MECHANICS OF MATERIALS

ELEVENTH EDITION

R. C. HIBBELER

Pearson

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PREFACE

It is intended that this book provide the student with a clear and thorough presentation of the theory and application of the principles of mechanics of materials. To achieve this objective, over the years this work has been shaped by the comments and suggestions of hundreds of reviewers in the teaching profession, as well as many of the author's students. The eleventh edition has been significantly enhanced from the previous edition, and it is hoped that both the instructor and student will benefit greatly from these improvements.

New to this Edition

- **Expanded Answer Section.** The answer section in the back of the book now includes additional information related to the solution of select Fundamental Problem in order to offer the student some guidance in solving the problems.

- **Re-writing of Text Material.** Further clarification of some concepts has been included in this edition, and throughout the book the accuracy has been enhanced, and important definitions are now in boldface throughout the book to highlight their importance.

- **New Photos.** The relevance of knowing the subject matter is reflected by the real-world applications depicted in the over 14 new or updated photos placed throughout the book. These photos generally are used to explain how the relevant principles apply to real-world situations and how materials behave under load.

- **New Problems.** There are approximately 30% new problems that have been added to this edition, which involve applications to many different fields of engineering.

- **New Videos.** Three types of videos are available that are designed to enhance the most important material in the book. Lecture videos serve to test the student's ability to understand the concepts. Example problem videos are intended to review these problems, and fundamental problem videos guide the student in solving these problems that are in the book.

Contents

The subject matter is organized into 14 chapters. Chapter 1 begins with a review of the important concepts of statics, followed by a formal definition of both normal and shear stress, and a discussion of normal stress in axially loaded members and average shear stress caused by direct shear.

In Chapter 2 normal and shear strain are defined, and in Chapter 3 a discussion of some of the important mechanical properties of materials is given. Separate treatments of axial load, torsion, and bending are presented in Chapters 4, 5, and 6, respectively. In each of these chapters, both linear-elastic and plastic behavior of the material covered in the previous chapters, where the state of stress results from combined loadings. In Chapter 9 the concepts for transforming multiaxial states of stress are presented. In a similar manner, Chapter 10 discusses the methods for strain transformation, including the application of various theories of failure. Chapter 11 provides a means for a further summary and review of previous material by covering design applications of beams and shafts. In Chapter 12 various methods for computing deflections of beams and shafts are covered. Also included is a discussion for finding the reactions on these members if they are statically indeterminate. Chapter 13 provides a discussion of column buckling, and lastly, in Chapter 14 the problem of impact and the application of various energy methods for computing deflections are considered.

Sections of the book that contain more advanced material are indicated by a star (*). Time permitting, some of these topics may be included in the course. Furthermore, this material provides a suitable reference for basic principles when it is covered in other courses, and it can be used as a basis for assigning special projects.

Alternative Method of Coverage. Some instructors prefer to cover stress and strain transformations *first,* before discussing specific applications of axial load, torsion, bending, and shear. One possible method for doing this would be first to cover stress and its transformation, Chapter 1 and Chapter 9, followed by strain and its transformation, Chapter 2 and the first part of Chapter 10. The discussion and example problems in these later chapters have been styled so that this is possible. Also, the problem sets have been subdivided so that this material can be covered without prior knowledge of the intervening chapters. Chapters 3 through 8 can then be covered with no loss in continuity.

Hallmark Elements

Organization and Approach. The contents of each chapter are organized into well-defined sections that contain an explanation of specific topics, illustrative example problems, and a set of homework problems. The topics within each section are placed into subgroups defined by titles. The purpose of this is to present a structured method for introducing each new definition or concept and to make the book convenient for later reference and review.

Chapter Contents. Each chapter begins with a full-page illustration that indicates a broad-range application of the material within the chapter. The "Chapter Objectives" are then provided to give a general overview of the material that will be covered.

Procedures for Analysis. Found after many of the sections of the book, this unique feature provides the student with a logical and orderly method to follow when applying the theory. The example problems are solved using this outlined method in order to clarify its numerical application. It is to be understood, however, that once the relevant principles have been mastered and enough confidence and judgment have been obtained, the student can then develop his or her own procedures for solving problems.

Photographs. Many photographs are used throughout the book to enhance conceptual understanding and explain how the principles of mechanics of materials apply to real-world situations.

Important Points. This feature provides a review or summary of the most important concepts in a section and highlights the most significant points that should be realized when applying the theory to solve problems.

Example Problems. All the example problems are presented in a concise manner and in a style that is easy to understand.

Homework Problems. Apart from of the preliminary, fundamental, and conceptual problems, there are numerous standard problems in the book that depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate the student's interest in the subject and provide a means for developing the skill to reduce any such problem from its physical description to a model or a symbolic representation to which principles may be applied. Throughout the book there is an approximate balance of problems using either SI or FPS units. Furthermore, in any set, an attempt has been made to arrange the problems in order of increasing difficulty. The answers to all but every fourth problem are listed in the back of the book. To alert the user to a problem without a reported answer, an asterisk (*) is placed before the problem number. Answers are reported to three significant figures, even though the data for material properties may be known with less accuracy. Although this might appear to be a poor practice, it is done simply to be consistent, and to allow the student a better chance to validate his or her solution.

Appendices. The appendices of the book provide a source for review and a listing of tabular data. Appendix A provides information on the centroid and the moment of inertia of an area. Appendices B and C list tabular data for structural shapes, and the deflection and slopes of various types of beams and shafts.

Accuracy. As with the previous editions, apart from the author, the accuracy of the text and problem solutions has been thoroughly checked in part by Kai Beng Yap, a practicing engineer, and a team of specialists at EPAM, including Georgii Kolobov, Ekaterina Radchenko, and Artur Akberov.

Acknowledgments

Over the years, this book has been shaped by the suggestions and comments of many of my colleagues in the teaching profession. Their encouragement and willingness to provide constructive criticism are very much appreciated and it is hoped that they will accept this anonymous recognition. There are a few people, however, that I feel deserve particular mention. They are S. Ahmad, A. Asgharatal, K. Dennehy, A. Lutz, M. Walter, and M. Zhang. A special note of thanks also goes to Jun Hwa Lee, who provided a careful reading of the manuscript, and checked many of the problems. Through the years, however, Kai Beng Yap supported me in this regard, but unfortunately his support has come to an end, due to his untimely passing. His contribution to this effort, and his friendship will be deeply missed.

During the production process I am also thankful for the assistance of Rose Kernan, my production editor for many years, and to my wife, Conny, for her help in proofreading of the manuscript during production.

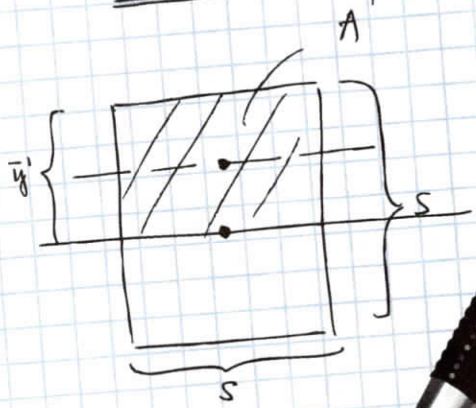
Finally, I would also like to thank all my students who have used the previous edition and have made comments to improve its contents.

I would greatly appreciate hearing from you if at any time you have any comments or suggestions regarding the contents of this edition.

Russell Charles Hibbeler
hibbeler@bellsouth.net

your work...

PART A



$s = 6.75 \text{ in}$

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$Q = \bar{y}'A' = 3.375 \text{ in} \times 22.8 \text{ in}^2 = 76.9 \text{ in}^3$

$\boxed{Q = 76.9 \text{ in}^3}$

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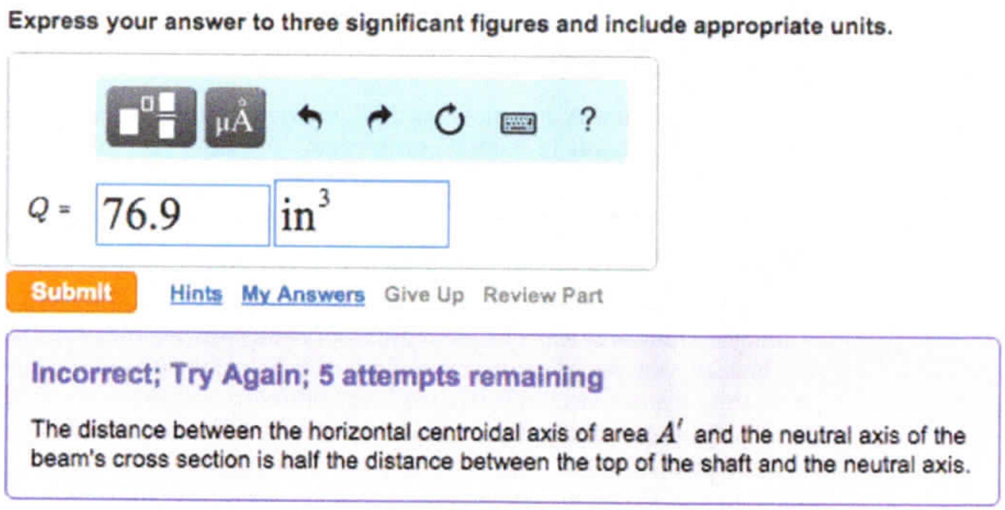

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PowerPoint Slides A complete set of all the figures and tables from the textbook are available in PowerPoint format.

Resources for Students

Videos Developed by the author, three different types of videos are now available to reinforce learning the basic theory and applying the principles. The first set provides a lecture review and a self-test of the material related to the theory and concepts presented in the book. The second set provides a self-test of the example problems and the basic procedures used for their solution. And the third set provides an engagement for solving the Fundamental Problems throughout the book. For more information on how to access these videos visit www.pearson.com/hibbeler.

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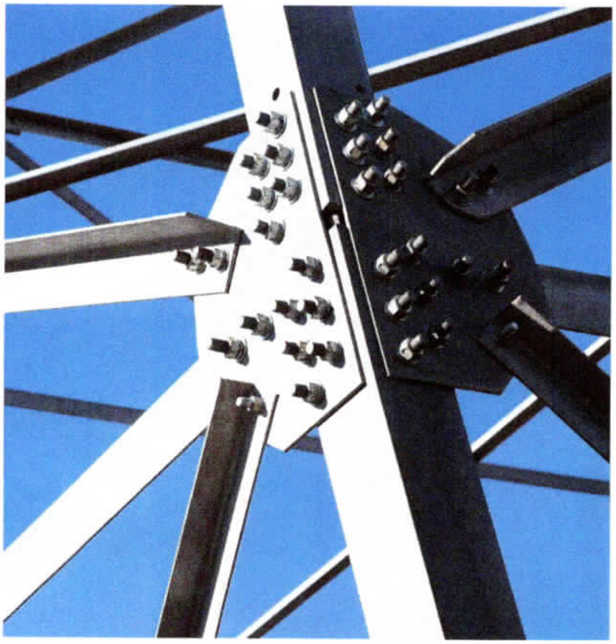

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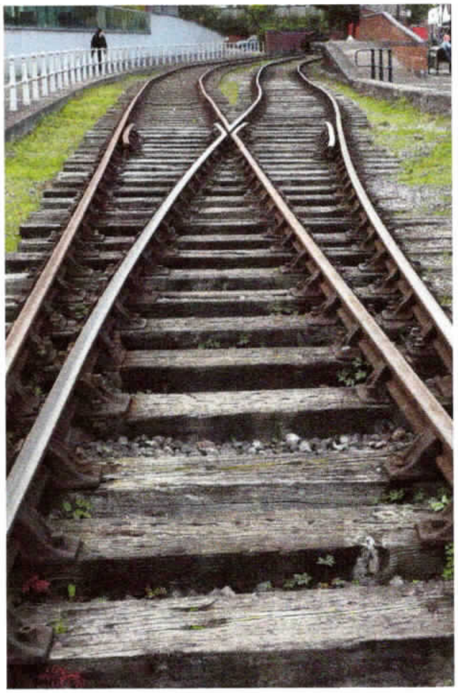

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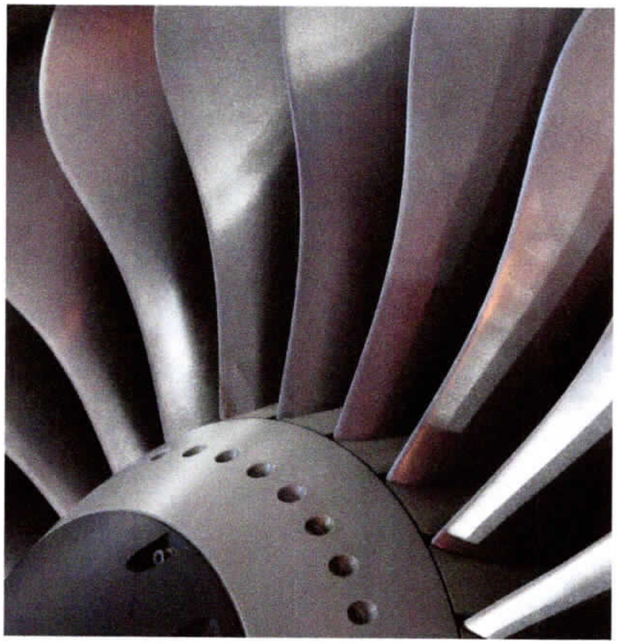

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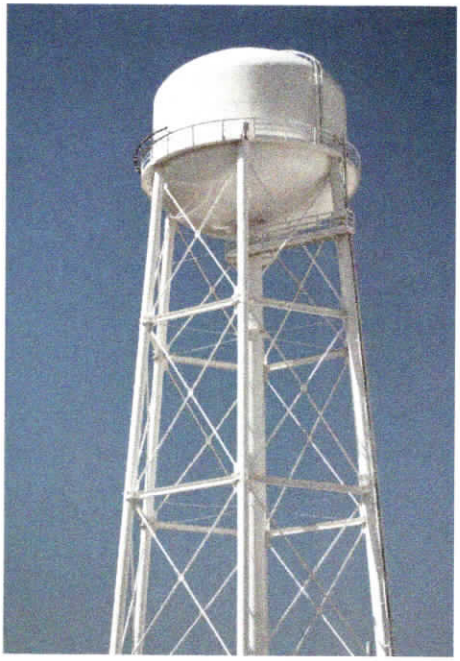

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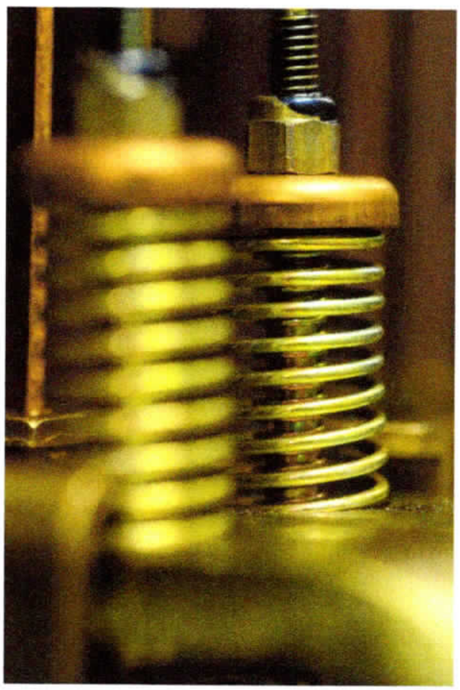

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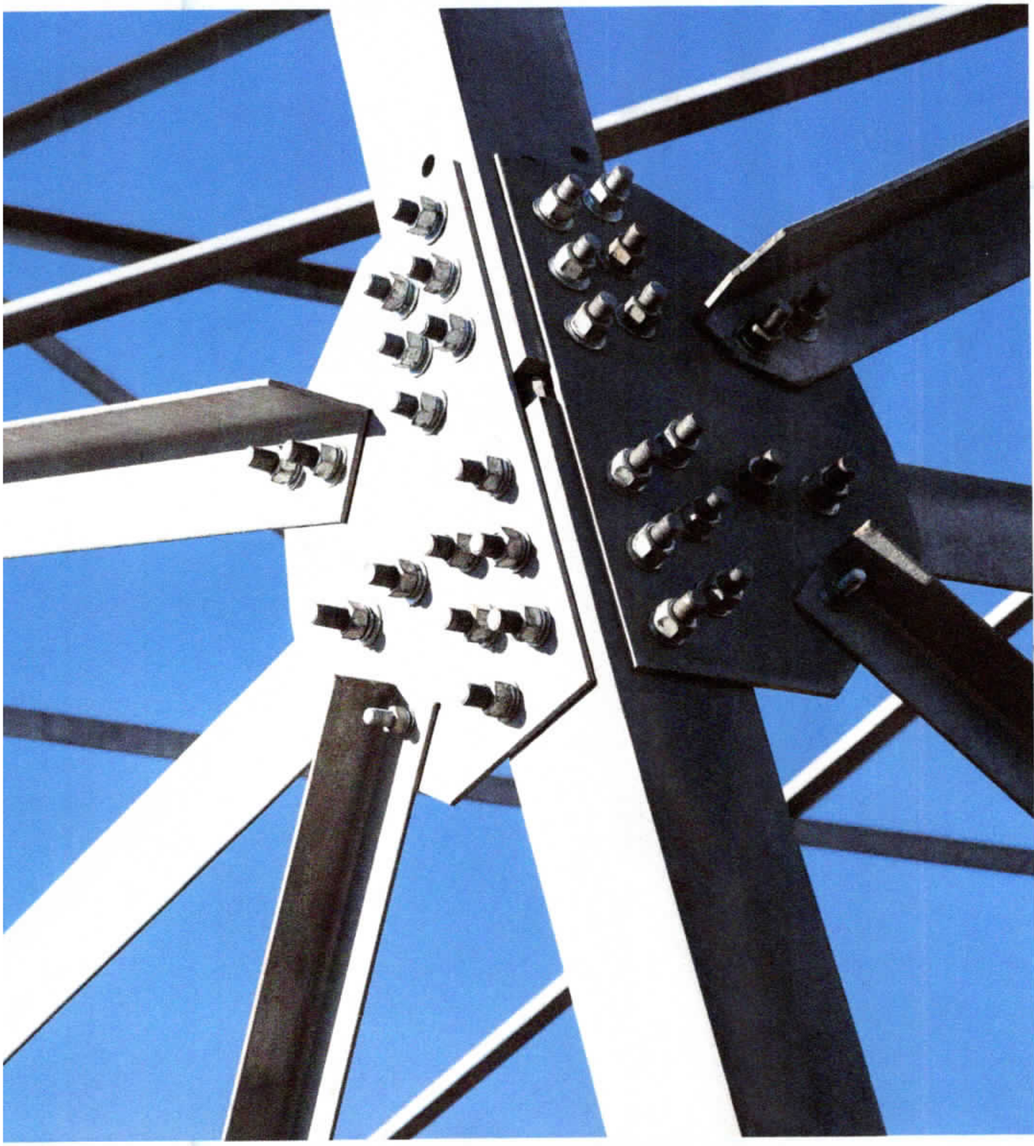
MECHANICS OF MATERIALS

ELEVENTH EDITION

CHAPTER 1



The bolts used for the connections of this steel framework are subjected to stress. In this chapter we will discuss how engineers design these connections and their fasteners.

STRESS

CHAPTER OBJECTIVES

- In this chapter we will review some of the important principles of statics and show how they are used to determine the internal loadings in a body. Afterwards the concepts of normal and shear stress will be introduced, and applications of the analysis and design of members subjected to an axial load or direct shear will be discussed.

1.1 INTRODUCTION

Mechanics of materials is a branch of mechanics that studies the internal effects of stress and strain in a solid body. Stress is the result of internal loading and so it is related to the strength of the material, while strain is a measure of the deformation produced by the internal loadings. A thorough understanding of the fundamentals of this subject is of vital importance for the design of any machine or structure, because many of the formulas and rules of design cited in engineering codes are based upon the principles of this subject.

1

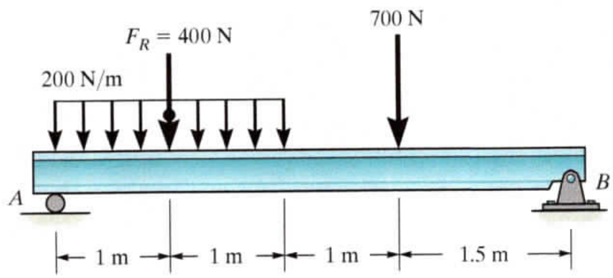
Historical Development. The origin of mechanics of materials dates back to the beginning of the seventeenth century, when Galileo Galilei performed experiments to study the effects of loads on rods and beams made of various materials. However, it was not until the beginning of the nineteenth century when experimental methods for testing materials were vastly improved. At that time many experimental and theoretical studies in this subject were undertaken, primarily in France, by such notables as Saint-Venant, Poisson, Lamé, and Navier.

Through the years, after many fundamental problems had been solved, it became necessary to use advanced mathematical and computer techniques to solve more complex problems. As a result, mechanics of materials has expanded into other areas of mechanics, such as the *theory of elasticity* and the *theory of plasticity*.

1.2 EQUILIBRIUM OF A DEFORMABLE BODY

Since statics plays an important role in both the development and application of mechanics of materials, it is very important to have a good understanding of its fundamentals. For this reason we will now review some of the main principles of statics that will be used throughout the book.

Loads. A body can be subjected to both surface loads and body forces. ***Surface loads*** that act on a small area of contact are reported by ***concentrated forces***, while ***distributed loadings*** act over a larger surface area of the body. When the loading is coplanar, as in Fig. 1–1*a*, then a resultant force $\mathbf{F}_R$ of a distributed loading is equal to the area under the distributed loading diagram, and this resultant acts through the geometric center or centroid of this area.

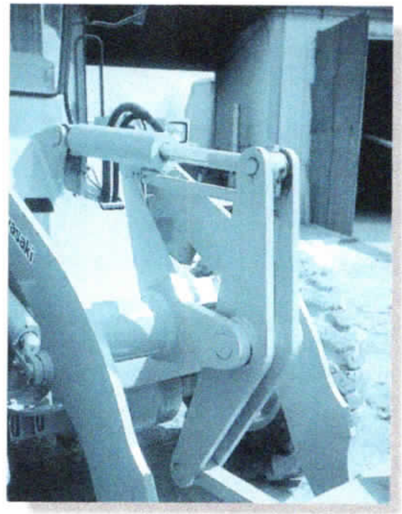


(a)

Fig. 1–1

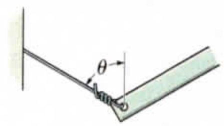
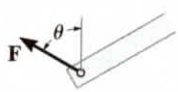
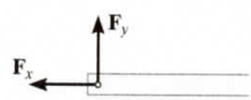
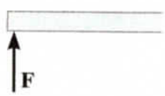
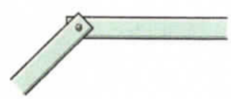
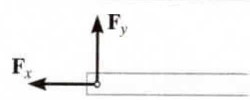
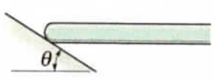
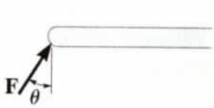
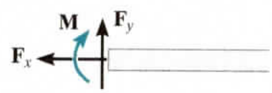
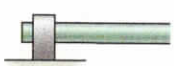
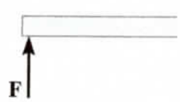
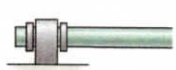
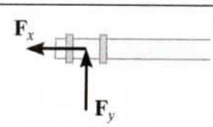
A ***body force*** is developed when one body exerts a force on another body without direct physical contact between the bodies. Examples include the effects caused by the earth's gravitation or by an electromagnetic field. Although these forces affect all the particles composing the body, they are normally represented by a single concentrated force acting on the body. In the case of gravitation, this force is called the ***weight* W** of the body and acts through the body's center of gravity.

Support Reactions. For bodies subjected to coplanar force systems, the supports most commonly encountered are shown in Table 1–1. Whatever the support, as a general rule, ***if the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member.*** For example, the roller support only prevents translation perpendicular or normal to the surface. Hence, the roller exerts a normal force **F** on the member at its point of contact. Since the member can freely rotate about the roller, a couple moment cannot be developed on the member.



Many machine elements are pin connected in order to enable free rotation at their connections. These supports exert a force on a member, but no moment.

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M
 Journal bearing	 One unknown: F	 Thrust bearing	 Two unknowns: F_x, F_y

In order to design the members of this building frame, it is first necessary to find the internal loadings at various points along their length.

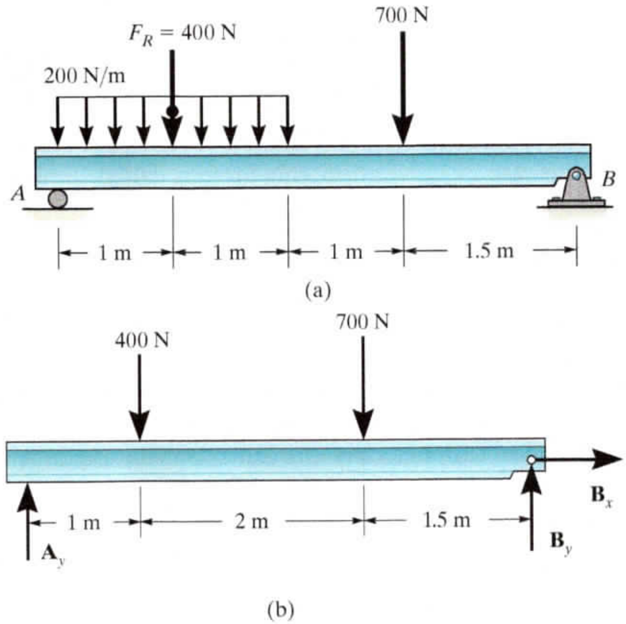


Fig. 1–1 (cont.)

Equations of Equilibrium. Equilibrium of a body requires both a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments***, to prevent the body from rotating. These conditions are expressed mathematically as the equations of equilibrium:

$$\boxed{\begin{aligned} \Sigma \mathbf{F} &= \mathbf{0} \\ \Sigma \mathbf{M}_O &= \mathbf{0} \end{aligned}} \qquad (1\text{–}1)$$

Here, $\Sigma \mathbf{F}$ represents the sum of all the forces acting on the body, and $\Sigma \mathbf{M}_O$ is the sum of the moments of all the forces about any point O either on or off the body.

If an x, y, z coordinate system is established with the origin at point O, the force and moment vectors can be resolved into components along each coordinate axis, and the above two equations can be written in scalar form as six scalar equations, namely,

$$\boxed{\begin{array}{lll} \Sigma F_x = 0 & \Sigma F_y = 0 & \Sigma F_z = 0 \\ \Sigma M_x = 0 & \Sigma M_y = 0 & \Sigma M_z = 0 \end{array}} \qquad (1\text{–}2)$$

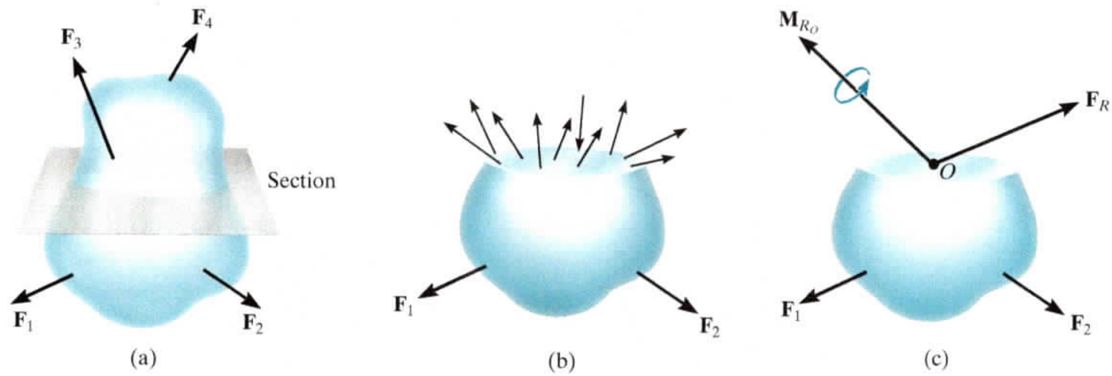


Fig. 1–2

Often in engineering practice the loading on a body can be represented as a system of *coplanar forces* in the x–y plane. In this case equilibrium of the body can be specified with only three scalar equilibrium equations, that is,

$$\boxed{\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma M_O &= 0\end{aligned}} \qquad (1\text{–}3)$$

Successful application of the equations of equilibrium must include all the known and unknown loadings that act on the body, and the best way to account for these loadings is to draw the body's free-body diagram first. For example, the free-body diagram of the beam in Fig. 1–1a is shown in Fig. 1–1b. Here each force is identified by its magnitude and direction. Included are the body's dimensions in order to sum the moments of the forces.

Internal Resultant Loadings. In mechanics of materials, statics is primarily used to determine the internal loadings that act within a body. This is done using the ***method of sections***. For example, consider the body shown in Fig. 1–2a, which is held in equilibrium by the four external forces.* In order to obtain the internal loadings acting on a specific region within the body, it is necessary to pass an imaginary section through the region where these loadings are to be determined. The two parts of the body are then separated, and a free-body diagram of one of the parts is drawn. If we consider the bottom segment, there will be a distribution of internal force acting on the "exposed" area of the section, Fig. 1–2b. These forces actually represent the effects of the material of the top section of the body acting on the bottom section.

Although the exact distribution of this internal loading may be *unknown*, its resultants $\mathbf{F}_R$ and $\mathbf{M}_{R_O}$ can be determined by applying the equations of equilibrium to the segment shown in Fig. 1–2c. Here these loadings act at point O; however, this point is often chosen at the centroid of the sectioned area.

*The body's weight is not shown, since it is assumed to be quite small, and therefore negligible compared with the other loads.

1

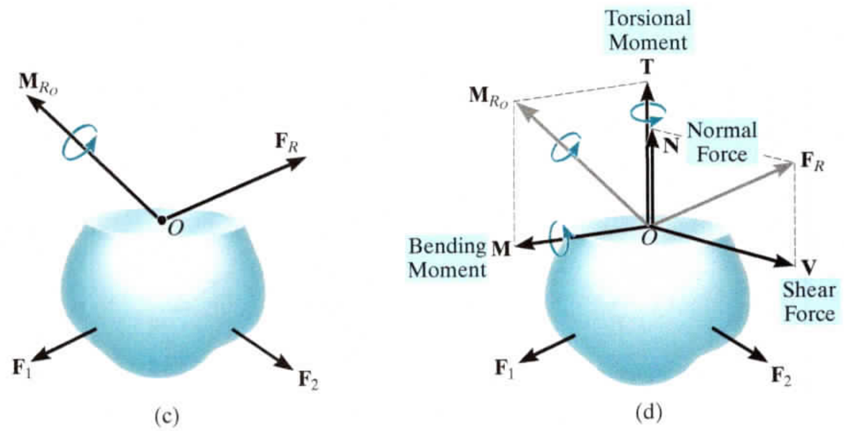


Fig. 1–2 (cont.)

The weight of this sign and the wind loadings acting on it will cause normal and shear forces and bending and torsional moments in the supporting column.

Three Dimensions. For later application of the formulas for mechanics of materials, we will consider the components of $\mathbf{F}_R$ and $\mathbf{M}_{R_O}$ acting both normal and tangent to the sectioned area, Fig. 1–2*d*. Four different types of resultant loadings can then be defined as follows:

Normal force, N. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.

Shear force, V. The shear force lies in the plane of the area, and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

Torsional moment or torque, T. This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

Bending moment, M. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

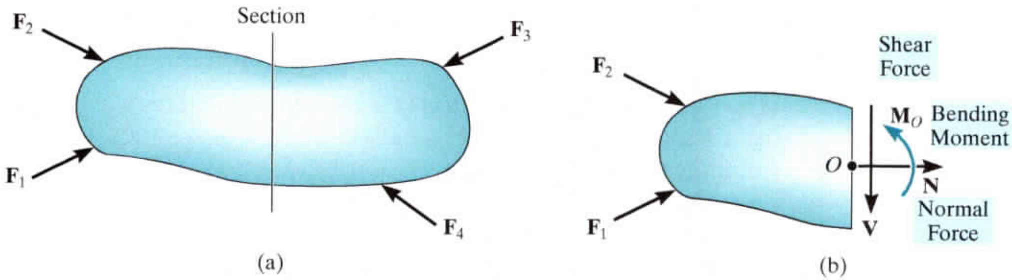


Fig. 1–3

Coplanar Loadings. If the body is subjected to a *coplanar system of forces*, Fig. 1–3*a*, then only normal-force, shear-force, and bending-moment components will exist at the section, Fig. 1–3*b*.

IMPORTANT POINTS

- External forces can be applied to a body as *distributed* or *concentrated surface loadings*, or as *body forces* that act throughout the volume of the body.
- Distributed loadings produce a *resultant force* having a *magnitude* equal to the *area* under the distributed load diagram, and act at a *location* that passes through the *centroid* of this area.
- A support produces a *force* in a particular direction on its attached member if it *prevents translation* of the member in that direction, and it produces a *couple moment* on the member if it *prevents rotation*.
- The equations of equilibrium $\Sigma \mathbf{F} = 0$ and $\Sigma \mathbf{M} = 0$ must be satisfied in order to prevent a body from translating with accelerated motion and from rotating.
- The method of sections is used to determine the internal resultant loadings acting on the surface of a sectioned body. In general, these resultants consist of a normal force, shear force, torsional moment, and bending moment.

1

PROCEDURE FOR ANALYSIS

The resultant *internal* loadings at a point located on the section of a body can be obtained using the method of sections. This requires the following steps.

Support Reactions.

- Before sectioning the body first decide which segment of the body is to be considered. If the segment has a support or connection to another body, then it will be necessary to determine the support reactions acting on the segment. To do this, draw the free-body diagram of the *entire body* and then apply the necessary equations of equilibrium to obtain these reactions.

Free-Body Diagram.

- Be sure to keep all external distributed loadings, couple moments, torques, and forces in their *exact locations*, before passing the section through the body at the point where the resultant internal loadings are to be determined.

- Draw a free-body diagram of one of the segments and indicate the unknown resultants **N**, **V**, **M**, and **T** at the section. These resultants are normally placed at the point representing the geometric center or *centroid* of the sectioned area.

Equations of Equilibrium.

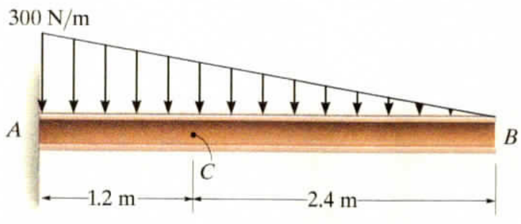
- Moments should be summed at the centroid of the section, about the established coordinate axes. Doing this eliminates the unknown forces **N** and **V** and allows a direct solution for **M** and **T**.

- If the solution of the equilibrium equations yields a negative value for a resultant, the *directional sense* of the resultant is *opposite* to that shown on the free-body diagram.

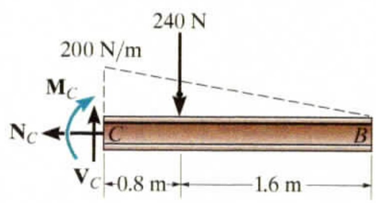
Lecture videos that cover the material in this and most of the other sections of the book can be obtained at www.pearson.com/hibbeler.

EXAMPLE 1.1

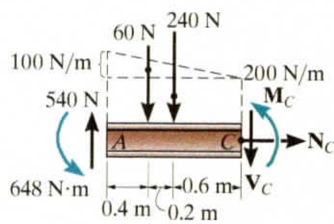
Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam shown in Fig. 1–4a.



(a)



(b)



(c)

Fig. 1–4

SOLUTION

Support Reactions. The support reactions at A do not have to be determined if segment CB is considered.

Free-Body Diagram. The free-body diagram of segment CB is shown in Fig. 1–4b. It is important to keep the distributed loading on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity w of the distributed loading at C is found by proportion, i.e., from Fig. 1–4a, $w/2.4\text{ m} = (300\text{ N/m})/3.6\text{ m}$, $w = 200\text{ N/m}$. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, $F = \frac{1}{2}(200\text{ N/m})(2.4\text{ m}) = 240\text{ N}$, which acts $\frac{1}{3}(2.4\text{ m}) = 0.8\text{ m}$ from C as shown in Fig. 1–4b.

Equations of Equilibrium. Applying the equations of equilibrium we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N_C = 0$$

$$N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C - 240\text{ N} = 0$$

$$V_C = 240\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 240\text{ N}(0.8\text{ m}) = 0$$

$$M_C = -192\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite sense of direction to that shown on the free-body diagram. Try solving this problem using segment AC, by first checking the support reactions at A, which are given in Fig. 1–4c.

EXAMPLE 1.2

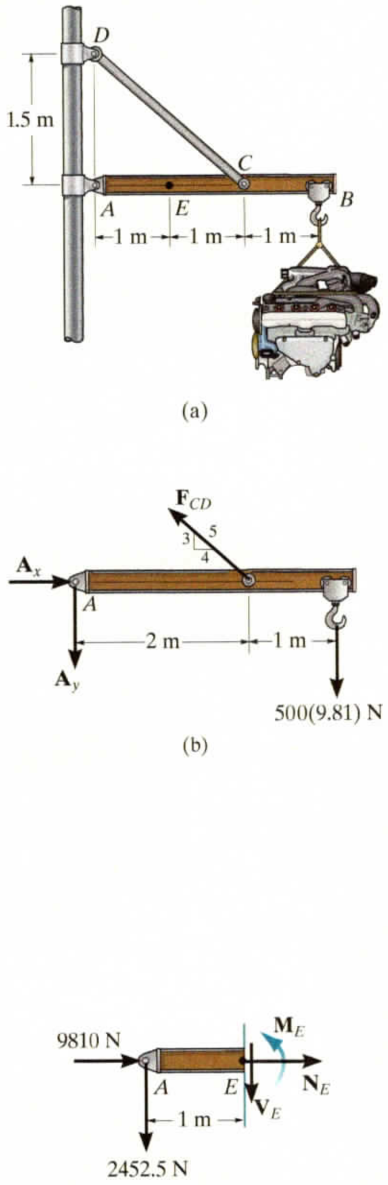


Fig. 1–5

The 500-kg engine is suspended from the crane boom in Fig. 1–5*a*. Determine the resultant internal loadings acting on the cross section of the boom at *E*.

SOLUTION

Support Reactions. We will consider segment *AE* of the boom, so we must first determine the pin reactions at *A*. Since member *CD* is a two-force member, it acts like a cable, and therefore exerts a force $\mathbf{F}_{CD}$ having a known direction. The free-body diagram of the boom is shown in Fig. 1–5*b*. Applying the equations of equilibrium,

$$\curvearrowleft + \Sigma M_A = 0; \qquad F_{CD}\left(\tfrac{3}{5}\right)(2\text{ m}) - [500(9.81)\text{ N}](3\text{ m}) = 0$$

$$F_{CD} = 12\ 262.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - (12\ 262.5\text{ N})\left(\tfrac{4}{5}\right) = 0$$

$$A_x = 9810\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + (12\ 262.5\text{ N})\left(\tfrac{3}{5}\right) - 500(9.81)\text{ N} = 0$$

$$A_y = 2452.5\text{ N}$$

Free-Body Diagram. The free-body diagram of segment *AE* is shown in Fig. 1–5*c*.

Equations of Equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_E + 9810\text{ N} = 0$$

$$N_E = -9810\text{ N} = -9.81\text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -V_E - 2452.5\text{ N} = 0$$

$$V_E = -2452.5\text{ N} = -2.45\text{ kN} \qquad \textit{Ans.}$$

$$\curvearrowleft + \Sigma M_E = 0; \qquad M_E + (2452.5\text{ N})(1\text{ m}) = 0$$

$$M_E = -2452.5\text{ N}\cdot\text{m} = -2.45\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

EXAMPLE 1.3

Determine the resultant internal loadings acting on the cross section at C of the beam shown in Fig. 1–6*a*.

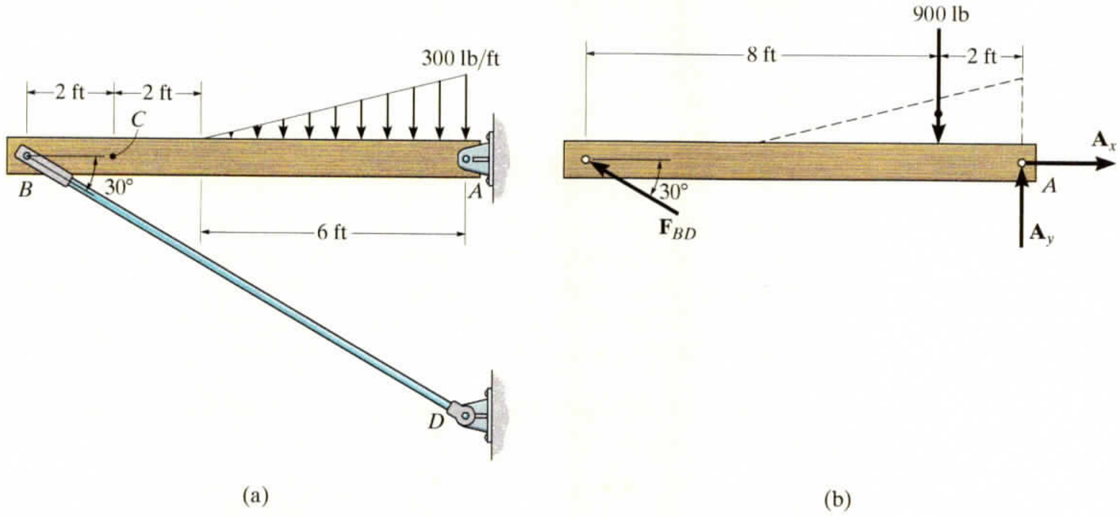


Fig. 1–6

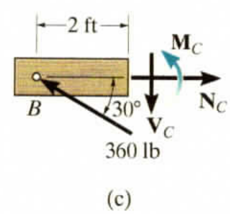
SOLUTION

Support Reactions. Here we will consider segment BC. The free-body diagram of the *entire* beam is shown in Fig. 1–6*b*. Since member BD is a two-force member, the force at B has a known direction.

$$\circlearrowleft + \Sigma M_A = 0; \quad (900 \text{ lb})(2 \text{ ft}) - (F_{BD} \sin 30^\circ)(10 \text{ ft}) = 0 \quad F_{BD} = 360 \text{ lb}$$

Free-Body Diagram. Using this result, the free-body diagram of segment BC is shown in Fig. 1–6*c*.

Equations of Equilibrium.



$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C - (360 \text{ lb}) \cos 30^\circ = 0$$

$$N_C = 312 \text{ lb} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad (360 \text{ lb}) \sin 30^\circ - V_C = 0$$

$$V_C = 180 \text{ lb} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M_C - (360 \text{ lb}) \sin 30^\circ (2 \text{ ft}) = 0$$

$$M_C = 360 \text{ lb} \cdot \text{ft} \qquad \textit{Ans.}$$

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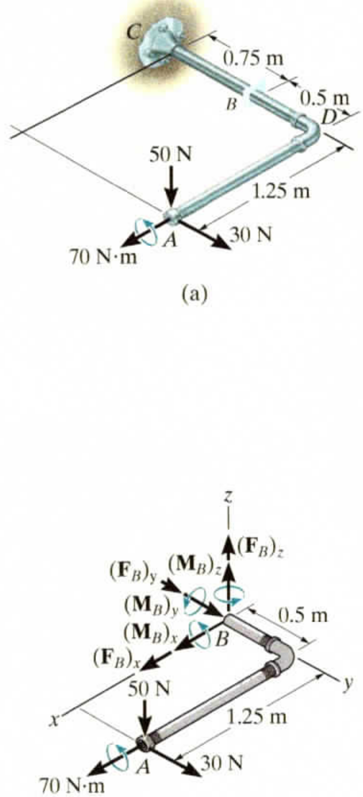
EXAMPLE 1.4



(a)

(b)

Fig. 1–7

Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 1–7*a*. End A is subjected to a vertical force of 50 N, a horizontal force of 30 N, and a couple moment of $70\ \text{N}\cdot\text{m}$. Neglect the pipe's mass.

SOLUTION

The problem can be solved by considering segment AB, so we do not need to calculate the support reactions at C.

Free-Body Diagram. The free-body diagram of segment AB is shown in Fig. 1–7*b*, where the x, y, z axes are established at B. The resultant force and moment components at the section are assumed to act in the *positive coordinate directions* and to pass through the *centroid* of the cross-sectional area at B.

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have*

$$\Sigma F_x = 0; \qquad (F_B)_x = 0 \qquad \textit{Ans.}$$

$$\Sigma F_y = 0; \qquad (F_B)_y + 30\ \text{N} = 0 \qquad (F_B)_y = -30\ \text{N} \qquad \textit{Ans.}$$

$$\Sigma F_z = 0; \qquad (F_B)_z - 50\ \text{N} = 0 \qquad (F_B)_z = 50\ \text{N} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_x = 0; \qquad (M_B)_x + 70\ \text{N}\cdot\text{m} - 50\ \text{N}\,(0.5\ \text{m}) = 0$$

$$(M_B)_x = -45\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

$$\Sigma (M_B)_y = 0; \quad (M_B)_y + 50\ \text{N}(1.25\ \text{m}) = 0 \quad (M_B)_y = -62.5\ \text{N}\cdot\text{m} \quad \textit{Ans.}$$

$$\Sigma (M_B)_z = 0; \quad (M_B)_z + 30\ \text{N}(1.25) = 0 \quad (M_B)_z = -37.5\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

NOTE: The normal force $N_B = |(F_B)_y| = 30\ \text{N}$, whereas the shear force is $V_B = \sqrt{(0)^2 + (50)^2} = 50\ \text{N}$. Also, the torsional moment is $T_B = |(M_B)_y| = 62.5\ \text{N}\cdot\text{m}$, and the bending moment is $M_B = \sqrt{(45)^2 + (37.5)^2} = 58.6\ \text{N}\cdot\text{m}$.

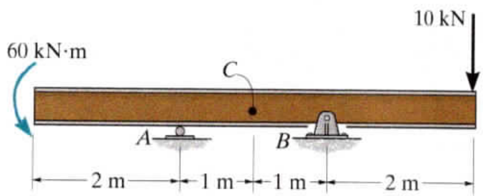
*The *magnitude* of each moment about the x, y, or z axis is equal to the magnitude of each force times the perpendicular distance from the axis to the line of action of the force. The *direction* of each moment is determined using the right-hand rule, with positive moments (thumb) directed along the positive coordinate axes.

Videos that "Test your Understanding" of these and other example problems throughout the book are available at www.pearson.com/hibbeler.

FUNDAMENTAL PROBLEMS

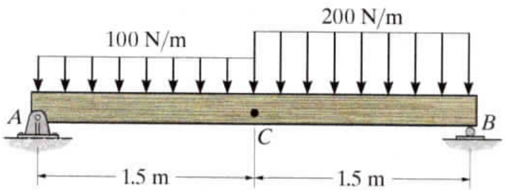
Review videos of these and other fundamental problems are available at www.pearson.com/hibbeler.

F1–1. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



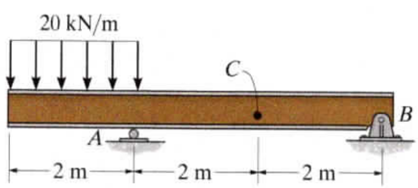
Prob. F1–1

F1–2. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



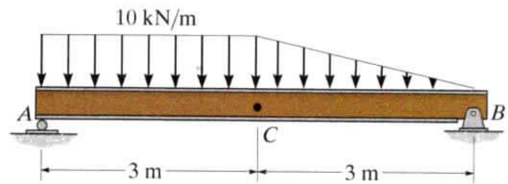
Prob. F1–2

F1–3. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



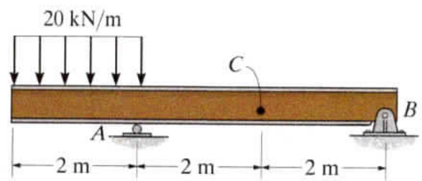
Prob. F1–3

F1–4. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



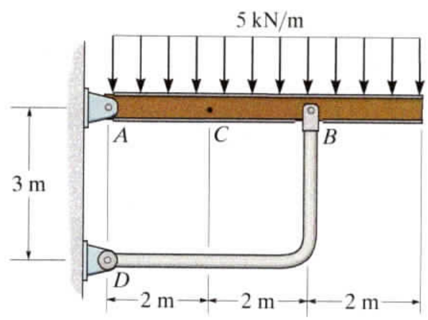
Prob. F1–4

F1–5. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



Prob. F1–5

F1–6. Determine the resultant internal normal force, shear force, and bending moment at point C in the beam.



Prob. F1–6

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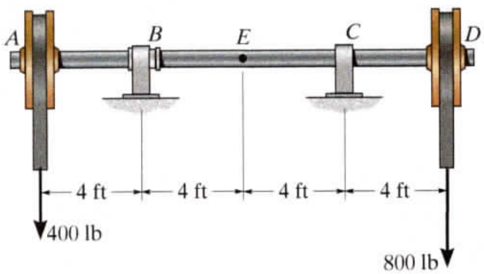
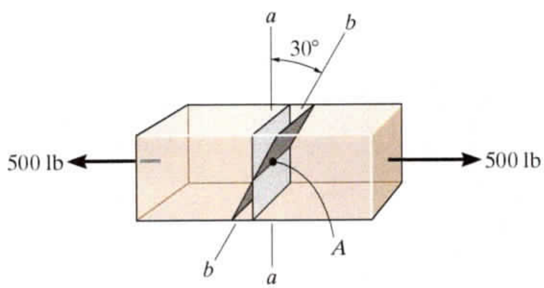
PROBLEMS

1–1. The shaft is supported by a smooth thrust bearing at B and a journal bearing at C. Determine the resultant internal loadings acting on the cross section at E.



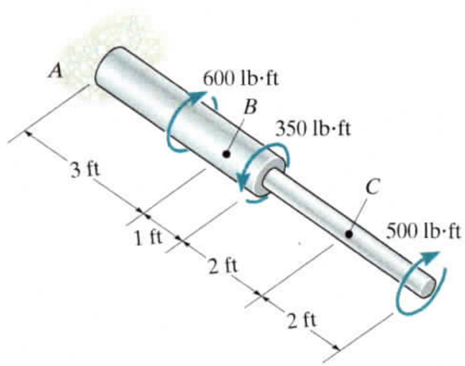
Prob. 1–1

1–2. Determine the resultant internal normal and shear force in the member on (a) section a–a and (b) section b–b, each of which passes through the centroid A. The 500-lb load is applied along the centroidal axis of the member.



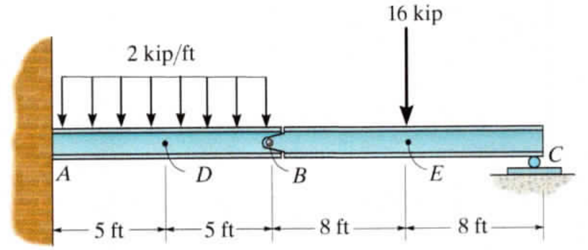
Prob. 1–2

1–3. Determine the resultant internal torque acting on the cross sections through points B and C.



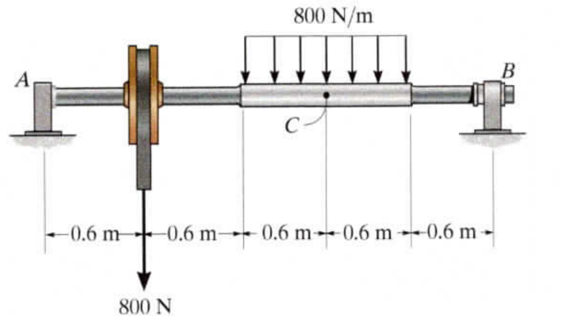
Prob. 1–3

***1–4.** Determine the resultant internal loadings in the beam at cross sections through points D and E. Point E is just to the left of the 16-kip load.



Prob. 1–4

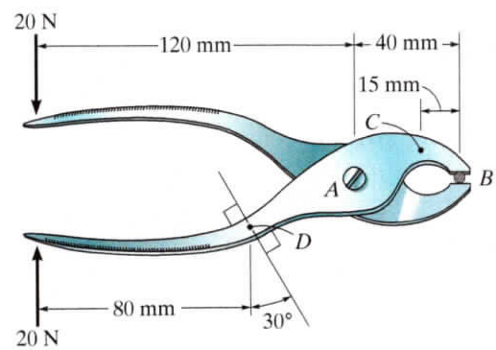
1–5. The shaft is supported by a smooth thrust bearing at B and a smooth journal bearing at A. Determine the resultant internal loadings acting on the cross section at C.



Prob. 1–5

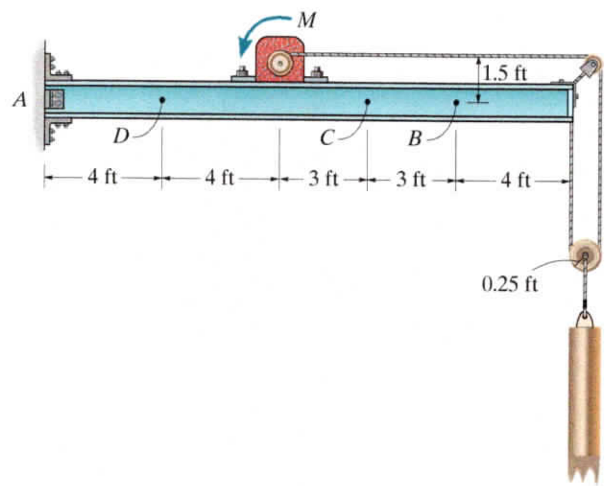
1–6. Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A, and the jaws at B are smooth.

1–7. Determine the resultant internal loading on the cross section through point D of the pliers. There is a pin at A, and the jaws at B are smooth.



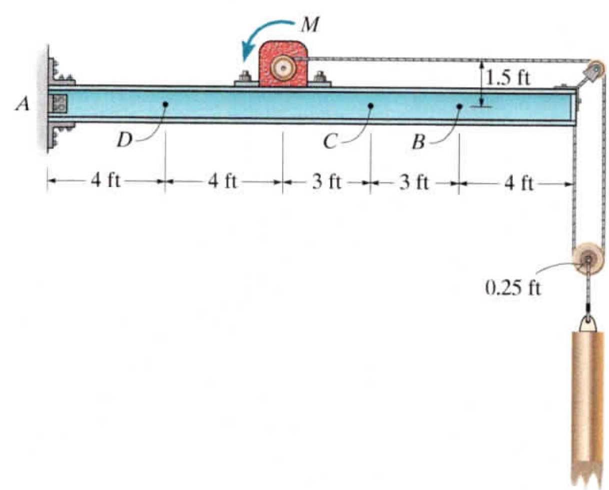
Probs. 1–6/7

*1–8. The 800-lb load is being hoisted at a constant speed using the motor M, which has a weight of 90 lb. Determine the resultant internal loadings acting on the cross section at point B in the beam. The beam has a weight of 40 lb/ft and is fixed to the wall at A.



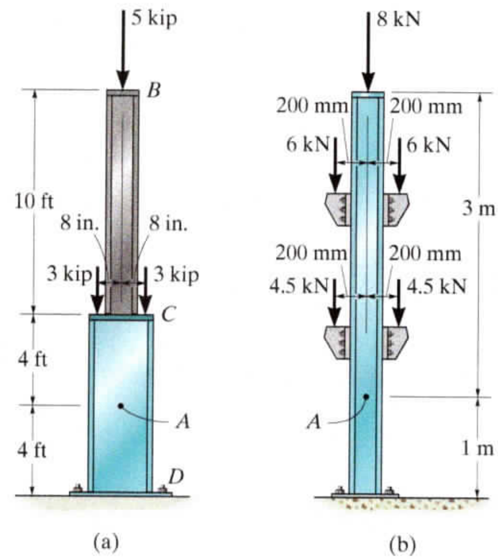
Prob. 1–8

1–9. Determine the resultant internal loadings acting on the cross section at points C and D of the beam.



Prob. 1–9

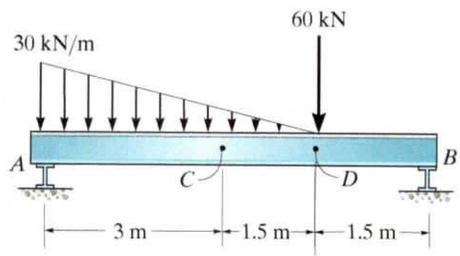
1–10. Determine the resultant internal normal force acting on the cross section at point A in each column. In (a) segment BC weighs 180 lb/ft and segment CD weighs 250 lb/ft. In (b) the column has a mass of 200 kg/m.



Prob. 1–10

1–11. Determine the resultant internal loadings on the cross section at point C. Assume the support reactions at A and B are vertical.

*1–12. Determine the resultant internal loadings on the cross section at point D. Assume point D is just to the left of the 60-kN force. Assume the support reactions at A and B are vertical.



Probs. 1–11/12

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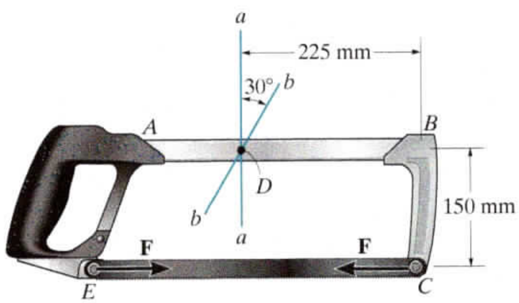
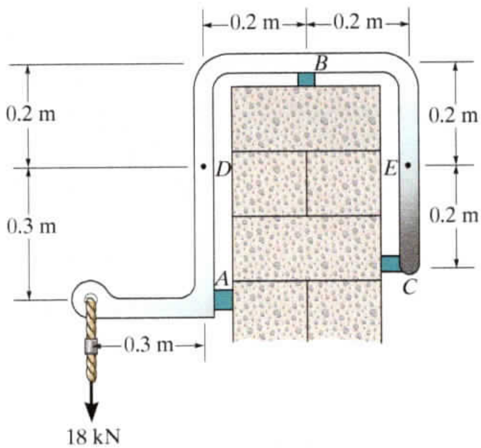
1–13. The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section a–a that passes through point D.

1–14. The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section b–b that passes through point D.



Probs. 1–13/14

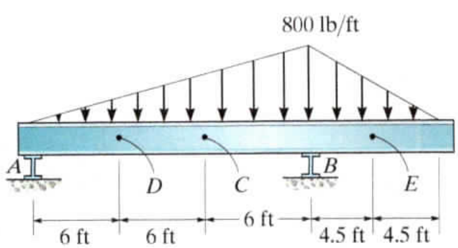
1–17. The sky hook is used to support the cable of a scaffold over the side of a building. If it consists of a smooth rod that contacts the parapet of a wall at points A, B, and C, determine the normal force, shear force, and moment on the cross section at points D and E.



Prob. 1–17

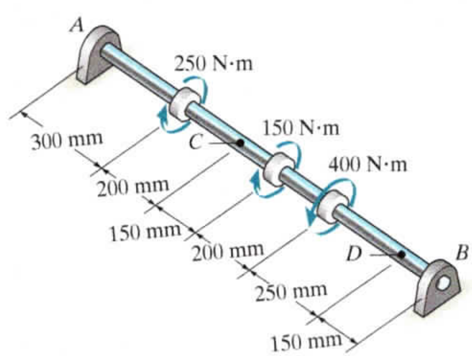
1–15. Determine the resultant internal loadings on the cross section at point C. Assume the reactions at the supports A and B are vertical.

***1–16.** Determine the resultant internal loadings on the cross section at points D and E. Assume the reactions at the supports A and B are vertical.



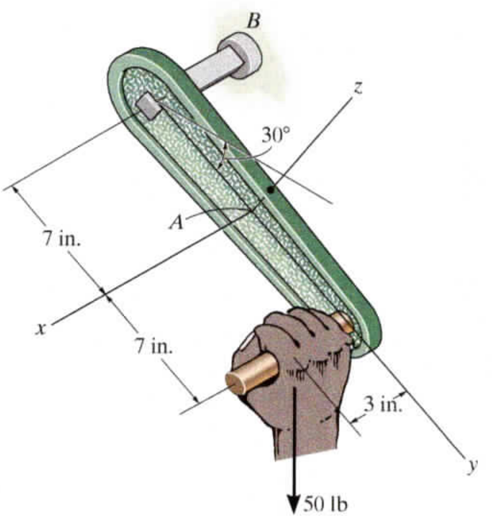
Probs. 1–15/16

1–18. Determine the resultant internal torque acting on the cross section at points C and D. The support bearings at A and B allow free turning of the shaft.



Prob. 1–18

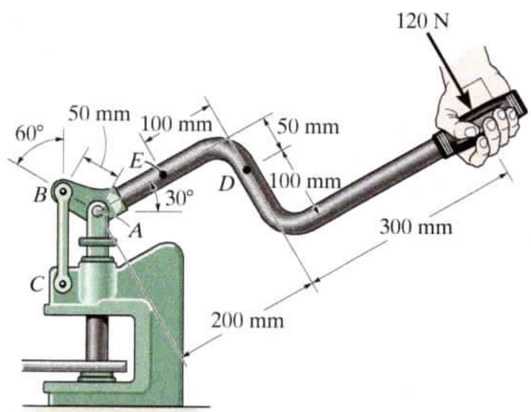
1–19. Determine the resultant internal loadings acting on the cross section of the hand crank at point A if a vertical force of 50 lb is applied to the handle. Assume the crank is fixed to the shaft at B.



Prob. 1–19

1–22. The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC. Also, determine the resultant internal loadings acting on the cross section at point D.

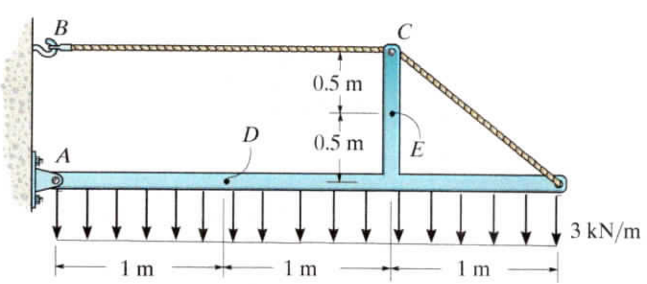
1–23. The metal stud punch is subjected to a force of 120 N on the handle. Determine the resultant internal loadings acting on the cross section of the handle at point E, and on the cross section of the short link BC.



Probs. 1–22/23

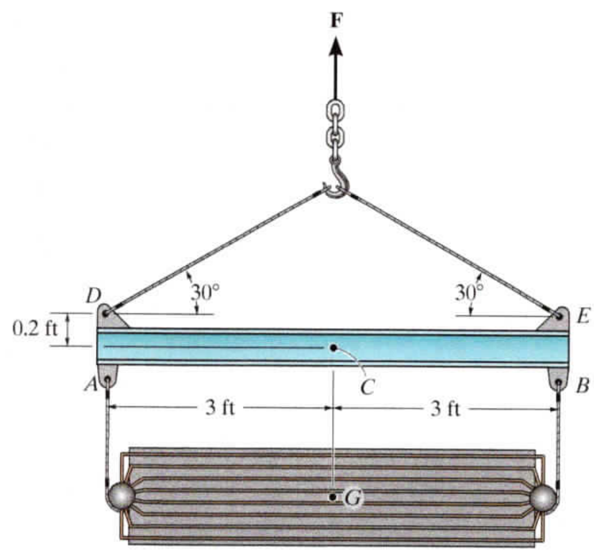
***1–20.** Determine the resultant internal loadings on the cross section at point D. The cable passes over a smooth peg at C.

1–21. Determine the resultant internal loadings on the cross section at point E. The cable passes over a smooth peg at C.



Probs. 1–20/21

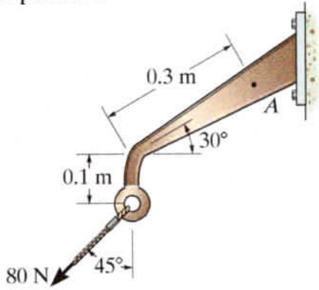
***1–24.** Determine the resultant internal loadings acting on the cross section at point C. The cooling unit has a total weight of 52 kip and a center of gravity at G.



Prob. 1–24

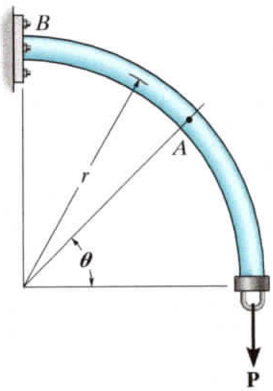
1

1–25. A force of 80 N is supported by the bracket. Determine the resultant internal loadings acting on the cross section at point A.



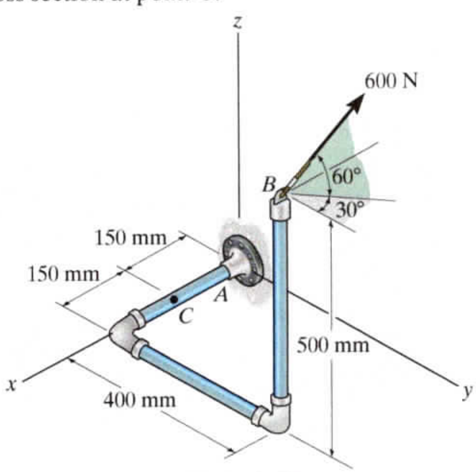
Prob. 1–25

1–26. The curved rod has a radius r and is fixed to the wall at B. Determine the resultant internal loadings acting on the cross section at point A, which is located at an angle θ from the horizontal.



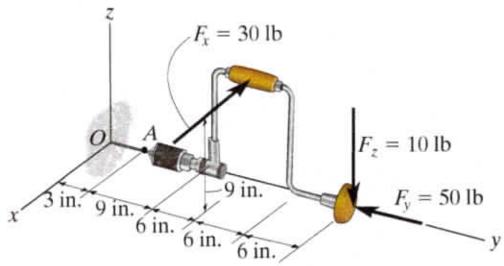
Prob. 1–26

1–27. The pipe assembly is subjected to a force of 600 N at B. Determine the resultant internal loadings acting on the cross section at point C.



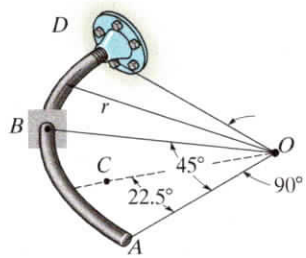
Prob. 1–27

***1–28.** If the drill bit jams when the handle of the hand drill is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



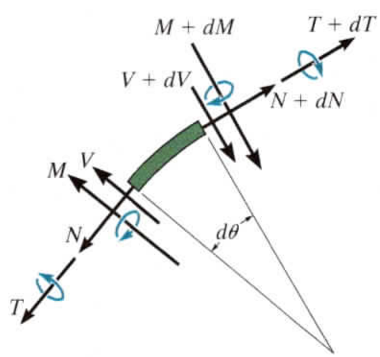
Prob. 1–28

1–29. The curved rod AD of radius r has a weight per length of w. If it lies in the horizontal plane, determine the resultant internal loadings acting on the cross section at point B. *Hint:* The distance from the centroid C of segment AB to point O is $CO = 0.9745r$.



Prob. 1–29

1–30. A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.



Prob. 1–30

1.3 STRESS

As noted in Sec. 1.2, the resultant internal force and moment acting at a specified point O on the sectioned area of the body, Fig. 1–8, represent the effects of the *distribution of loading* that acts over the sectioned area, Fig. 1–9*a*. Obtaining this *distribution* is of primary importance in mechanics of materials, and to solve this problem it is first necessary to establish the concept of stress.

We begin by considering the entire sectioned area to be subdivided into small elements, such as ΔA shown in Fig. 1–9*a*. As we reduce ΔA to a smaller and smaller size, we will make two assumptions regarding the properties of the material. We will consider it to be ***continuous***, that is, to consist of a *continuum* or uniform distribution having no voids. Also, it must be ***cohesive***, meaning that all portions are connected together, without having breaks, cracks, or separations.

A typical finite yet very small force $\Delta \mathbf{F}$, acting on ΔA, is shown in Fig. 1–9*a*. This force, like all the others, will have a unique magnitude and direction, but to compare it with all the other forces, we will replace it by its *three components*, $\Delta \mathbf{F}_x$, $\Delta \mathbf{F}_y$, and $\Delta \mathbf{F}_z$. As ΔA approaches zero, the quotient of the force and area will approach a finite limit. This quotient is called ***stress***, and it describes the *intensity of the internal force* acting on the sectioned area at a point.

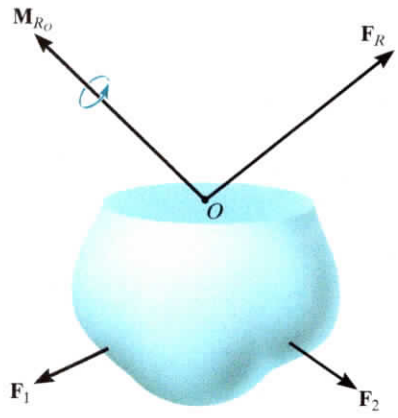


Fig. 1–8

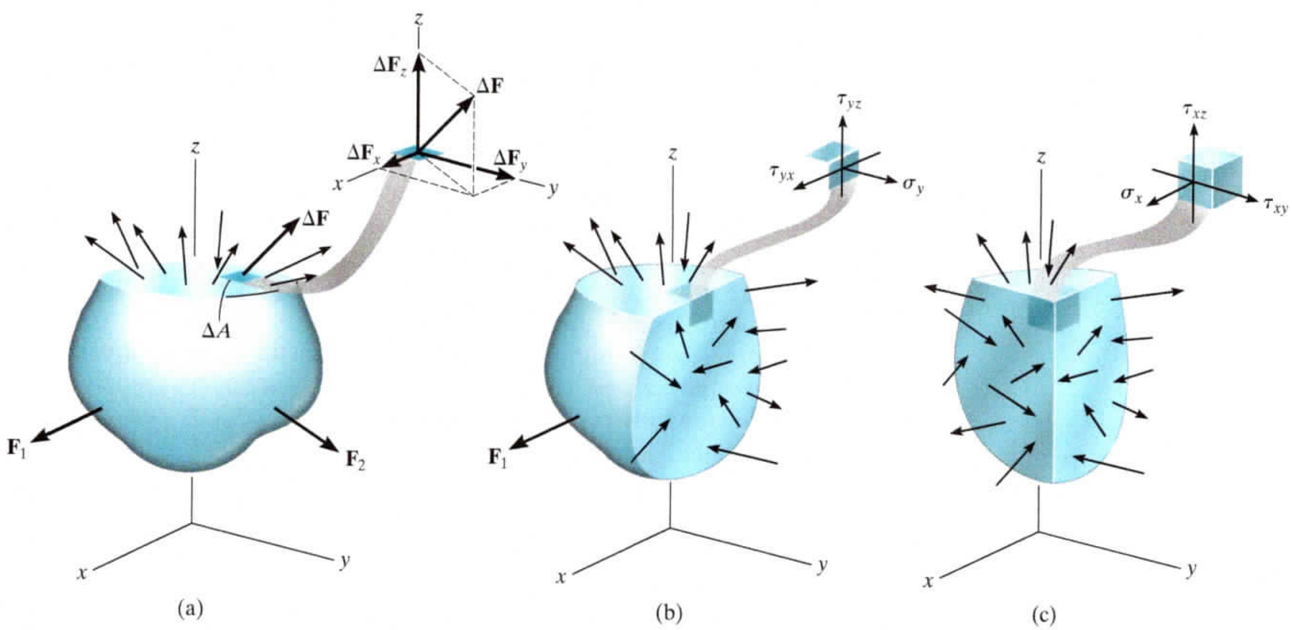


Fig. 1–9

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Normal Stress. The *intensity* of the force acting normal to ΔA is referred to as the ***normal stress***, σ (sigma). Since $\Delta \mathbf{F}_z$ is normal to the area then

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A} \tag{1–4}$$

If this stress "pulls" on ΔA as shown in Fig. 1–9*a*, it is *tensile stress*, whereas if it "pushes" on ΔA it is *compressive stress*.

Shear Stress. The intensity of force acting tangent to ΔA is called the ***shear stress***, τ (tau). Here we have two shear stress components,

$$\begin{aligned} \tau_{zx} &= \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A} \\ \tau_{zy} &= \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A} \end{aligned} \tag{1–5}$$

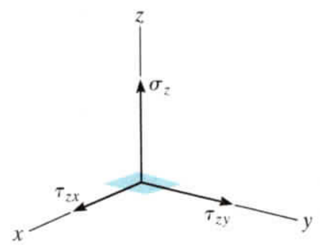

Fig. 1–10

The subscript notation z specifies the orientation of the area ΔA, Fig. 1–10, and x and y indicate the axes along which each shear stress acts.

General State of Stress. If the body is further sectioned by planes parallel to the x–z plane, Fig. 1–9*b*, and then the y–z plane, Fig. 1–9*c*, we can then "cut out" a cubic volume element of material that represents the ***state of stress*** acting around a point within the body. This state of stress is then characterized by three components acting on each face of the element, Fig. 1–11.

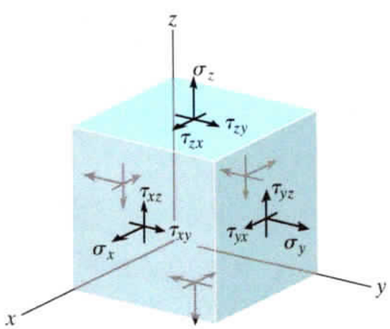

Fig. 1–11

Units. Since stress represents a force per unit area, in the International Standard or SI system, the magnitudes of both normal and shear stress are specified in base units of newtons per square meter (N/m^2). This combination is called a pascal ($1 \text{ Pa} = 1 \text{ N/m}^2$), and because it is rather small, prefixes such as kilo- (10^3), symbolized by k, mega- (10^6), symbolized by M, or giga- (10^9), symbolized by G, are used in engineering to represent larger, more realistic values of stress.* In the Foot-Pound-Second (FPS) system of units, engineers usually express stress in pounds per square inch (psi) or kilopounds per square inch (ksi), where 1 kilopound (kip) = 1000 lb.

*Sometimes stress is expressed in units of N/mm^2, where $1 \text{ mm} = 10^{-3}$ m. However, in the SI system, prefixes are not allowed in the denominator of a fraction, and therefore it is better to use the equivalent $1 \text{ N/mm}^2 = 1 \text{ MN/m}^2 = 1 \text{ MPa}$.

1.4 AVERAGE NORMAL STRESS IN AN AXIALLY LOADED BAR

We will now determine the average stress distribution acting over the cross-sectional area of an axially loaded bar such as the one shown in Fig. 1–12*a*. Specifically, the ***cross section*** is the section taken *perpendicular* to the longitudinal axis of the bar, and since the bar is prismatic all cross sections are the same throughout its length. Provided the material is ***homogeneous*** so that it has the same physical and mechanical properties throughout its volume, and it is also ***isotropic***, meaning it has the same properties in all directions, then when the load P is applied to the bar through the centroid of its cross-sectional area, the bar will deform uniformly throughout its central region, Fig. 1–12*b*.

Many engineering materials can be approximated as being both homogeneous and isotropic. Steel, for example, contains thousands of randomly oriented crystals in each cubic millimeter of its volume, and since most objects made of this material have a physical size that is very much larger than a single crystal, the above assumption regarding the material's composition is quite realistic.

By comparison, ***anisotropic materials***, such as wood, have different properties in different directions; and although this is the case, if the grains of wood are oriented along the bar's axis (as for instance in a typical wood board), then the bar will also deform uniformly when subjected to the axial load P.

Average Normal Stress Distribution. If we pass a section through the bar, and separate it into two parts, then equilibrium of the bottom segment requires the resultant normal force N at the section to be equal to P, Fig. 1–12*c*. And because the material undergoes a *uniform* deformation, it is necessary that the cross section be subjected to a *uniform* normal stress distribution.

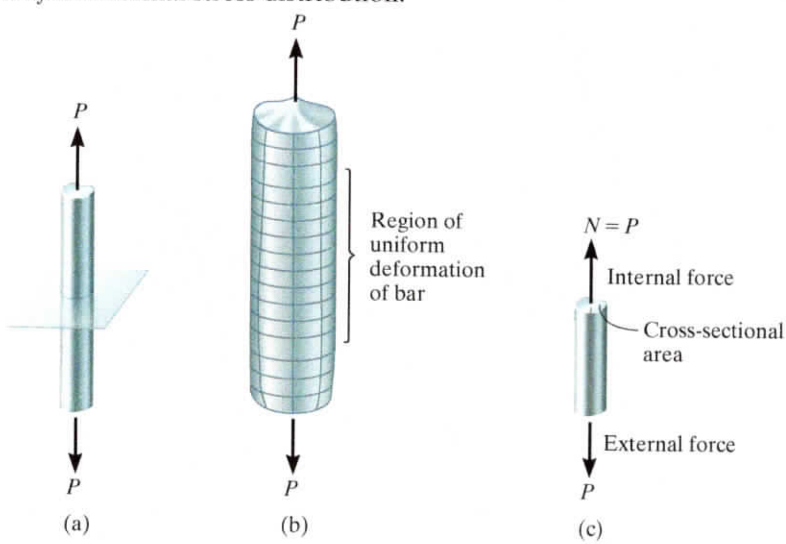

Fig. 1–12

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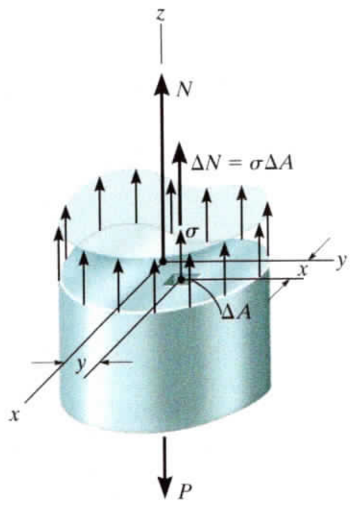


(d)

Fig. 1–12 (cont.)

As a result, each small area ΔA on the cross section is subjected to a force $\Delta N = \sigma\,\Delta A$, Fig. 1–12$d$, and the *sum* of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force $\mathbf{N}$ at the section. If we let $\Delta A \rightarrow dA$ and therefore $\Delta N \rightarrow dN$, then, recognizing σ is *constant*, we have

$$+\uparrow F_{Rz} = \Sigma F_z; \qquad \int dN = \int_A \sigma\, dA$$

$$N = \sigma A$$

$$\boxed{\sigma = \frac{N}{A}} \tag{1–6}$$

Here

σ = average normal stress at any point on the cross-sectional area

N = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. N is determined using the method of sections and the equation of force equilibrium, where for this case $N = P$.

A = cross-sectional area of the bar where σ is determined

Equilibrium. The stress distribution in Fig. 1–12 indicates that only a normal stress exists on the top of any small volume element of material located at each point on the cross section. If we consider vertical equilibrium of an element of material and then apply the equation of force equilibrium to its free-body diagram, Fig. 1–13, we get

$$\Sigma F_z = 0; \qquad \sigma(\Delta A) - \sigma'(\Delta A) = 0$$

$$\sigma = \sigma'$$

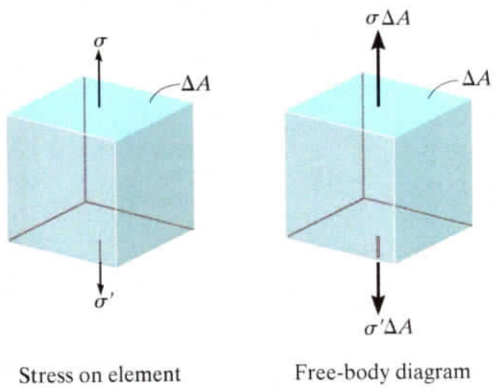


Fig. 1–13

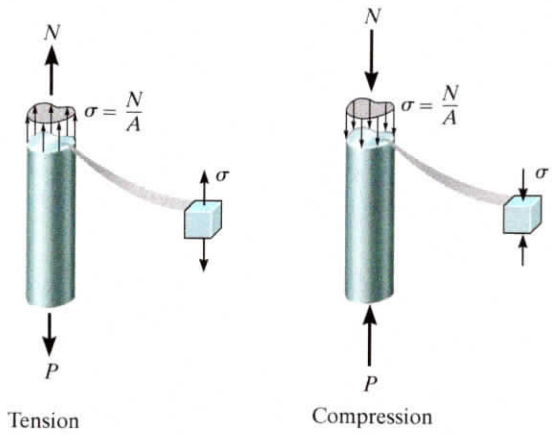
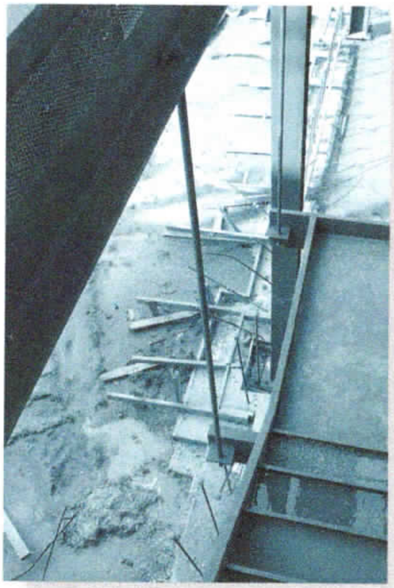


Fig. 1–14

In other words, the normal stress components on the top and bottom of the element must be equal in magnitude but opposite in direction. This is called ***uniaxial stress***, and this analysis applies to members subjected to either tension or compression, as shown in Fig. 1–14.

Although we have developed this analysis for *prismatic* bars, this assumption can be relaxed somewhat to include bars that have a *slight taper*. For example, it can be shown, using the more exact analysis of the theory of elasticity, that for a tapered bar of rectangular cross section, where the angle between two adjacent sides is 15°, the average normal stress, as calculated by $\sigma = N/A$, is only 2.2% *less* than its more exact maximum value found from the theory of elasticity.



This steel tie rod is used as a hanger to suspend a portion of a staircase. As a result it is subjected to tensile stress.

Maximum Average Normal Stress. For our analysis, both the internal force N and the cross-sectional area A were *constant* along the longitudinal axis of the bar, and as a result the normal stress $\sigma = N/A$ will be *constant* throughout the bar's length. Occasionally, however, the bar may be subjected to *several external axial loads*, or a sudden change in its cross-sectional area may occur. When this is the case, the normal stress along the bar may be different from one section to the next, and, if the *maximum* average normal stress is to be determined, then it becomes important to find the location where the ratio N/A is a *maximum*. Example 1.5 illustrates the procedure. Once the internal loading throughout the bar is known, the maximum ratio N/A can then be identified.

IMPORTANT POINTS

- When a body subjected to external loads is sectioned, there is a distribution of force acting over the sectioned area which holds each segment of the body in equilibrium. The intensity of this internal force per unit area at a point in the body is referred to as *stress*.
- The magnitude of each stress component at a point depends upon the type of loading acting on the body, and the orientation of the element at the point.
- When a prismatic bar is made of homogeneous and isotropic material, and is subjected to an axial force acting through the centroid of the cross-sectional area, then the bar will deform uniformly. As a result, the material will be subjected *only to normal stress*. This stress is uniform or *averaged* over the cross section.

PROCEDURE FOR ANALYSIS

To obtain the average stress requires two steps.

Internal Loading.

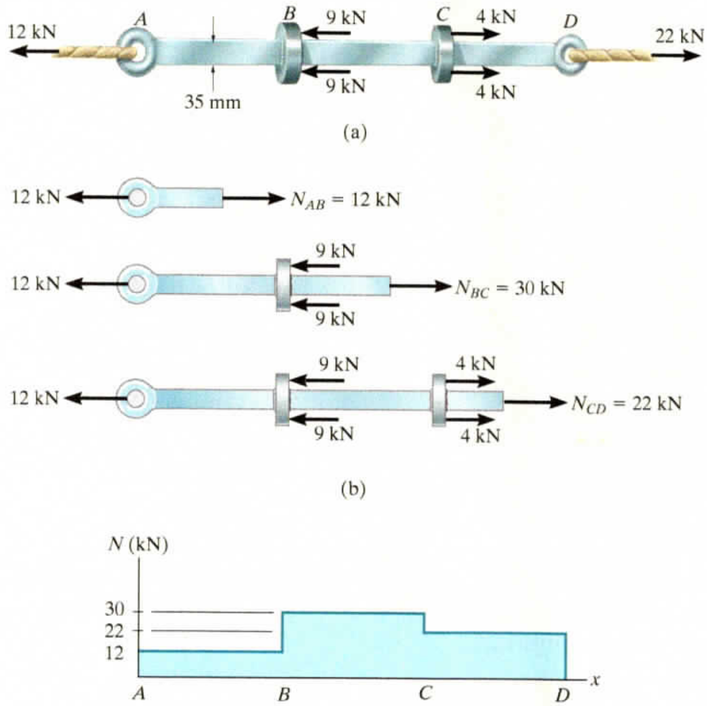
- Section the member *perpendicular* to its longitudinal axis at the point where the average normal stress is to be determined, and draw the free-body diagram of one of the segments. Apply the force equation of equilibrium to obtain the *internal axial force* N at the section.

Average Normal Stress.

- Determine the member's cross-sectional area at the section and calculate the average normal stress $\sigma = N/A$.
- It is suggested that σ be shown acting on a small volume element of the material located at a point on the section where the stress is calculated. To do this, first draw σ on the face of the element coincident with the sectioned area A. Here σ acts in the *same direction* as the internal force $\mathbf{N}$ since it is the resultant of the normal stresses on the cross section. The normal stress σ on the opposite face of the element acts in the opposite sense of direction.

EXAMPLE 1.5

The bar in Fig. 1–15*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



(c)

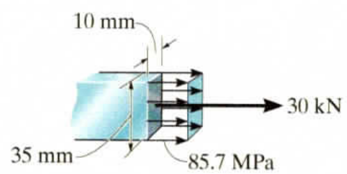
SOLUTION

Internal Loading. By inspection, the internal axial forces in regions *AB*, *BC*, and *CD* are all constant yet have different magnitudes. Using the method of sections, these loadings are shown on the free-body diagrams of the left segments shown in Fig. 1–15*b*.* The ***normal force diagram***, which represents these results graphically, is shown in Fig. 1–15*c*. The largest loading is in region *BC*, where $N_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the maximum average normal stress also occurs within this region of the bar.

Maximum Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{30(10^3)\,\text{N}}{(0.035\,\text{m})(0.010\,\text{m})} = 85.7\,\text{MPa} \qquad \textit{Ans.}$$

The stress distribution acting on an arbitrary cross section of the bar within region *BC* is shown in Fig. 1–15*d*.



(d)

Fig. 1–15

*Show that you get these *same results* using the right segments.

1

EXAMPLE 1.6

The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 1–16a. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.

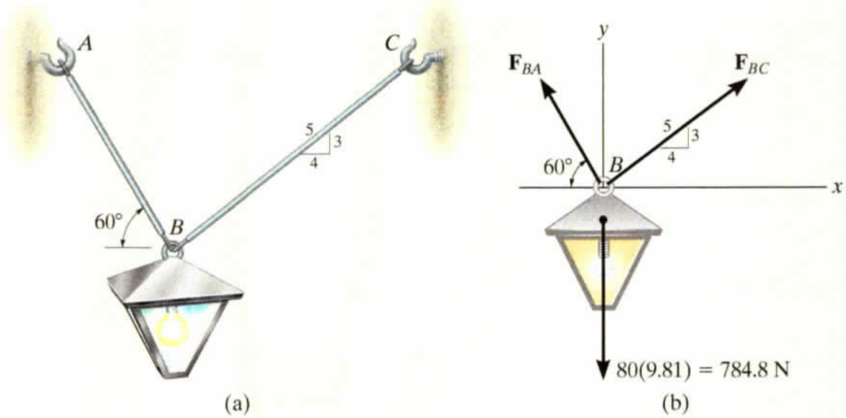


Fig. 1–16

SOLUTION

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–16b. Applying the equations of force equilibrium,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

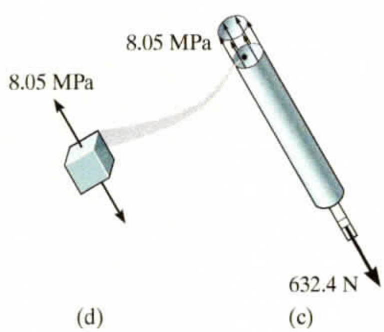
$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

By Newton's third law of action-reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6,

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$



The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–16c, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–16d.

1

EXAMPLE 1.7

The cylinder shown in Fig. 1–17*a* is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$. Determine the average compressive stress acting at points A and B.

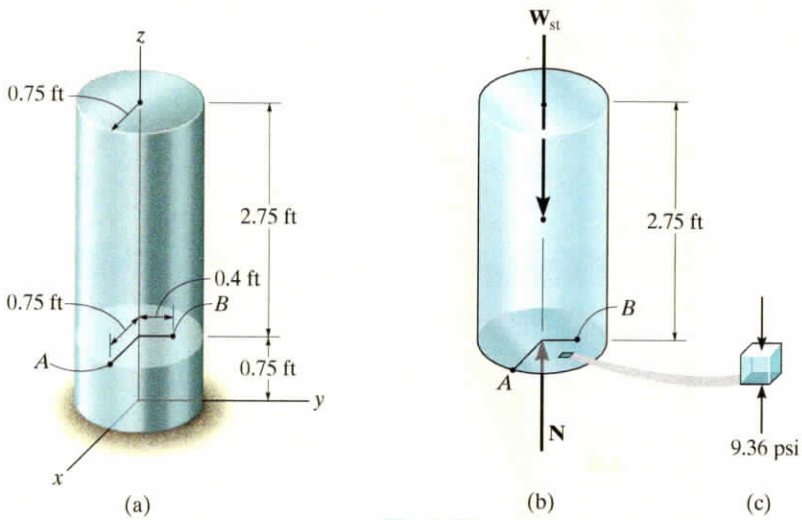


Fig. 1–17

SOLUTION

Internal Loading. A free-body diagram of the top segment of the cylinder is shown in Fig. 1–17*b*. The weight of this segment is determined from $W_{st} = \gamma_{st} V_{st}$. Thus the internal axial force N at the section is

$$+\uparrow \Sigma F_z = 0; \qquad N - W_{st} = 0$$

$$N - (490\,\text{lb/ft}^3)(2.75\,\text{ft})\left[\pi(0.75\,\text{ft})^2\right] = 0$$

$$N = 2381\,\text{lb}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\ \text{ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{N}{A} = \frac{2381\,\text{lb}}{\pi(0.75\,\text{ft})^2} = 1347.5\,\text{lb/ft}^2$$

$$\sigma = 1347.5\,\text{lb/ft}^2\,(1\text{ft}/12\,\text{in.})^2 = 9.36\,\text{psi} \qquad \textit{Ans.}$$

The stress shown on the volume element of material in Fig. 1–17*c* is representative of the conditions at either point A or B. Notice that this stress acts *upward* on the bottom or shaded face of the element since this face forms part of the bottom surface area of the section, and on this surface, the normal force **N** is pushing upward.

EXAMPLE 1.8

Member AC shown in Fig. 1–18a is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.

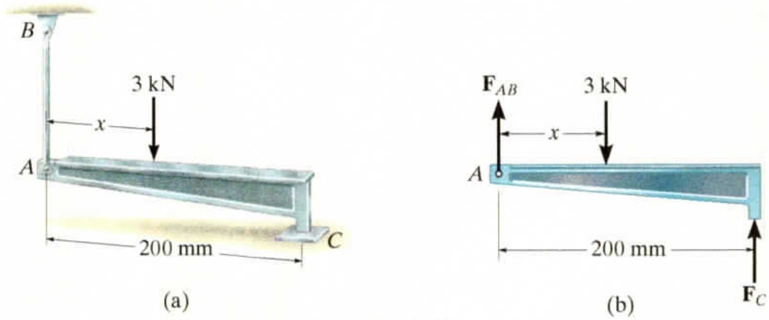


Fig. 1–18

SOLUTION

Internal Loading. The forces at A and C can be related by considering the free-body diagram of member AC, Fig. 1–18b. There are three unknowns, namely, F_{AB}, F_C, and x. To solve we will work in units of newtons and millimeters.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000\text{ N} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -3000\text{ N}(x) + F_C(200\text{ mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the rod AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400\text{ mm}^2} = \frac{F_C}{650\text{ mm}^2}$$

$$F_C = 1.625 F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143\text{ N}$$

$$F_C = 1857\text{ N}$$

The position of the applied load is determined from Eq. 2,

$$x = 124\text{ mm} \qquad \textit{Ans.}$$

As required, $0 < x < 200$ mm.