mechanics insists that there are situations in which we cannot think of light as a wave, but must think of it as a collection of particles, like bullets shot out of the source at the speed of light. These particles are termed ***photons***. Each photon "contains" a particular amount of energy, E, that depends on the frequency it possesses when it exhibits its wavelike properties:

$$E = h\nu = \frac{hc}{\lambda} \tag{1.4}$$

Here h is Planck's constant (6.626×10^{-34} J s) and ν is the frequency of the wave. Thus a single radio photon (low frequency) contains a small amount of energy, and a single gamma-ray photon (high frequency) contains a lot. A convenient unit for the energy of a photon is the ***electronvolt*** (1 eV = 1.602×10^{-19} J)

The quantum theory of light gives an elegant and successful picture of the interaction between light and matter on the microscopic scale. In this view, atoms no longer have electrons bound to nuclei by springs or (what is equivalent in classical physics) electric fields. Electrons in an atom, rather, have certain permitted energy states described by a wave function – in this theory, everything, including electrons, has a wave as well as a particle nature. An electron changing from one of these permitted states to another explains the generation or absorption of light by atoms. Energy is conserved: energy lost when an atom makes the transition from a higher to a lower state is exactly matched by the energy of the photon emitted. In summary, the quantum theory says:

1. Light exhibits all the properties described in the wave theory in situations where wave properties are measured.
2. Light behaves, in other circumstances, as if it were composed of massless particles called photons, each containing an amount of energy equal to its frequency times Planck's constant.
3. The interaction between light and matter involves creation and destruction of individual photons and the corresponding changes of energy states of charged particles (usually electrons).

We will make great use of the quantum theory in later chapters, but for now, our needs are more modest.

1.2.3 A geometric approximation: light rays

> By Light Rays I understand its least Parts . . . Mathematicians usually consider the Rays of Light to be Lines reaching from the luminous Body to the Body illuminated . . .
>
> – Isaac Newton, *Opticks,* 1704

Since the quantum picture of light is as close as we can get to the real nature of light, you might think quantum mechanics would be the only theory worth considering. However, except in simple situations, application of the theory demands complex and lengthy computation. Fortunately, it is often possible to ignore much of what we know about light and use a very rudimentary picture which pays attention only to those few properties of light necessary to understand much of the information brought to us by photons from out there. In this geometric approximation, we treat light as if it traveled in "rays" or streams that obey the laws of reflection and refraction as described by geometrical optics. It is helpful to imagine a ray as the path taken by a single photon of a particular wavelength.

We might then imagine a stream of photons, each tracing a ray from the source to an observer's detector. Sometimes it is essential to recognize the discrete nature of the particles. We might then think of astronomical measurements as acts of *counting* and classifying the individual photons as they hit our detector like sparse raindrops tapping on a tin roof.

Sometimes, we can ignore the lumpy nature of the photon stream and just assume it behaves like a smooth fluid that carries energy from source to detector along the rays. In this case, we think of astronomical measurements as recording smoothly varying quantities – like measuring the volume of rain that falls into a bucket in one day. We might be aware that the rain arrived as discrete drops, but it is safe to ignore the fact.

We will adopt this simplified ray picture for much of the discussion that follows, adjusting our awareness of the discrete nature of the photon stream or its wave properties as circumstances warrant. For the rest of this chapter, we use the ray picture to discuss two of the basic measurements important in astronomy: ***photometry,*** which measures the amount of energy arriving from a source, and ***spectrometry,*** which measures the distribution of this energy with wavelength. Incidentally, our use of the word "wavelength" does not mean we are going to think deeply about the wave theory just yet. It will be sufficient to think of wavelength as a property of a light ray that can be measured – by noting which ray a photon follows when sent through a spectrograph, for example.

Besides photometry and spectroscopy, the other general categories of measurement are ***imaging*** and ***astrometry***, which are concerned with the appearance and positions of objects in the sky, and ***polarimetry***, which is concerned with the polarization of light from the source.

1.3 Measurements of light rays

> Twinkle, twinkle, little star,
> Flux says just how bright you are.
> – Anonymous, *c.* 1980

1.3.1 Luminosity and brightness

Astronomers have to construct the story of a distant object using only the tiny whisper of electromagnetic radiation it sends us. We define the (electromagnetic) luminosity, L, as the total amount of energy that leaves the surface of the source per unit time in the form of photons. Energy per unit time is called power, so we can measure L in physicists' units for power (SI units), joules per second or watts. Alternatively, it might be useful to compare the object with the Sun, and we then might measure the luminosity in solar units:

$$L = \textit{Luminosity} = \text{Energy per unit time emitted by the entire source}$$

$$L_{\odot} = \text{Luminosity of the sun} = 3.827 \times 10^{26}\,\text{W}.$$

The luminosity of a source is an important clue about its nature. One way to measure luminosity is to surround the source completely with a box or (since this is physics) sphere of perfectly energy-absorbing material, then use an "energy gauge" to measure the total amount of energy intercepted by this enclosure during some time interval. Figure 1.3 illustrates the method. Luminosity is the amount of energy absorbed divided by the time interval over which the energy accumulates. The astronomer, however, cannot measure luminosity in this way. She is too distant from the source to put it inside a sphere, even in the unlikely case she has one big enough. Fortunately, there is a quantity related to luminosity, called the ***apparent brightness*** of the source, which is much easier to measure.

Measuring apparent brightness is a local operation. The astronomer holds up a scrap of perfectly absorbing material of known area so that its surface is perpendicular to the line of sight to the source. She measures how much energy from the source accumulates in this material in a known time interval. Apparent brightness, F, is defined as the total energy per unit time per unit area that arrives from the source:

$$F = \frac{E}{tA} \tag{1.5}$$

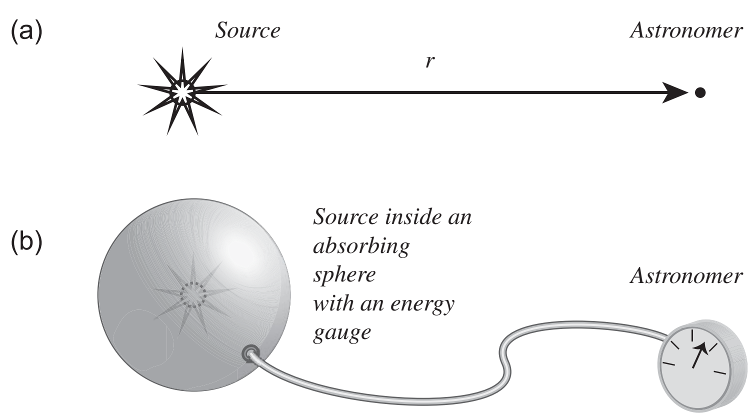


Fig. 1.3 Measuring luminosity by intercepting all the power from a source.

This quantity, F, is usually known as the ***flux*** or the ***flux density*** in the astronomical literature. In the physics literature, the same quantity is usually called the ***irradiance*** (or, in studies restricted to visual light, the ***illuminance***). To make matters not only complex but also confusing, what astronomers call luminosity, L, physicists call the ***radiant flux***.

Whatever one calls it, F will have units of power per unit area, or $\mathrm{Wm^{-2}}$. For example, the average flux from the Sun at the top of the Earth's atmosphere (the apparent brightness of the Sun) is about 1361 $\mathrm{Wm^{-2}}$, a quantity known as the ***solar constant.*** The instantaneous value of the flux from the Sun, the ***total solar irradiance,*** varies by about 7% because of the Earth's elliptical orbit and by perhaps 0.2% over longer historical periods because of intrinsic solar variations.

1.3.2 The inverse square law of brightness

Refer to Figure 1.4 to derive the relationship between the flux from a source and the source's luminosity. We choose to determine the flux by measuring the power intercepted by the surface of a very large sphere of radius r centered on the source. The astronomer is on the surface of this sphere. Since this surface is everywhere perpendicular to the line of sight to the source, the apparent brightness, according to Equation (1.5) is simply the total power absorbed by the large sphere divided by its area. But surrounding the source with a sphere is exactly what we did in Figure 1.3, so the total power absorbed by the large sphere must be the luminosity, L, of the source. We assume that there is nothing located between the source and the spherical surface that absorbs light – no dark cloud or planet. The brightness, averaged over the whole sphere, then, is:

$$\langle F \rangle = \frac{L}{4\pi r^2}$$

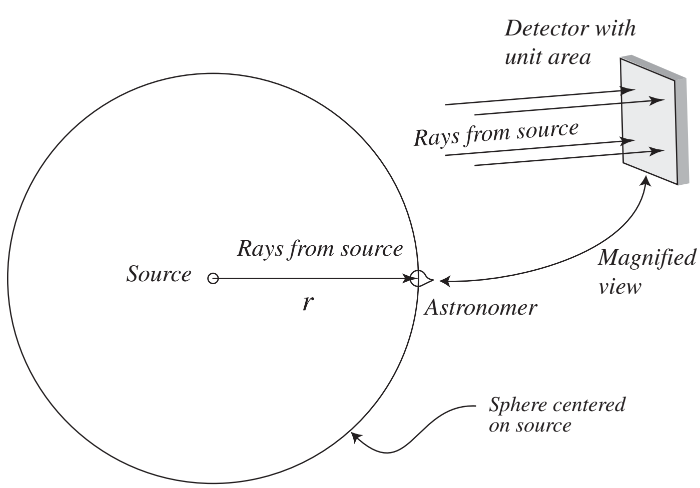


Fig. 1.4 Measuring the apparent brightness of an isotropic source that is at distance r. The astronomer locally detects the power reaching a unit area oriented perpendicular to the direction of the source. If there is no intervening absorber, then the source luminosity is equal to the apparent brightness multiplied by the area of a sphere of radius r.

Now make the additional assumption that the radiation from the source is isotropic (the same in all directions). Then the *average* brightness is the same as the brightness measured locally, using any convenient small surface:

$$F = \frac{L}{4\pi r^2} \tag{1.6}$$

Both assumptions, isotropy and the absence of absorption, can be violated in reality. Nevertheless, in its simple form, Equation (1.6) not only represents one of the fundamental relationships in astronomy, it also reveals one of the central problems in our science.

The problem is that the left-hand side of Equation (1.6) is the flux, a quantity that can be determined by direct observation. However, the right-hand side contains two unknowns, luminosity and distance. Without further information these two cannot be disentangled. This is a frustration – you can't say, for example, how much power a quasar is producing without knowing its distance, and you can't know its distance unless you know how much power it is producing. A fundamental problem in astronomy is determining the third dimension.

1.3.3 Surface brightness

One observable quantity that does not depend on the distance of a source is its surface brightness on the sky. Consider the simple case of a uniform spherical source of radius a and luminosity L. On the surface, the amount of power leaving a unit area is called the ***radiant exitance***:

$$s = \frac{L}{4\pi a^2} \tag{1.7}$$

Note that s has the same dimensions, W m^{-2}, as F, the apparent brightness seen by a distant observer. The two are very different quantities, however. The value of s is characteristic only of the object itself, whereas F changes with distance. Now, suppose that our sphere has a detectable angular size – that our eye or telescope can distinguish it from a point source: it looks like a disk. The ***solid angle***, in ***steradians*** subtended by a disk of radius a and distance r is (for $a \ll r$):

$$\Omega \cong \frac{\pi a^2}{r^2}\,[\text{steradians}]. \tag{1.8}$$

Now we write down σ, the apparent ***surface brightness*** of the source on the sky, that is, the flux from the disk divided by the solid angle it subtends:

$$\sigma = \frac{F}{\Omega} = \frac{s}{\pi} \tag{1.9}$$

So, in this example, σ depends only on the radiant exitance of the source and so is ***independent of distance***. A more careful analysis of non-spherical, non-uniform resolved objects supports the same conclusion: σ does not change with distance. Ordinary optical telescopes can measure Ω with accuracy only if it has a value larger than a few square arc seconds (about 10^{-10} steradians), mainly because of turbulence in the Earth's atmosphere. Space telescopes and ground-based systems with adaptive optics can resolve solid angles perhaps 100 times smaller. Unfortunately, the majority of even the nearest stars have angular sizes too small (diameters of a few milliarc seconds) to resolve with present instruments, so that for them Ω (and therefore σ) cannot be measured directly. Astronomers do routinely measure σ for "extended" or "non-stellar" images of objects like planets, gaseous nebulae, and galaxies, and find these values immensely useful.

1.4 Spectra

> If a question on an astronomy exam starts with the phrase 'how do we know . . .' then the answer is probably 'spectrometry.'
>
> – Anonymous, *c.* 1950

Astronomers usually learn most about a source not from its flux, surface brightness, or even luminosity, but from its spectrum: the way in which light is distributed with wavelength. Measuring its luminosity is like reading the title of a book about the source. Measuring its spectrum is like opening the book and skimming a few chapters, chapters that might explain the source's chemical composition, pressure, density, temperature, rotation speed, or radial velocity. (You seldom get to read the whole book.) Although evidence in astronomy is usually meager, the most satisfying and eloquent evidence is spectroscopic.

1.4.1 Monochromatic flux

Consider measuring the flux from a source in the usual fashion, with a set-up like the one in Figure 1.4. Arrange our detector to register only photons that have frequencies between ν and ν+dν, where dν is an infinitesimally small frequency interval. Write the result of this measurement as $F(\nu, \nu + \mathrm{d}\nu)$. Keep in mind that d$\nu$ and $F(\nu, \nu + \mathrm{d}\nu)$ are the *limits* of finite quantities called $\Delta\nu$ and $F(\nu, \nu + \Delta\nu)$. We then define ***monochromatic flux*** or ***monochromatic brightness*** as

$$f_\nu = \frac{F(\nu, \nu + \mathrm{d}\nu)}{\mathrm{d}\nu} = \operatorname*{limit}_{\Delta\nu \to 0} \frac{F(\nu, \nu + \Delta\nu)}{\Delta\nu} \tag{1.10}$$

The complete function, f_ν, running over all frequencies, (or even over a limited range of frequencies) is called the ***spectrum*** of the object. It has units [$\mathrm{Wm^{-2}Hz^{-1}}$]. The extreme right-hand side of Equation (1.10) reminds us that f_ν is the limiting value of the ratio as the quantity $\Delta\nu$ (and correspondingly, $F(\nu, \nu + \Delta\nu)$) become indefinitely small. In practice, $\Delta\nu$ must have a finite size,

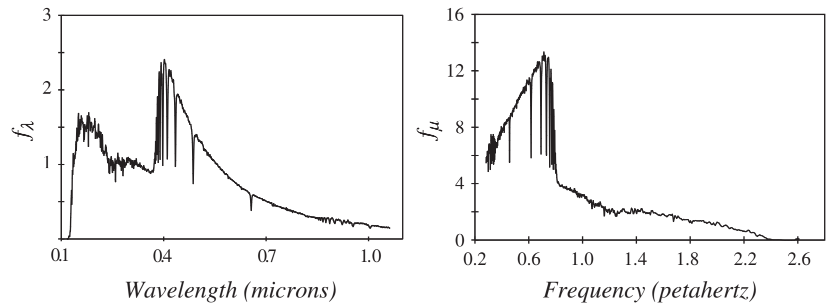


Fig. 1.5 Two forms of the ultraviolet and visible outside-the-atmosphere spectrum of Vega. The two curves convey the same information, but have very different shapes. Units on the vertical axes are arbitrary.

since $F(\nu, \nu + \Delta\nu)$ must be large enough to register on a detector. If $\Delta\nu$ is large, the detailed wiggles and jumps in the spectrum will be smoothed out, and one is said to have measured a ***low-resolution spectrum***. Likewise, a ***high-resolution spectrum*** will more faithfully show the details of the limiting function, f_ν.

If we choose the wavelength as the important characteristic of light, we can define a different monochromatic brightness, f_λ. Symbolize the flux between wavelengths λ and λ+dλ as $F(\lambda, \lambda + \mathrm{d}\lambda)$ and write

$$f_\lambda = \frac{F(\lambda, \lambda + \mathrm{d}\lambda)}{\mathrm{d}\lambda} \tag{1.11}$$

Although the functions f_ν and f_λ are each called the spectrum, they differ from one another in numerical value and overall appearance for the same object. Figure 1.5 shows schematic low-resolution spectra of the bright star, Vega, plotted over the same range of wavelengths, first as f_λ, then as f_ν.

1.4.2 Flux within a band

> Less is more.
>
> – Robert Browning, "Andrea del Sarto," 1855, often quoted by L. Mies van der Rohe

An ***ideal bolometer*** is a detector that responds to all wavelengths with perfect efficiency. In a unit time, a bolometer would record every photon reaching it from a source, regardless of wavelength. We could symbolize the ***bolometric flux*** thereby recorded as the integral:

$$F_{\mathrm{bol}} = \int_0^\infty f_\lambda \mathrm{d}\lambda \tag{1.12}$$

Real bolometers operate by monitoring the temperature of a highly absorbing (i.e. black) object of low thermal mass. They are imperfect in part because it is difficult to design an object that is "black" at all wavelengths. More commonly, practical instruments for measuring brightness can only detect light within a limited range of wavelengths or frequencies. Suppose a detector registers light between wavelengths λ_1 and λ_2, and nothing outside this range. In the notation of the previous section, we might then write the flux in the 1, 2 pass-band as:

$$F(\lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} f_\lambda \mathrm{d}\lambda = F(\nu_2, \nu_1) = \int_{\nu_2}^{\nu_1} f_\nu \mathrm{d}\nu \tag{1.13}$$

Usually, the situation is even more complex. Practical detectors vary in detecting efficiency over any band. If $R_{\mathrm{A}}(\lambda)$ is the fraction of the incident flux of wavelength λ that is eventually detected by instrument A, then the flux actually recorded by such a system might be represented as

$$F_{\mathrm{A}} = \int_0^\infty R_A(\lambda) f_\lambda \mathrm{d}\lambda \tag{1.14}$$

The function $R_{\mathrm{A}}(\lambda)$ may be imposed in part by the environment rather than by the instrument. The Earth's atmosphere, for example, is (imperfectly) transparent only in the visible and near-infrared pass-band between about 0.32 and 1 micron (extending in restricted bands to 25 μm at very dry, high-altitude sites), and in the microwave–radio pass-band between about 0.5 mm and 50 m.

Astronomers routinely restrict the range of a detector's sensitivity intentionally, by using a ***filter*** to control the form of the function R_{A}. Why? First, a well-defined standard band makes it easier for different astronomers to compare measurements. Second, a filter can block troublesome wavelengths, ones where the background is high, perhaps, or where atmospheric transmission is low. Finally, comparison of two or more different bandpass fluxes for the same source is akin to measuring a very low-resolution spectrum, and thus can provide some of the information, like temperature or chemical composition, that a spectrum conveys.

Hundreds of bands have found use in astronomy. Table 1.2 lists the broad-band filters (i.e. filters where the bandwidth, $\Delta\lambda = \lambda_2 - \lambda_1$, is large) that are most commonly encountered in the visible–near-infrared window. Standardization of bands is less common in radio and high-energy observations.

1.4.3 Spectrum analysis

> [With regard to stars] . . . we would never know how to study by any means their chemical composition. . .. In a word, our positive knowledge with respect to stars is necessarily limited solely to geometrical and mechanical phenomena. . ..
>
> – Auguste Comte, *Cours de Philosophie Positive* II, 19th Lesson, 1835

> . . . I made some observations which disclose an unexpected explanation of the origin of Fraunhofer's lines, and authorize conclusions therefrom respecting the material constitution of the atmosphere of the sun, and perhaps also of that of the brighter fixed stars.
>
> - Gustav R. Kirchhoff, Letter to the Academy of Science at Berlin, 1859

Table 1.2 *Common broad bandpasses in the visible (UBVRI) and short- (JHK), mid- (LM), long- (N), and very long- (Q) wavelength infrared. Chapter 10 discusses standard bands in greater detail.*

Name	λ_c (μm)	Width (μm)	Rationale
U	0.365	0.068	Ultraviolet
B	0.44	0.098	Blue
V	0.55	0.089	Visual
R	0.70	0.22	Red
I	0.90	0.24	Infrared
J	1.25	0.38	SWIR
H	1.63	0.31	SWIR
K	2.2	0.48	SWIR
L	3.4	0.70	MWIR
M	5.0	1.123	MWIR
N	10.2	4.31	LWIR
Q	21.0	8	VLWIR

Astronomers are fond of juxtaposing Comte's pronouncement about the impossibility of knowing the chemistry of stars with Kirchhoff's breakthrough a generation later. Me too. Comte deserves better, since he wrote quite thoughtfully about the philosophy of science and would certainly have been among the first to applaud the powerful new techniques of spectrum analysis developed later in the century. Nevertheless, the failure of his dictum about what is knowable is a caution against pomposity for all.

Had science been quicker to investigate spectra, Comte might have been spared posthumous deflation. In 1666, Newton observed the dispersion of visible "white" sunlight into its component colors by glass prisms, but subsequent applications of spectroscopy were very slow to develop. It was not until 1802 that the English physicist William Wollaston noted the presence of dark ***lines*** in the visible solar spectrum. Lines are very narrow wavelength bands where the value of function f_ν drops almost discontinuously, then rises back to the previous "continuum." (see Figures 1.6 and 1.7). The term "line" arises because in visual spectroscopy, one actually examines the image of a narrow slit at each wavelength. If the intensity is unusually low at a particular wavelength, then the image of the slit there looks like a dark line.

The Fraunhofer spectrum

Unaware of Wollaston's work, Joseph von Fraunhofer (1787–1826), used a much superior spectroscope to produce an extensive map of the solar absorption lines in around 1812.

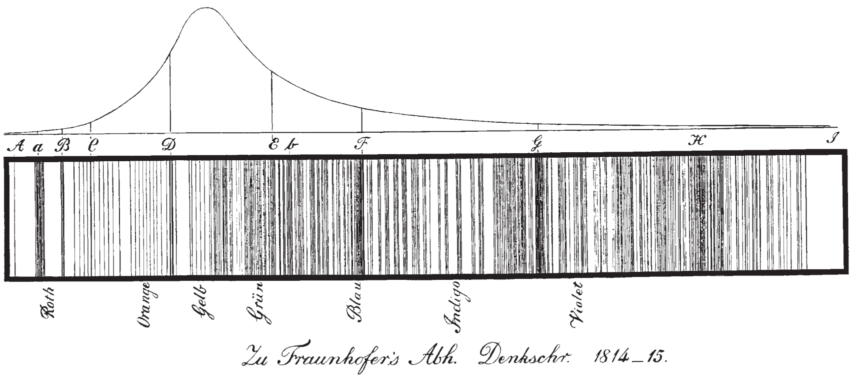


Fig. 1.6 A much-reduced reproduction of one of Fraunhofer's drawings of the solar spectrum. Frequency increases to the right, and the stronger absorption lines are labeled with his designations. Modern designations differ slightly.

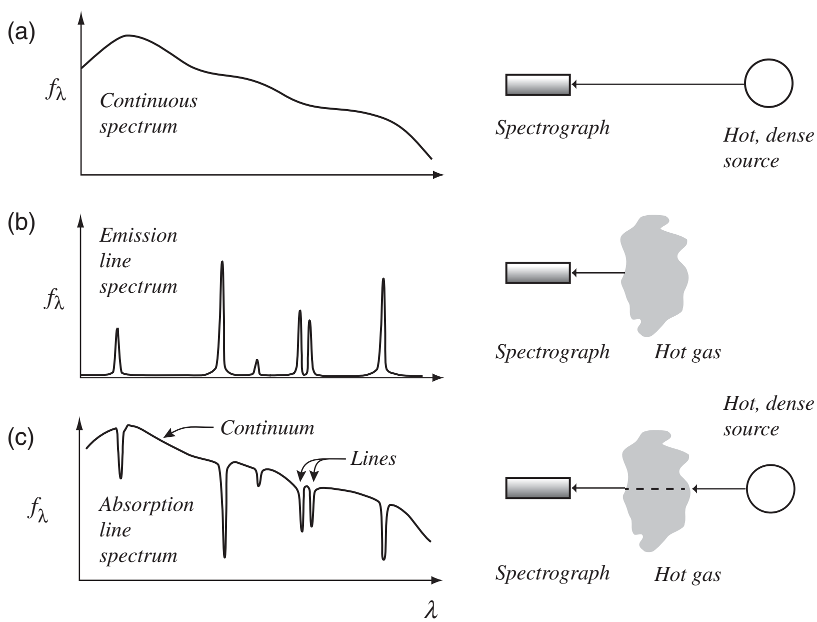


Fig. 1.7 Three types of spectra and the situations that produce them. (a) A solid, liquid, or dense gas produces a continuous spectrum. (b) A rarefied hot gas like a flame or a spark produces an emission-line spectrum. (c) A continuous spectrum viewed through a rarefied gas produces an absorption line spectrum.

Of humble birth, Fraunhofer began his career as an apprentice at a glassmaking factory located in an abandoned monastery in Benediktbeuern, outside Munich. By talent and fate (he survived a serious industrial accident), he advanced quickly in the firm. The business became quite successful and famous because of a secret process for making large blanks of high-quality crown and flint glass, which had important military and civil uses. Observing the solar spectrum with the ultimate goal of improving optical instruments, Fraunhofer pursued what he believed to be his discovery of solar absorption lines with characteristic enthusiasm and thoroughness. By 1814, he had given precise positions for 350 lines and approximate positions for another 225 fainter lines (see Figure 1.6). By 1823, Fraunhofer was reporting on the spectra of bright stars and planets, although his main enthusiasm was the creation of high-quality optical instruments. Tragically, Fraunhofer died from tuberculosis at the age of 39 and one can only speculate on the development of astrophysics had he remained active for another quarter century.

Fraunhofer designated the ten most prominent of the dark lines he observed in the solar spectrum with letters (Figure 1.6, and Appendix B.2). He noted that the two dark lines he labeled with the letter *D* occurred at wavelengths identical to the two bright emission lines produced by a candle flame. (***Emission lines*** are narrow wavelength regions where the value of function f_λ increases almost discontinuously, then drops back down to the continuum. See Figure 1.7.) Soon several observers noted these two bright lines, which occur in the yellow part of the spectrum at wavelengths of 589.0 and 589.6 nanometers, always arise from the presence of sodium in a flame. Several researchers (John Herschel, W.H. Fox Talbot, David Brewster) in the 1820s and 1830s suggested that there was a connection between the composition of an object and its flame spectrum, but none could describe it precisely. At about this same time, still others noted that heated solids, unlike the gases in flames, produce ***continuous spectra*** (no bright or dark lines – again, see Figure 1.7).

The Kirchhoff–Bunsen results

Spectroscopy languished for the 30 years following Fraunhofer's death in 1826. Then, in Heidelberg in 1859, physicist Gustav Kirchhoff and chemist Robert Bunsen performed a crucial experiment. They passed a beam of sunlight through a sodium flame, initially to measure the precision with which the solar absorption and flame emission lines coincided. What they observed instead was that the bright lines faded, and the dark *D* lines became darker still. Kirchhoff reasoned that the hot gas in the flame had both absorbing and emitting properties at the *D* wavelengths, but that the absorption became more apparent as more light to be absorbed was supplied, whereas the emitting properties remained constant. This suggested that absorption lines would always be seen in situations like that sketched on the right of Figure 1.7c, so long as a sufficiently bright source were observed through a gas. If the background source were too weak or altogether absent, then the situation sketched in Figure 1.7b would hold and emission lines would be prominent.

Kirchhoff, moreover, proposed an explanation of all the Fraunhofer lines: The Sun consists of a bright source – a very dense gas, as it turns out – that emits a continuous spectrum. A low-density, gaseous atmosphere overlies this dense region. As the light from the continuous source passes through the atmosphere, its chemicals each absorb their characteristic wavelengths. One could then conclude, for example, that the Fraunhofer *D* lines demonstrated the presence of sodium in the solar atmosphere. Identification of other chemicals in the solar atmosphere became a matter of obtaining an emission line "fingerprint" from a laboratory flame or spark spectrum, then searching for the corresponding absorption line or lines at identical wavelengths in the Fraunhofer spectrum. Kirchhoff and Bunsen quickly confirmed the presence of potassium, iron, and calcium and the absence (or very low abundance) of lithium in the solar atmosphere. The Kirchhoff–Bunsen results were not limited to the Sun. The spectra of almost all stars turned out to be absorption spectra, and it was easy to identify many of the lines present.

Quantitative chemical analysis of solar and stellar atmospheres became possible in the 1940s, after the development of astrophysics in the early twentieth century. At that time astronomers showed that most stars were composed of hydrogen and helium in a roughly 12 to 1 ratio by number, with small additions of other elements. However, the early qualitative results of Kirchhoff and Bunsen had already demonstrated to the world that stars were made of ordinary matter, and that one could hope to learn their exact composition by spectrometry. By the 1860s they had replaced the "truth" that stars were inherently unknowable with the new "truth" that stars were made of ordinary stuff.

Blackbody spectra

In 1860, Kirchhoff discussed the ratio of absorption to emission in hot objects by first considering the behavior of a perfect absorber, an object that would absorb all light falling on its surface. He called such an object a ***blackbody*** (*kürzer schwarze*), since it by definition would reflect nothing. Blackbodies, however, must emit (otherwise their temperatures would always increase from absorbing ambient radiation). One can construct a simple blackbody by drilling a small hole into a uniform oven. The hole is the blackbody. The black walls of the oven will always absorb light entering the hole, so that the hole is a perfect absorber. The spectrum of the light *emitted* by the hole will depend on the temperature of the oven (and, it turns out, on nothing else). The blackbody spectrum is usually a good approximation to the spectrum emitted by any solid, liquid, or dense gas. (Low-density gases produce line spectra.)

In 1878, Josef Stefan found experimentally that the surface brightness of a blackbody (total power emitted per unit area – the luminous exitance) depends only on the fourth power of its temperature, and, in 1884, Ludwig Boltzmann supplied a theoretical understanding of this relation. **The *Stefan–Boltzmann law*** is

$$s = \sigma T^4 \tag{1.15}$$

where σ = the Stefan–Boltzmann constant = 5.6696×10^{-8} W m^{-2} K^{-4}.

Laboratory studies of blackbodies at about this time showed that although their spectra change with temperature, all have a similar shape: a smooth curve with one maximum (see Figure 1.8). This peak in the monochromatic flux curve (either f_λ or f_ν) shifts to shorter wavelengths with increasing

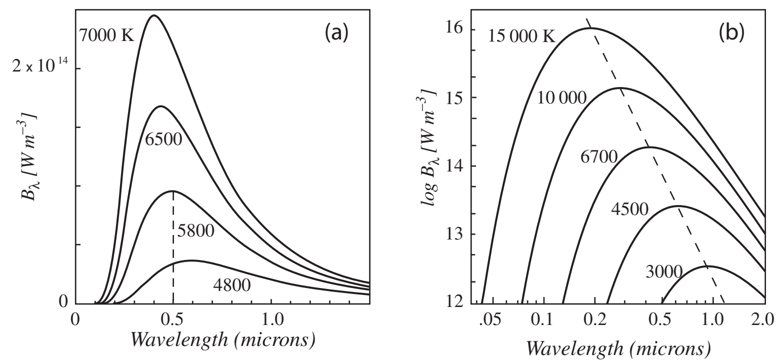


Fig. 1.8 Blackbody spectra $B(\lambda, T)$ for objects at several different surface temperatures. (a) Both axes are linear. The wavelength at which the 5800 K (effective surface temperature of the Sun) spectrum peaks is indicated. (b) Both axes are logarithmic. The dashed line traces the peaks in the spectra.

temperature, following a relation called ***Wien's displacement law*** (1893). Wien's law states that for f_λ

$$T\lambda_{\mathrm{MAX}} = 2.8979 \times 10^{-3} m{\cdot}K \tag{1.16}$$

or equivalently, for f_ν:

$$\frac{T}{\nu_{\mathrm{MAX}}} = 1.7344 \times 10^{-11} Hz^{-1} K \tag{1.17}$$

Thus blackbodies are never black: the color of a glowing object shifts from red to yellow to blue as it is heated.

Max Planck presented the actual functional form for the blackbody spectrum at a Physical Society meeting in Berlin in 1900. His subsequent attempt to supply a theoretical understanding of the empirical ***"Planck function"*** led him to introduce the quantum hypothesis – that energy can only radiate in discrete packets. Later work by Einstein and Bohr eventually showed the significance of this hypothesis as a fundamental principle of quantum mechanics.

The Planck function gives the ***specific intensity***, that is, the monochromatic flux per unit solid angle, usually symbolized as $B(\nu.T)$ or $B_\nu(T)$. The total power emitted by a unit surface blackbody over all angles is just $s(\nu,T) = \pi B(\nu.T)$. The Planck function is

$$B(\nu, T) = \frac{2h\nu^3}{c^2}\left(e^{\left(\frac{h\nu}{kT}\right)} - 1\right)^{-1} \tag{1.18}$$

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\left(\mathrm{e}^{\left(\frac{hc}{\lambda kT}\right)} - 1\right)^{-1} \tag{1.19}$$

For astronomers, this means that we can observe the shape of an object's spectrum and from it deduce the object's temperature (provided the object behaves like a blackbody). Figure 1.8 shows the Planck function for several temperatures. Note that even if the wavelength of the peak of the spectrum cannot be observed, the slope of the spectrum gives a measure of the temperature.

It is useful to note that at long wavelengths or very high temperatures, a blackbody's specific intensity (and therefore angular surface brightness) is directly proportional to its temperature. This is the ***Rayleigh–Jeans approximation*** to the tail of the Planck function

$$B(\lambda, T) = 2\lambda^{-4}ckT \tag{1.20}$$

$$B(\lambda, T) = 2c^{-2}k\nu^2 T \tag{1.21}$$

In radio astronomy, where this approximation holds, and where one often observes extended sources, these equations suggest that observed brightness is a linear measure of the ***brightness temperature*** of a source.

1.4.4 Spectra of stars

When astronomers first examined the great variety in the absorption line spectra of different stars, they did not fully understand what they saw. Flooded with puzzling but presumably significant observations, most scientists have the (good) impulse to look for similarities and patterns: to sort the large number of observations into a small number of classes. Astronomers based their initial sorting of stellar spectra on the overall simplicity of the pattern of lines, assigning the simplest to class A, next simplest to B, and so on through the alphabet. Only after a great number of stars had been so classified from photographs[3] (in the production of the Henry Draper Catalog – see Chapters 4 and 11) did astronomers come to understand, through the new science of astrophysics, that the great variety arises mainly from temperature differences.

There is an important secondary effect due to surface gravity, as well as some subtle effects due to variations in chemical abundance. The chemical differences usually involved only the minor constituents – the elements other than hydrogen and helium.

The spectral type of a star, then, is basically an indication of its ***effective temperature*** – that is, the temperature of the blackbody that would produce the same amount of radiation per unit surface area as the star does. If the spectral type is sufficiently precise, it might also indicate the surface gravity or relative diameter or luminosity (if two stars have the same temperature and mass, the one with the larger diameter has the lower surface gravity as well as the higher luminosity). In order of decreasing effective temperature, the modern spectral classes are:

O ($T > 30\,000$ K)**, B, A, F, G, K, M, L, T, Y** ($T < 500$ K)

[3] Antonia Maury suggested in 1897 that the correct sequence of types should be O through M (the first seven in Table 11.1 – although Maury used a different notation scheme). Annie Cannon, in 1901, justified this order on the basis of continuity, not temperature. Cannon's system solidified the modern notation and was quickly adopted by the astronomical community. In 1921, Megh Nad Saha used atomic theory to explain the Cannon sequence as one of stellar temperature.

The ***spectral type*** of a star consists of three designations: (a) a letter, indicating the general temperature class, (b) a decimal subclass number between 0 and 9.9 refining the temperature estimate (0 indicating the hottest subclass), and (c) a roman numeral indicating the relative surface gravity or luminosity class. Luminosity class I (the ***supergiants***) is most luminous, III (the ***giants***) is intermediate, and V (the ***dwarves***) is least luminous and most common. The Sun has spectral type G2 V. We discuss stellar spectra in greater detail in Chapter 11, and give more detailed data on spectral types in Table 11.1 and in Appendix K.

1.5 Magnitudes

1.5.1 Apparent magnitudes

When Hipparchus of Rhodes (*c.* 190–120 BCE), arguably the greatest astronomer in the Hellenistic school, published his catalog of 600 stars, he included an estimate of the brightness of each – our quantity F. Strictly, what Hipparchus and all visual observers estimate is F_{vis}, the flux in the visual bandpass, the band corresponding to the response of the human eye. The eye has two different response functions, corresponding to two different types of receptor cells – rods and cones. At high levels of illumination, only the cones operate (***photopic vision***), and the eye is relatively sensitive to red light. At low light levels (***scotopic vision***) only the rods operate, and sensitivity shifts to the blue. Greatest sensitivity is at about 555 nm (yellow) for cones and 505 nm (green) for rods. Except for extreme red wavelengths, scotopic vision is more sensitive than photopic and most closely corresponds to the Hipparchus system. See Appendix B3.

Hipparchus cataloged brightness by assigning each star to one of six classes, the first class (or first ***magnitude***) being the brightest, the sixth class the faintest. The choice of six classes, rather than some other number – ten, for example – is a curious one, and may be tied to earlier Babylonian mysticism, which held six to be a significant number. For the next two millennia, astronomers perpetuated this system, eventually extending it to fainter stars at higher magnitudes: magnitudes 7, 8, 9, etc. could only be seen with a telescope. With the introduction of photometers in the nineteenth century, William Pogson (*c.* 1856 CE) discovered that Hipparchus' classes were in fact approximately a geometric progression in F, with each class two or three times fainter than the preceding. Pogson proposed regularizing the system so that ***a magnitude difference of 5 corresponds to a brightness ratio of* 100:1**, a proposal eventually adopted by international agreement early in the twentieth century.

Astronomers who observe in the visual and near infrared persist in using this system. It has advantages: for example, all astronomical bodies have apparent magnitudes that fall in the restricted and easy-to-comprehend range of about

−26 (the Sun) to +32 (the faintest telescopic objects). The faintest magnitude detected with the unaided eye depends critically on sky brightness (and whose eye) but the limit seems to be about 8.0 under the very best conditions.

However, when Hipparchus and Pogson assigned the more positive magnitudes to the fainter objects, they were asking for trouble. Avoid the trouble and remember that *smaller* (more negative) magnitudes mean *brighter* objects. Those who work at other wavelengths are less burdened by tradition and use less confusing (but sometimes less convenient) units. Such units linearly relate to the apparent brightness, F, or to the monochromatic brightness, f_λ. In radio astronomy, for example, one often encounters the ***Jansky*** (1 Jy = 10^{-26} W m^{-2} Hz^{-1}) as a unit for f_ν.

The relationship between apparent magnitude, m, and brightness, F, is:

$$m = -2.5\log_{10}(F) + K \tag{1.22}$$

The constant K is often chosen so that modern measurements agree, more or less, with the older catalogs, all the way back to Hipparchus. For example, the bright star, Vega, has $m \approx 0$ in modern magnitude systems. If the flux in Equation (1.22) is the total or bolometric flux (see Section 1.4) then the magnitude defined is called the ***apparent bolometric magnitude***, m_{bol}. Most practical measurements are made in a restricted bandpass, but the definition of such a bandpass magnitude remains as in Equation (1.22), even to the extent that K is often (not always) chosen so that Vega has $m \approx 0$ in any band. This standardization has many practical advantages but is potentially confusing, since the integral in Equation (1.14) will be very different in different bands for Vega (see Figure 1.5) or any other star. In most practical systems, the constant K is specified by defining the values of m for some set of ***standard stars***. Absolute calibration of such a system, so that magnitudes can be converted into energy units, requires comparison of at least one of the standard stars to a source of known brightness, like a blackbody at a known, stable temperature.

Equation (1.22) implies the *magnitude difference* between two sources is

$$\Delta m = m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right) \tag{1.23}$$

This equation holds for both bolometric and bandpass magnitudes. It should be clear from Equation (1.23) that once you define the magnitudes of a set of standard stars, measuring the magnitude of an unknown is a matter of measuring a flux ratio – or magnitude difference – between the standard and the unknown. Magnitudes are almost always measured in this ***differential*** fashion without resorting to absolute energy units. Inverting Equation (1.23) gives the flux ratio as a function of magnitude difference:

$$\frac{F_1}{F_2} = 10^{-0.4(m_1 - m_2)} = 10^{-0.4\Delta m} \tag{1.24}$$

A word about notation: You can write magnitudes measured in a bandpass in two ways, by (1) using the bandpass name as a subscript to the letter "m", or (2) by using the name itself as the symbol. So for example, the B band apparent magnitudes of a certain star could be written as $m_B = 5.67$ or as $B = 5.67$.

1.5.2 Absolute magnitudes

The magnitude system can also be used to express the luminosity of a source. The ***absolute magnitude*** of a source is defined to be the apparent magnitude (either bolometric or bandpass) that the source would have if it were at the standard distance of 10 parsecs in empty space. (1 parsec = 3.086×10^{16} m = 206 265 au – see Chapter 3). The relation between the apparent and absolute magnitudes of the same object is

$$m - M = 5\log r - 5 \tag{1.25}$$

where M is the absolute magnitude and r is the actual distance to the source in parsecs. The quantity $(m - M)$ on the left-hand side of Equation (1.25) is called the ***distance modulus*** of the source. You should recognize this equation as the equivalent of the inverse square law relation between apparent brightness and luminosity. Equation (1.25) must be modified if the source is not isotropic or if there is absorption along the path between it and the observer.

To symbolize the absolute magnitude in a bandpass, use the band name as a subscript to the symbol M. The Sun, for example, has absolute magnitudes: $M_B = 5.515$, $M_v = 4.862$. The International Astronomical Union (IAU) has defined the bolometric absolute magnitude scale so that $M_{\mathrm{bol}} = 0$ corresponds to $L = 3.055 \times 10^{28}$ W = $79.8\ L_\odot$. This implies that for the Sun, $M_{\mathrm{bol}} = 4.756$

1.5.3 The bolometric correction

For a particular source in any magnitude system, the difference between the bolometric magnitude (either apparent or absolute) and the magnitude in a particular band is termed the ***bolometric correction***. The bolometric correction is usually tabulated for the V band

$$BC = M_{\mathrm{bol}} - M_v \tag{1.26}$$

and is a strong function of spectral type. Some authors vary in their definition of the BC, but the values tabulated in Appendix K are consistent with the *IAU* definition of the absolute magnitude zero point, which implies a V-band BC for the Sun of -0.11.

1.5.4 Apparent magnitudes from images

In Chapter 10 we will consider in detail how to measure the apparent magnitude of a source. Right now, it is helpful to have at least a simplified description.

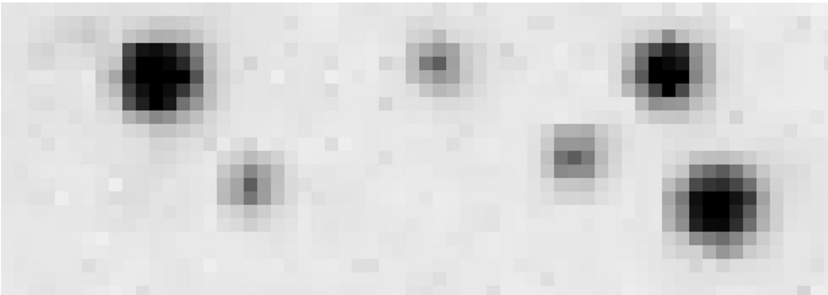


Fig. 1.9 A digital image. The width of each star image, despite appearances, is the same. Brighter stars have a higher peak and half-peak gray level, but every star image has the same width at the half-peak level.

Imagine a detector similar to the sensor in a black-and-white digital camera. Attach this to a telescope and take a picture.

Our picture, shown in Figure 1.9, is a visual light image of a few stars, and is composed of a grid of many little square elements called *pic*ture *el*ements or ***pixels***. Each pixel corresponds to a detector in the sensor, and stores a number that is proportional to the energy that reaches that small detector during the exposure. Figure 1.9 displays these data by mimicking a photographic negative: each pixel location is painted a shade of gray, with the pixels that store the largest numbers painted darkest.

The image does *not* show the surfaces of the stars. The images of the stars are most intense in the center and fade out over several pixels. Several effects cause this – the finite resolving power of any lens, turbulence in the Earth's atmosphere, scattering of photons by particles in the air, or the scattering of photons from pixel to pixel within the detector itself. The apparent "diameter" of a star image in the picture has to do with the strength of the blurring effect and the choice of gray-scale mapping – not with the physical size of the star. The size of every star image is actually the same (about 4 pixels) when scaled by its peak brightness.

Suppose we manage to take a single picture with our camera that records an image of the star Vega (or some other standard star) as well as that of a star whose brightness we wish to measure. To compute the brightness of any star, we can add up the energies it deposits in each pixel. If E_{xy} is the energy recorded in pixel x, y due to light rays *from the star*, then the brightness of the star will be

$$F = \frac{1}{tA} \sum_{x,y}^{image} E_{xy} \tag{1.27}$$

where t is the exposure time in seconds, A is the area of the telescope lens or mirror, and the sum is understood to include only pixels in the star image. Measuring F is just a matter of adding up those E_{xy} s.

Of course things are not quite so simple. One problem is that the detector cannot distinguish between light rays coming from the star and light rays coming from any other source in the same general direction. A faint star or galaxy nearly in the same line of sight as the star, or a moonlit terrestrial dust grain or air molecule floating in front of the star, can make unwelcome additions to the signal. All such sources contribute *background* light rays that reach the same pixels as the star's light. In addition, the detector itself may contribute a background that has nothing to do with the sky. Therefore, the actual signal we measure in pixel x, y, will be the sum of the signal from the star, E_{xy}, and that from the background, B_{xy}, or

$$S_{xy} = E_{xy} + B_{xy} \tag{1.28}$$

The task then is to determine B_{xy} so we can subtract it from each S_{xy}. You can do this by measuring the energy reaching some pixels near but not within a star image, and taking some appropriate average (call it B). Then assume that everywhere in the star image, B_{xy} is equal to B. Sometimes this is a good assumption, sometimes not so good. Granting the assumption, then the brightness of the star is

$$F = \frac{1}{tA} \sum_{x,y}^{image} \left[S_{xy} - B\right] \tag{1.29}$$

If we acknowledge that the estimated backgrounds for the two might be different, then the apparent magnitude difference between the unknown star and the standard star is

$$m_{star} - m_{std} = -2.5\log_{10}\frac{F_{star}}{F_{std}} = -2.5\log_{10}\left\{\frac{\sum_{x,y}^{star}\left[S_{xy} - B_{star}\right]}{\sum_{x,y}^{std}\left[S_{xy} - B_{std}\right]}\right\} \tag{1.30}$$

Notice that the exposure time t, and the area, A, cancel on the right side of Equation (1.30). Also notice that since a ratio is involved, the pixels need not record units of energy – anything proportional to energy will do. Since scaled star images are the same size, the sums in Equation (1.30) should contain the same number of pixels.

The operation described in Equation (1.30) is called ***differential photometry.*** Because they appear in the same picture, the images of both the standard and the unknown star are subject to identical effects: atmospheric transparency, exposure time, telescope condition, sensor temperature, etc.

Differential photometry is only possible if a standard and unknown can be recorded on the same image. The alternative, ***all-sky photometry,*** requires two images, one of the unknown, the second of a standard star, with conditions as similar as possible in the two exposures. Ground-based all-sky photometry can be difficult in the optical-IR window, because you must look through different paths in the air to observe the standard and program stars. Any variability in the atmosphere (e.g. clouds) defeats the technique. In the radio window, clouds are less problematic and the all-sky technique is usually appropriate. In space, of course, there are no atmospheric effects, and all-sky photometry is usually appropriate at every wavelength.

1.5.5 Example problem

The bright star Betelgeuse is 150 pc from the Sun. It is a massive red supergiant that will probably end its evolution in a supernova explosion of type II. These supernovae, when brightest, reach a bolometric luminosity of 6×10^{35} W, with a spectrum similar to a blackbody of effective temperature 15 000 K. Assume Betelgeuse explodes as a supernova with these characteristics. At maximum light compute (a) its absolute bolometric magnitude, (b) its physical radius (assume a sphere) in solar units, and (c) its apparent visual magnitude, m_V, as seen from Earth. Compare the latter with the apparent brightness of the full moon (−12.74) and Venus (−4.6 at brightest). Assume the bolometric correction in the V band at maximum light is −1.3.

(a) Since we are given the bolometric luminosity, we can compute the absolute magnitude by comparing the supernova with the IAU standard for $M_{bol} = 0$. Or, since we eventually want to compare the SN with the Sun

$$M_{\text{bol}} - M_{\text{sun}} = -2.5 \log\left[\frac{L}{L_{\text{sun}}}\right] = -2.5 \log\left[\frac{6 \times 10^{35}\ \text{W}}{3.8 \times 10^{26}\ \text{W}}\right] = -23.0 \tag{1.31}$$

$$M_{\text{bol}} = -23 + 4.76 = -18.24$$

(b) The effective temperature is that of a blackbody of the same size and luminosity, so that by Stefan's Law: $L = (\text{surface area})\sigma T_{\text{eff}}^4 = 4\pi R_{sn}^2 \sigma T_{\text{eff}}^4$. Again, it is helpful to compare the luminosities of the supernova and the Sun (T_{sun} = 5800 K):

$$\frac{L}{L_{\text{sun}}} = \frac{R_{\text{sn}}^2 T_{\text{eff}}^4}{R_{\text{sun}}^2 T_{\text{sun}}^4}$$

$$R_{\text{sn}} = \left[\frac{T_{\text{sun}}}{T_{\text{sn}}}\right]^2 \left[\frac{L}{L_{\text{sun}}}\right]^{\frac{1}{2}} R_{\text{sun}} = \left[\frac{15}{5.8}\right]^2 (1.58 \times 10^9)^{\frac{1}{2}} R_{\text{sun}}$$

$$R_{\text{sn}} = 2.7 \times 10^5 R_{\text{sun}} = 1.85 \times 10^{11}\text{km} = 1200\ \text{au}$$

(c) The absolute visual magnitude of the supernova follows from the bolometric correction: $M_v = M_{\text{bol}} - BC = -16.9$, and the apparent visual magnitude follows from the distance modulus:

$$m_v = 5\log r - 5 + M_v = 5.88 - 16.9 = -11.0$$

This is 2.76 magnitudes fainter than the full moon, (a factor of 12.7 in brightness) and 6.4 magnitudes (a factor of 360) brighter than Venus.

Summary

- The wave theory formulated by Maxwell envisions light as a self-propagating disturbance in the electric and magnetic fields. A light wave is characterized by wavelength or frequency and exhibits the properties of diffraction and interference. Concepts:
  $\lambda = c/\nu$ *electromagnetic spectrum*
  Radio, microwave, infrared, visible, ultraviolet, X-ray, gamma ray
  polarization *phase* *amplitude*
- Quantum mechanics improves on the wave theory and describes light as a stream of photons, massless particles that can exhibit wave properties. Concept:
  energy of a photon: $E = h\nu$
- A simple but useful description of light postulates a set of geometric rays that carry luminous energy. Concepts:
  luminosity *apparent brightness = flux = irradiance*
  inverse square law $F = L/(4\pi r^2)$
- The surface brightness of a resolved source is invariant with distance. Concepts:
  radiant exitance *brightness per unit solid angle*
- The spectrum of an object gives its brightness as a function of wavelength or frequency. Concepts:
  monochromatic flux *high- (or low-) resolution spectrum*
  flux within a band *standard bands*
  UBVRI(JHKLMNQ)
- The Kirchhoff–Bunsen rules specify the circumstances under which an object produces an emission line, absorption line, or continuous spectrum.
- Line spectra contain information about (among other things) chemical composition. However, although based on patterns of absorption lines, the spectral types of stars depend primarily on stellar temperatures. Concepts:
  OBAFGKMLTY *luminosity class*
- A blackbody emits a continuous spectrum whose shape depends only on its temperature as described by Planck's Law. Concepts:
  Wien's law: $T\lambda_{\mathrm{MAX}} = 2.8979 \times 10^{-3}\,\mathrm{m{\cdot}K}$
  Stefan–Boltzmann law: $s = \sigma T^4$
- The astronomical magnitude system uses apparent and absolute magnitudes to quantify brightness measurements on a logarithmic scale. Concepts:
  $$\Delta m = m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$$
  distance modulus $m - M = 5\log r - 5$
  parsec *bolometric magnitude*
  differential photometry *all-sky photometry*

Exercises

A bear, however hard he tries, gets tubby without exercise.

– A.A. Milne, *Winnie the Pooh*, 1926

1. Propose a definition of astronomy that distinguishes it from other sciences like physics and geology.
2. (a) What wavelength photon would you need to ionize a hydrogen atom (ionization energy = 13.6 eV)? (b) Compute the temperature of the blackbody whose spectrum peaks at the wavelength you found in (a).
3. What are the units of the monochromatic brightness, f_λ?
4. What is the value of the ratio f_λ/f_ν for any source?
5. Consider an eclipsing binary star system viewed by a terrestrial observer located in the orbital plane of the two stars. The orbit is so small that the observer sees this system as a single object of apparent brightness, F. The larger star of the pair has a radius $R = 1$ and effective temperature T_1, the smaller has radius a and temperature T_2. Because of mutual eclipses, F is a periodic function of time called the light curve, $F(t)$. The orbit is circular. There are two brightness minima per orbit. (a) Regardless of the value of a, the primary (deeper) minimum always occurs when the hotter star is behind the cooler. Explain why. (b) Assume both stars are Lambertian (i.e. appear, in projection, to be uniform disks). If F_{max} is the maximum brightness of the system, show that the relative brightness at the two minima of the light curve are:

   $$\frac{F_1}{F_{max}} = \frac{1}{1+b}, \text{ when the smaller star is eclipsed by the larger}$$

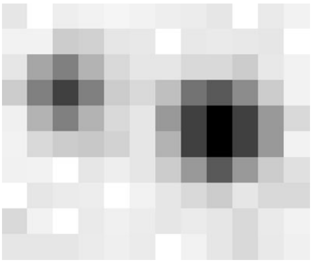
   $$\frac{F_2}{F_{max}} = 1 - \frac{a^2}{1+b}, \text{ when the smaller transits in front of the larger}$$

   Here, $b = a^2\left(\frac{T_2}{T_1}\right)^4$
6. A certain radio source has a monochromatic flux density of 1 Jy at a frequency of 1 MHz. What is the corresponding flux density in photon number? (How many photons arrive per m^2 in one second with frequencies between 1 000 000 Hz and 1 000 001 Hz?)
7. The bolometric flux from a star with $m_{bol} = 0$ is about 2.65×10^{-8} W m^{-2} outside the Earth's atmosphere. Compute the value of the constant K in Equation (1.22) for bolometric magnitudes.
8. The monochromatic flux at the center of the B bandpass (440 nm) for a certain star is 375 Jy. (a) If this star has a blue magnitude of $m_B = 4.71$, what is the monochromatic flux, in Jy, at 440 nm for star X which has $m_B = 22.5$? (b) If the width of the B band is 2.5×10^{14} Hz, about how many photons from star X can be collected in 100 seconds by a telescope with a collecting area of 5 square meters?
9. A double star has two components of equal brightness, each with a magnitude of 8.34. If these stars are so close together that they appear to be one object, what is the apparent magnitude of the combined object?

10. A uniform gaseous nebula has an average surface brightness of 17.77 magnitudes per square second of arc.
    (a) If the nebula has an angular area of 144 square arcsec, what is its total apparent magnitude?
    (b) If the nebula were moved to twice its original distance, what would happen to its angular area, total apparent magnitude, and surface brightness?
11. Assume the nebula in 10(a) were moved to 100 times its original distance. If you observed this object with a telescope whose resolving power is 1.2 arc seconds (i.e. even point sources have an apparent diameter of 1.2 arc seconds), what would be the apparent angular area and apparent surface brightness of the nebula?
12. At maximum light, the brightest Type Ia supernovae are believed to have an absolute visual magnitude of -19.60. A supernova in the Pigpen Galaxy is observed to reach apparent visual magnitude of 13.25 at its brightest. (a) Compute the distance to the Pigpen Galaxy. (b) If you believe the Pigpen Galaxy contains clouds of dust that absorb 1.5 magnitudes of light in the V band, re-compute the distance to the galaxy.
13. Derive the distance modulus relation in Equation (1.25) from the inverse square law relation in Equation (1.6).
14. Show that, for small values of Δm, the difference in magnitude is approximately equal to the fractional difference in brightness, that is

$$\Delta m \approx \frac{\Delta F}{F} \tag{1.32}$$

    Hint: consider the derivative of m with respect to F.
15. An astronomer is performing synthetic aperture photometry on a single unknown star and standard star (review Section 1.5) in the same field. The data frame is in the figure below. The unknown star is the fainter one. If the magnitude of the standard is 9.000, compute the magnitude of the unknown. Actual data numbers are listed for the frame in the table. Assume these are proportional to the number of photons counted in each pixel, and that the bandpass is narrow enough that all photons can be assumed to have the same energy. Remember that photometrically both star images have the same size.



34	16	26	33	37	22	25	25	29	19	28	25
22	20	44	34	22	26	14	30	30	20	19	17
31	70	98	66	37	25	35	36	39	39	23	20
34	99	229	107	38	28	46	102	159	93	37	22
33	67	103	67	36	32	69	240	393	248	69	30
22	33	34	29	36	24	65	241	363	244	68	24
28	22	17	16	32	24	46	85	157	84	42	22
18	25	27	26	17	18	30	29	35	24	30	27
32	23	16	29	25	24	30	28	20	35	22	23
28	28	28	24	26	26	17	19	30	35	30	26

Chapter 2
Uncertainty

Errare humanum est.

– Anonymous Latin saying

Upon foundations of evidence, astronomers erect splendid narratives about the lives of stars, the prevalence of habitable planets, or the fate of the universe. Inaccurate or imprecise evidence weakens the foundation and imperils the astronomical story it supports. Incorrect ideas and theories are vital to science, which normally works by proving many, many ideas to be wrong until only one remains. Wrong data, on the other hand, are deadly.

As an astronomer you need to know how far to trust the data you have, or how much observing you need to do to achieve a particular level of trust. This chapter describes the formal distinction between accuracy and precision in measurement, and methods for estimating both. It then introduces the concepts of a population, a sample of a population, and the statistical descriptions of each. Any characteristic of a population (e.g. the masses of stars) can be described by a probability distribution (e.g. low-mass stars are more probable than high-mass stars), so we next will consider a few probability distributions important in astronomical measurements. Finally, armed with new statistical expertise, we revisit the question of estimating uncertainty.

2.1 Accuracy and precision

In common speech, we often do not distinguish between these two terms, but we will see that establishing a clear distinction is useful. An example will help.

2.1.1 An example

In the distant future, a very smart theoretical astrophysicist determines that the star Malificus might soon implode to form a black hole, and in the process destroy all life on its two inhabited planets. Careful computations show that if Malificus is fainter than magnitude $14.190 \pm .003$ by July 24, as seen from the observing station orbiting Pluto, then the implosion will not take place and its

Table 2.1 *Results of trials by four astronomers. The values for σ and s are computed from Equations (2.6) and (2.1), respectively.*

Astronomer	A	B	C	D
Trial 1	14.115	14.495	14.386	14.2
Trial 2	14.073	14.559	14.322	14.2
Trial 3	14.137	14.566	14.187	14.2
Trial 4	14.161	14.537	14.085	14.2
Trial 5	14.109	14.503	13.970	14.2
Mean	14.119	14.532	14.190	14.2
Deviation from truth	−0.004	+0.409	+0.067	+0.077
Spread	0.088	0.071	0.418	0
s	0.033	0.032	0.174	0
σ	0.029	0.029	0.156	0
Uncertainty of the mean	0.013	0.013	0.070	(0.05)
Decision	Evacuate	Stay	Uncertain	Uncertain
Accuracy?	accurate	inaccurate	accurate	inaccurate
Precision?	precise	precise	imprecise	imprecise

planets will be spared. The Galactic government is prepared to spend the ten thousand trillion dollars necessary to construct a wormhole and evacuate the doomed populations but needs to know if the effort is really called for; it funds some astronomical research. Four astronomers and a demigod each set up experiments on the Pluto station to measure the apparent magnitude of Malificus.

The demigod performs photometry with divine perfection, obtaining a result of 14.123 010 (all the remaining digits are zeros). The truth, therefore, is that Malificus is brighter than the limit and will implode. The four astronomers, in contrast, are only human, and, fearing error, repeat their measurements – five times each. I'll refer to a single one of these five as a ***trial.*** Table 2.1 lists the results of each trial, and Figure 2.1 illustrates them.

2.1.2 Accuracy and systematic error

In our example, we are fortunate a demigod participates, so we feel perfectly confident to tell the government, sorry, Malificus is doomed, and those additional taxes are necessary. The ***accuracy*** of a measurement describes (usually numerically) how close it is to the "true" value. The demigod measures with perfect accuracy.

What is the accuracy of the human results? First, decide what we mean by "a result": Since each astronomer made five trials, we choose a single value that

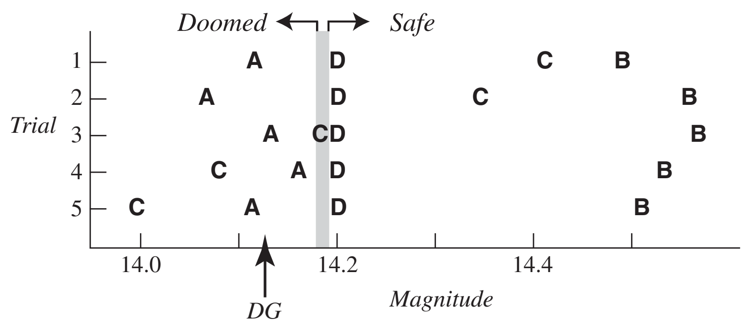


Fig. 2.1 Apparent magnitude measurements. The arrow points to the demigod's result, which is the true value. The thick gray line marks the critical limit and its uncertainty.

summarizes these five measurements. In this example, each astronomer chooses to compute the mean – or average – of the five, a reasonable choice. (We will see there are others.) Table 2.1 lists the mean values from each astronomer – a ***statistic*** that summarizes the five trials each has made.

Since we know how much each result deviates from the truth, we could express its accuracy with a sentence like: "The result of Astronomer A is 0.004 magnitude smaller than the true value." This statement is easy to make if a demigod tells us the truth, but in the real universe, how could you determine the "true" value, and hence the accuracy? In science, after all, the whole point is to discover values that are unknown at the start, and (the self-images of some astronomers notwithstanding) no demigods work at observatories.

How, then, can we judge accuracy? The alternative to divinity is variety. We can only repeat measurements using different devices, assumptions, strategies, and observers, and then check for general agreements (and disagreements) among the results. We suspect a particular set-up of inaccuracy if it disagrees with all other experiments. For example, the result of Astronomer B differs appreciably from those of his colleagues. Even in the absence of the demigod result, we would suspect that B's result is the least accurate.

If a particular set-up always produces consistent inaccuracies, if its result is always biased by about the same amount, then we say it produces a ***systematic error***. Although Astronomer B's trials do not have identical outcomes, they all tend to be much too large, and are, we suspect, subject to a systematic error of around +0.4 magnitude. Systematic errors are due to some instrumental or procedural fault, or some mistake in modeling the phenomena under investigation. Astronomer B, for example, used the wrong magnitude for the standard star in his measurements. He could not improve his measurement just by repeating it – making more trials would give the same general result, and B would continue to recommend against evacuation.

In a second example of inaccuracy, suppose the astrophysicist who computed the critical value of $V = 14.190$ had made a mistake because he neglected the