FOURTH EDITION

Mathematics FOR THE Clinical Laboratory

Lorraine J. Doucette

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Preface

Mathematics has a reputation as a subject that is difficult to understand and utilize. This perception may lead students to avoid careers that rely on mathematics even if they enjoy science in general. That is unfortunate, because solving mathematical problems should not be a barrier to a career choice.

Students who do not feel comfortable with math may struggle with the math that is associated with the clinical laboratory. This book is written with those students in mind. It will also benefit the student who is comfortable with math, the laboratory professional who may need some review of math concepts, and staff in the physician office laboratory (POL) or clinic who may not have been formally taught any laboratory math.

ORGANIZATION

Content is divided into three sections, beginning with a review of math and calculation basics, followed by coverage of particular areas of the clinical laboratory (including immunohematology and microbiology), and ending with statistical calculations. There is a new section that contains calculations that are not commonly performed in a typical clinical laboratory but may be still taught to students.

Readers who are already proficient in basic mathematics may want to review the introductory chapters quickly and then proceed to the chapters focusing on clinical laboratory skills.

DISTINCTIVE FEATURES OF THIS BOOK

Readers will find recurring margin notes indicating **WHAT** and **HOW** of important information. These margin notes guide the reader through the process of identifying in which situation each formula should be used and how to correctly analyze the outcome.

Readers will also notice that the answer for each problem is marked in **boldface** to aid in following the steps used in calculating the unknown. When appropriate, the unknown value to be determined will be noted as a bold-faced green X. As the calculation proceeds step by step, the green boldface will be used to highlight the progress of the calculation.

Examples of calculations for each type of math concept are worked out in a step-by-step manner. A substantial set of practice problems is included in each chapter.

NEW FEATURES INCLUDE:

- A full-color design with a more accessible look and feel.
- Topical margin notes that indicate **WHAT** and **HOW**, linking each math concept with its implementation.
- The topics of molarity and normality, and calculations associated with solutions combined into one chapter for a more cohesive presentation.

- A new chapter that includes calculations performed by students but not frequently found in the clinical laboratory.
- A glossary with definitions of important mathematical terms.
- A Greek symbol appendix for quick reference.

EVOLVE RESOURCES

http://evolve.elsevier.com/Doucette/math/

For Students:
- Glossary
- Practice Problems plus Answers

For Instructors:
- Image Collection
- PPT Slides
- Test Bank (not in ExamView)

NOTE TO THE STUDENT

Congratulations on deciding to become a valuable member of the healthcare team as a clinical laboratory professional! Enjoy the sense of accomplishment that will come with learning and performing the math that is essential for accurate and precise laboratory results. Remember, this book will be a valuable resource and companion well into your career.

Welcome to the profession of clinical laboratory science!

Acknowledgements

I would like to thank my family and friends, especially my sons, Steven and Kenneth, who provided love and support to me, as well as my mother who is my inspiration for how to live life to its fullest. It has been a journey for me through the years from when I wrote my first edition in 1997 when my sons were still in grade school to now as they have grown into fine, young men. I lost my husband in 2007 and worked on the second book in the months following his death of complications from kidney failure that began with strep throat at age 9 and ended with peritonitis by *C. difficile* at age 52. I wrote the third edition in a very different emotional place, but I have grown from that experience while I wrote my fourth edition.

I would like to thank laboratory professionals in general for their hard work and dedication to their profession as well as students of clinical laboratory science who use this book to help them learn laboratory mathematics concepts. And last, I would like to thank the staff at Elsevier for helping to make this fourth edition a reality.

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Contents

CHAPTER 1

Basic Arithmetic, Rounding Numbers, and Significant Figures

OUTLINE

OBJECTIVES

At the end of this chapter, the reader should be able to do the following:

1. Perform basic arithmetic calculations, including addition, subtraction, multiplication, and division with positive and negative numbers.
2. Perform calculations that require multiple steps in the correct order.
3. Convert fractions to their lowest equivalent form.
4. Perform calculations involving fractions using ratio and proportion.
5. Convert numbers between their percentage and decimal forms.
6. State the rules for rounding numbers in the following situations:
   a. The number to be rounded ends in a number less than 5.
   b. The number to be rounded ends in a number greater than 5.
   c. The number to be rounded ends in 5.
7. State the rules for addition, subtraction, multiplication, and division using significant figures.
8. Perform calculations (incorporating rounding numbers), including addition, subtraction, multiplication, and division to achieve the correct result.
9. Determine the number of significant figures that are in a number.

BASIC ARITHMETIC

This chapter as well as Chapters 2 and 3 are designed as a review of basic mathematical concepts. Students already proficient in these concepts may wish to briefly review them and begin at Chapter 4. Note: Significant figures are not used in Examples 1-1 through 1-16g.

WHAT Positive and Negative Numbers

A positive number is a number that has a value greater than zero; a negative number is a number with a value less than zero. Figure 1-1 is a number line that demonstrates this concept. A plus sign (+) is used to identify a positive number, and a negative, or minus, sign (−) is used to identify a negative number. If only positive numbers are used in an equation, the (+) sign is usually omitted.

WHAT Addition of Positive Numbers

The sum of two or more positive numbers will also be a positive number:

$$(+13) + (+16) + (+22) = +51$$

$$(+12) + (+4) = +16$$

$$(+6) + (+5) + (+28) + (+32) = +71$$

$$(+4) + (+3) + (+3) + (+2) = +12$$

HOW ***Example 1-1***

What is the sum of +64 and +45?

Because both numbers are a positive number, the sum will also be a positive number. Therefore, the sum of +64 and +45 is +109.

Example 1-1a What is the sum of +45 and +38?

The sum of these numbers is +83.

Example 1-1b What is the sum of +105 and +107?

The sum of these numbers is +212.

Example 1-1c What is the sum of +3 and +86?

The sum of these numbers is +89.

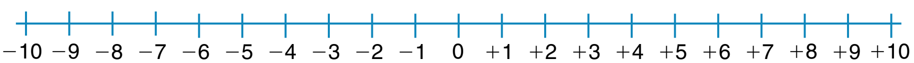

FIGURE 1-1 Number line.

HOW **_Example 1-2_**

What is the sum of +22, +37, +61, +18, and +44?

The sum of this group of numbers will also be a positive number and is determined by simple addition:

$$(+22) + (+37) + (+61) + (+18) + (+44) = +182$$

Therefore, the sum of this group of numbers is +182.

Example 1-2a Using Example 1-2 as a guide, what is the sum of +2, +8, +11, and +23?

The sum of this group of numbers is +44.

Example 1-2b Using Example 1-2 as a guide, what is the sum of +45, +83, +37, +75, and +105?

The sum of this group of numbers is +345.

Example 1-2c Using Example 1-2 as a guide, what is the sum of +71, +16, +38, and +104?

The sum of this group of numbers is +229.

WHAT Addition of Negative Numbers

The sum of two or more negative numbers will also be a negative number. That is because when two or more negative numbers are added together, they stay on the left (or negative) side of the number line, and the sum will always be a negative number.

$$(-5) + (-14) + (-2) = -21$$
$$(-7) + (-32) + (-18) = -57$$

HOW **_Example 1-3_**

What is the sum of −17 and −38?

Remember: The sum of two negative numbers will be a negative number.

$$(-17) + (-38) = -55$$

Therefore, the sum of these two negative numbers is −55.

Example 1-3a Using Example 1-3 as a guide, what is the sum of −2 and −7?

The sum of these numbers is −9.

Example 1-3b Using Example 1-3 as a guide, what is the sum of −15 and −65?

The sum of these numbers is −80.

Example 1-3c Using Example 1-3 as a guide, what is the sum of −14 and −55?

The sum of these numbers is −69.

Example 1-3d Using Example 1-3 as a guide, what is the sum of the following group of numbers?

$$-6, -2, -3, -9$$

The sum of this group of numbers is determined by simple addition:

$$(-6) + (-2) + (-3) + (-9) = -20$$

Therefore, the sum of this group of numbers is -20.

Example 1-3e Using Example 1-3 as a guide, what is the sum of -10, -18, -42, and -11?

The sum of this group of numbers is -81.

Example 1-3f Using Example 1-3 as a guide, what is the sum of -59, -48, -62, and -12?

The sum of this group of numbers is -181.

Example 1-3g Using Example 1-3 as a guide, what is the sum of -30, -18, -13, -26, and -84?

The sum of this group of numbers is -171.

WHAT Addition of Both Positive and Negative Numbers

The sum of an addition of both positive and negative numbers will be the sign of the larger number involved in the addition. If you think of the numbers on the number line as players in a "tug of war" game (Figure 1-2), the larger number will be able to pull the smaller number to the sign direction of the larger number; that is, if the larger number is positive, the smaller number is pulled to the positive direction. When a negative number is added to a positive number, it is actually subtracted from the positive number. For example: What is the sum of -23 and $+12$? Because the larger number is a negative number, the sum will have a negative number. By convention, when placing the numbers in correct order to perform the calculation, the

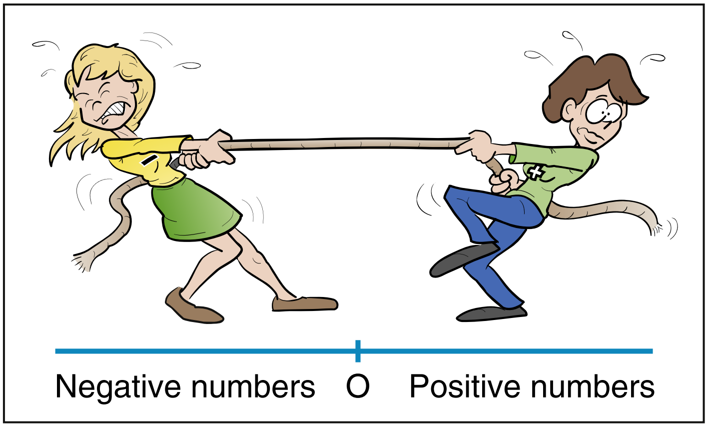

FIGURE 1-2 "Tug of war" between the positive and negative sides of the number line. (This figure was published in Doucette: *Basic Mathematics for the Health-Related Professions*, p. 18, copyright Elsevier, 2000.)

positive number is listed first, followed by the negative number. **The negative number is actually subtracted from the positive number to arrive at the sum.**

$$+12 - 23 = -11$$

The sum of these two numbers is -11, a negative number.

HOW ***Example 1-4***

What is the sum of +31 and −65?

When presented with an equation that contains both positive and negative numbers, first list the numbers in the equation so that the positive number(s) are listed first, followed by the negative number(s):

$$(+31) + (-65)$$

Then, solve the equation:

$$(+31) + (-65) = X$$
$$31 - 65 = -34$$

In this equation, the negative number was larger than the positive number, leading to the sum being a negative number. Remember: Think of the numbers as being a "tug of war." The sign of the larger number will determine the sign of the final sum (see Figure 1-2). In this example, since the number −65 is a larger number than +31, the final sum of 34 is a negative number (i.e., -34).

Example 1-4a Using Example 1-4 as a guide, what is the sum of −17 and +41?

The sum is $+24$ because the equation is set up as $+41 - 17 = X$.

Example 1-4b Using Example 1-4 as a guide, what is the sum of −32 and +14?

The sum is -18 because the equation is set up as $+14 - 32 = X$.

Example 1-4c Using Example 1-4 as a guide, what is the sum of −8 and +3?

The sum is -5 because the equation is set up as $+3 - 8 = X$.

Example 1-4d Using Example 1-4 as a guide, what is the sum of the following group of numbers?

$$-79,\ +11,\ -15,\ +43$$

When performing a calculation with both positive and negative numbers, always list the positive numbers first, followed by the negative numbers.

$$+11,\ +43,\ -79,\ -15$$

Then, determine the sum of the positive numbers:

$$+11 + 43 = +54$$

Next, determine the sum of the negative numbers:

$$-79 + -15 = -94$$

Last, set up the final equation of the sums of the positive and negative numbers:

$$(+54) + (-94) = X$$

Now solve for X:

$$54 - 94 = X$$
$$X = -40$$

Example 1-4e Using Example 1-4d as a guide, what is the sum of the following group of numbers: −66, −92, +83, +12?

The sum of this group of numbers is −63 because first determine the sum of the positive numbers, or +83 + 12 = +95. Next, determine the sum of the negative numbers, or − 66 + −92 = −158. Then, subtract −158 from +95 or 95 − 158 = X, which equals −63.

Example 1-4f Using Examples 1-4d and 1-4e as a guide, what is the sum of the following group of numbers: −21, +5, +8, −4?

The sum of this group of numbers is −12.

WHAT Subtraction of Two or More Positive Numbers

If the difference is greater than zero, it remains a positive number. However, if a larger positive number is subtracted from a smaller positive number, the difference will have a value less than zero and will be a negative number.

HOW ***Example 1-5***

What is the answer to the following equation?

$$(+34) - (+5) = X$$

The answer can be found using simple subtraction.

$$(+34) - (+5) = +29$$

The answer, +29, is a positive number, since it has a value greater than zero.

Example 1-5a Using Example 1-5 as a guide, what is the answer to the following equation?

$$(+59) - (+29) = X$$

59 − 29 = +30, a positive number, since 29 is less than 59.

Example 1-5b Using Example 1-5 as a guide, what is the answer to the following equation?

$$(+18) - (+4) = X$$

(+18) − (+4) = +14, a positive number, since 18 is a larger number than 4.

Example 1-5c Using Example 1-5 as a guide, what is the answer to the following equation?

$$(+101) - (+82) = X$$

$(+101) - (+82) = +19$, a positive number, since 101 is greater than 82.

Example 1-5d Using Example 1-5 as a guide, what is the answer to this equation?

$$(+56) - (+75) = X$$

Using simple subtraction:

$$56 - 75 = X$$

$X = -19$, a negative number, since 75 is greater than 56.

Example 1-5e Using Example 1-5 as a guide, what is the answer to this equation?

$$(+12) - (+38) = X$$

The answer is -26, since 38 is greater than 12.

Example 1-5f Using Example 1-5 as a guide, what is the answer to this equation?

$$(+55) - (+75) = X$$

The answer is -20, since 75 is greater than 55.

Example 1-5g Using Example 1-5 as a guide, what is the answer to this equation?

$$(+3) - (+7) = X$$

The answer is -4, since 7 is greater than 3.

WHAT Subtraction of Two or More Negative Numbers

When two negative numbers are subtracted from each other, the remainder remains negative as long as it is less than zero. However, as with positive numbers, if a larger negative number is subtracted from a smaller negative number, the difference will have a value greater than zero and be a positive number. Refer to the number line in Figure 1-1. This is because when a negative number is subtracted from a positive or negative number, it is actually added to the positive or negative number because of the following rule:

Two Negatives Rule

Two negatives become a positive (Figure 1-3).

Whether the final result is a positive or negative number depends on how much "pull" there is on the number line by the numbers in the equation.

HOW ***Example 1-6***

Solve the following equation:

$$(-53) - (-11) = X$$

In this equation, because the negative 11 is being subtracted from the negative 53, the double negatives convert to a positive (+) sign for the number 11.

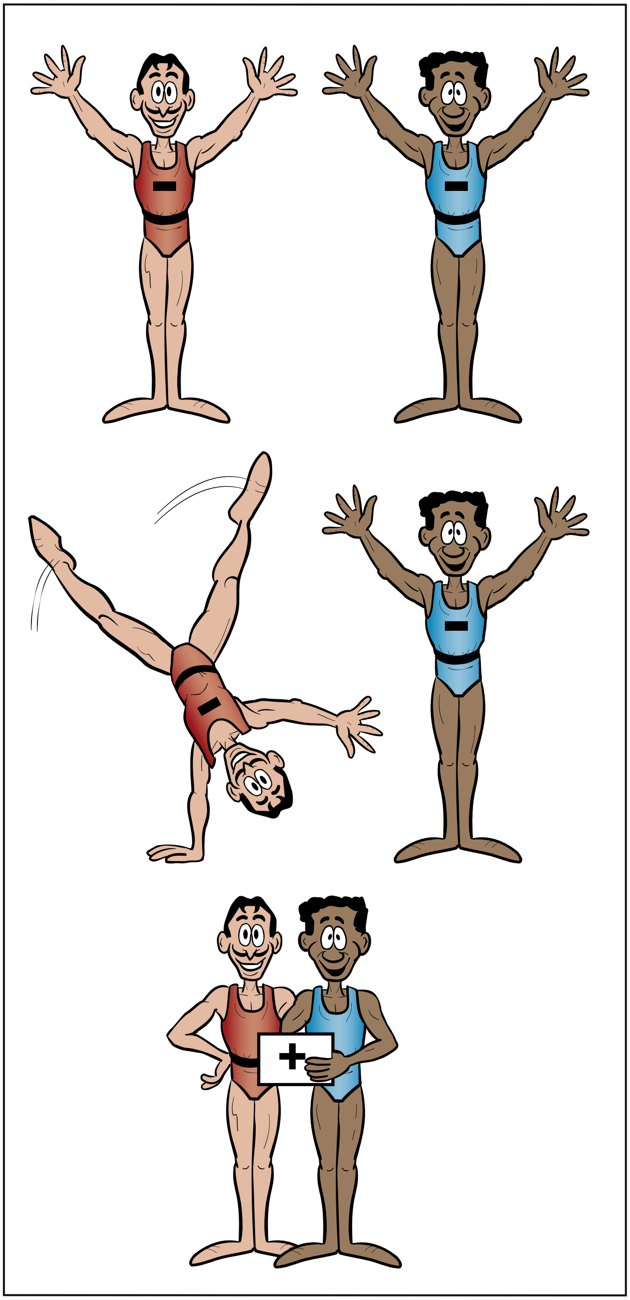

FIGURE 1-3 Two negatives become a positive. (This figure was published in Doucette: *Basic Mathematics for the Health-Related Professions*, p. 24, copyright Elsevier, 2000.)

$$(-53) + 11 = X$$

This equation can be rearranged to have the positive number listed first:

$$11 - 53 = X$$
$$\mathbf{X} = -42$$

The final answer is still a negative number because the (-11) did not have enough "pull" to pull the final result into the positive numbers.

Example 1-6a Using Example 1-6 as guide, solve the following equation:

$$(-21) - (-23) = X$$

Again, because two negatives make a positive, the sign associated with the number 23 is changed to a positive:

$$(-21) + (23) = X$$

The equation can be rearranged so that the positive number is listed first:

$$23 - 21 = X$$
$$\mathbf{X} = +2$$

In this case, the final result is a positive number because the number 23 had enough "pull" to pull the final result over the zero value in the number line.

Example 1-6b Using Example 1-6 as a guide, what is the answer to the following problem?

$$(-9) - (-5) = X$$

The problem is rearranged to be:

$$5 - 9 = X$$
$$\mathbf{X} = -4$$

Example 1-6c Using Example 1-6 as a guide, what is the answer to the following problem?

$$(-32) - (-37) = X$$

Again, by rearranging the problem:

$$37 - 32 = X$$
$$\mathbf{X} = +5$$

WHAT Subtraction of a Positive Number From a Negative Number

When a positive number is subtracted from a negative number, it is actually ADDED to the negative number.

HOW **Example 1-7**

Solve the following equation:

$$(-42) - (+15) = X$$

The number 15 is added to the 42 to have a final answer of -57.

Example 1-7a Using Example 1-7 as a guide, solve the following equation:

$$-85 - (+42) = X$$
$$X = -127$$

Example 1-7b Using Example 1-7 as a guide, solve the following equation:

$$(-124) - (+135) = X$$
$$X = -259$$

Example 1-7c Using Example 1-7 as a guide, solve the following equation:

$$(-38) - (+17) = X$$
$$X = -55$$

WHAT Subtraction of a Negative Number From a Positive Number

When a negative number is subtracted from a positive number, it is actually added to the positive number becase of the Two Negatives Rule (two negatives make a positive).

HOW **Example 1-8**

Solve the following equation:

$$(+40) - (-15) = X$$
$$X = +55$$

The result is a positive number, 55.

Example 1-8a Using Example 1-8 as a guide, solve the following equation:

$$(+15) - (-13) = X$$
$$X = +28$$

Example 1-8b Using Example 1-8 as a guide, solve the following equation:

$$(+26) - (-16) = X$$
$$X = +42$$

Example 1-8c Using Example 1-8 as a guide, solve the following equation:

$$(+82) - (-9) = X$$
$$X = +91$$

WHAT Multiplication or Division of Two or More Positive Numbers

When two or more positive numbers are multiplied or divided, the result will be a positive number.

HOW ***Example 1-9***

Solve the following equation:

$$(+14) \times (+2) = X$$

This equation of two positive numbers can be solved using simple multiplication:

$$14 \times 2 = 28$$

The product, 28, is also a positive number.

Example 1-9a Using Example 1-9 as a guide, what is the product of $(+47) \times (+5)$?

Using simple multiplication, the product is $+235$.

Example 1-9b Using Example 1-9 as a guide, what is the product of $(+7) \times (+43)$?

Using simple multiplication, the product is $+301$.

Example 1-9c Using Example 1-9 as a guide, what is the product of $(+24) \times (+8)$?

Using simple multiplication, the product is $+192$.

Example 1-9d Using Example 1-9 as a guide, solve the following equation:

$$+20 \div +4 = X \text{ OR } \frac{20}{4} = X$$

This is solved by simple division arithmetic:

$$X = +5$$

Example 1-9e Using Example 1-9d as a guide, solve the following equation:

$$+72 \div +6 = X$$
$$X = +12$$

Example 1-9f Using Example 1-9d as a guide, solve the following equation:

$$+50 \div +10 = X$$
$$X = +5$$

Example 1-9g Using Example 1-9d as a guide, solve the following equation:

$$+24 \div +3 = X$$
$$X = +8$$

WHAT Multiplication or Division of Two or More Negative Numbers

When two or more negative numbers are multiplied or divided, the result will be a positive number.

HOW

Example 1-10

Solve the following equation:

$$(-15) \times (-3) = X$$

This is solved by simple multiplication arithmetic:

$$X = +45$$

Example 1-10a Using Example 1-10 as a guide, solve the following equation:

$$(-55) \times (-6) = X$$
$$X = +330$$

Example 1-10b Using Example 1-10 as a guide, solve the following equation:

$$(-130) \times (-17) = X$$
$$X = +2,210$$

Example 1-10c Using Example 1-10 as a guide, solve the following equation:

$$(-8) \times (-23) = X$$
$$X = +184$$

Example 1-10d Using Example 1-10 as a guide, solve the following equation:

$$-45 \div -5 = X \quad \text{OR} \quad \frac{-45}{-5} = X$$

This is solved by simple arithmetic:

$$X = +9$$

Example 1-10e Using Example 1-10d as a guide, solve the following equation:

$$-68 \div -4 = X$$
$$X = +17$$

Example 1-10f Using Example 1-10d as a guide, solve the following equation:

$$-56 \div -8 = X$$
$$\mathbf{X} = +7$$

Example 1-10g Using Example 1-10d as a guide, solve the following equation:

$$-55 \div -11 = X$$
$$\mathbf{X} = +5$$

WHAT Multiplication or Division of Negative and Positive Numbers

The result of a mixture of both positive and negative numbers in both multiplication or division calculations will always be a negative number.

HOW ***Example 1-11***

Solve the following equation:

$$(-45) \times (+4) = X$$

This is a simple multiplication problem. Remember that the final result will be a negative number.

$$X = -180$$

Example 1-11a Using Example 1-11 as a guide, what is the product of −16 and +14?

The product is −224.

Example 1-11b Using Example 1-11 as a guide, what is the product of +15 and −6?

The product is −90.

Example 1-11c Using Example 1-11 as a guide, what is the product of −34 and +9?

The product is −306.

Example 1-11d Using Example 1-11 as a guide, solve the following equation:

$$(-28) \div (+4) = X$$

This equation can be solved by simple division; however, remember the final result will be a negative number.

$$(-28) \div (+4) = -7$$

Therefore, the result of this equation is −7.

Example 1-11e Using Example 1-11d as a guide, solve the following equation:

$$(-60) \div (+5) = X$$
$$X = -12$$

Example 1-11f Using Example 1-11d as a guide, solve the following equation:

$$(-75) \div (+5) = X$$
$$X = -15$$

Example 1-11g Using Example 1-11d as a guide, solve the following equation:

$$(-192) \div (+12) = X$$
$$X = -16$$

WHAT Order of Calculations

When an equation contains numbers within parentheses, the calculation within the parenthesis is performed first. For example, the equation $23 + (43 \times 3)$ is solved by first performing the calculation within the parenthesis. The equation then becomes $23 + 129$. The sum of this simple addition problem is 152.

HOW ***Example 1-12***

Solve the following problem:

$$45 + (11 \times 10) = X$$

First solve what is in the parentheses:

$$(11 \times 10) = 110$$

Next add 110 to 45:

$$45 + 110 = 155$$
$$X = 155$$

Example 1-12a Using Example 1-12 as a guide, solve the following problem:

$$2 + (9 \times 3) = X$$
$$X = 29$$

Example 1-12b Using Example 1-12 as a guide, solve the following problem:

$$16 - (2 \times 5) = X$$
$$X = 6$$

Example 1-12c Using Example 1-12 as a guide, solve the following problem:

$$42 \times (2 \times 5) = X$$
$$X = 420$$

Example 1-12d Now that you have mastered equations within parentheses, solve the following problem:

$$350 \div [4.0 + (275 \div 5.0)] = X$$

In this example, first solve the division calculation within the parentheses:

$$(275 \div 5.0) = 55$$

Next, solve the equation within the brackets:

$$[4.0 + (55)] = 59$$

Now, solve for X:

$$350 \div 59 = X$$
$$X = 5.9$$

Example 1-12e Using Example 1-12d as a guide, solve the following problem:

$$125 \div [82 - (70 \div 7.0)] = X$$
$$X = 1.7$$

Example 1-12f Using Example 1-12d as a guide, solve the following problem:

$$40 + [25 + (3 \times 15)] = X$$
$$X = 110$$

WHAT Fractions

A fraction is a mathematical way to represent the amount of "parts" within a number or substance. The top number, called the numerator, represents the "part"; the bottom number of the fraction, called the denominator, represents the entire amount of parts. For example, a pizza can be cut into four equal slices, or that same pizza can be cut into eight equal slices, each of which would be half as wide as the four slices (Figure 1-4). If the pizza was cut into four equal slices and you ate one of the slices, then you ate one of the four slices of pizza or ¼ of the pizza (Figure 1-5).

HOW ***Example 1-13***

Using the pizza example in Figure 1-4, what fraction would be made if two slices were eaten?

The 2 becomes the numerator, and the 4 becomes the denominator:

$$X = 2/4 \text{ or equivalent fraction } 1/2$$

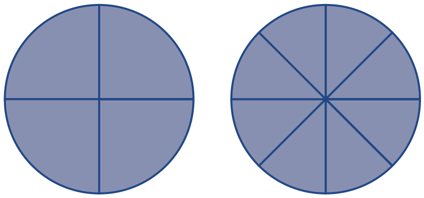

FIGURE 1-4 (A) Four slices of pizza. (B) Eight slices of pizza.

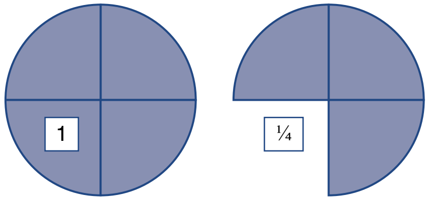

FIGURE 1-5 (A) One whole, (B) ¼.

Example 1-13a Using Example 1-13 as a guide, if you ate three pieces of the pizza, what fraction would be made?

$$X = 3/4$$

Example 1-13b Using Example 1-13 as a guide, if you ate all four slices of the pizza, what fraction would be made?

$$X = 4/4 \text{ or equivalent of } 1.0$$

WHAT ***Equivalent Fractions***

If the pizza discussed in Example 1-13 was cut into 8 equal slices (Figure 1-5; see Figure 1-4), and you ate two of them, then you also ate 1/4 of the pizza (or 2/8, which can be reduced to 1/4). Typically, fractions are written so that the numerator is equal to or as close to 1.0 as possible. Therefore, 2/8 becomes 1/4, 2/6 becomes 1/3, 10/100 becomes 1/10, and so forth. These new fractions are called equivalent fractions because the actual overall concentration is the same. Fractions are reduced to their lowest equivalent fraction by using division and algebra.

HOW ***Example 1-14***

Reduce the fraction 5/100 to its lowest equivalent fraction.

The numerator of a fraction should be as close to 1.0 as possible. A shortcut way is to divide the denominator, 100, by the numerator value 5 to obtain the equivalent fraction 1/20. To check the accuracy of your calculation, multiply both sides of the new fraction by 5—that is, $5/5 \times 1/20$—and the original fraction of 5/100 can be obtained.

Algebra can also be used, especially when the numerator does not divide easily into the denominator. Using Example 1-14 and algebra, the following equation is constructed:

$$1/X = 5/100$$

Crossmultiplying both sides yields $(1)(100) = (5)X$:

$$100 = 5X$$
$$100/5 = X$$
$$20 = X$$

Therefore, $5/100 = 1/20$.

HOW

Example 1-14a Using Example 1-14 as a guide, reduce the fraction $4/80$ to its lowest equivalent fraction.

The fraction $4/80$ can be reduced to its equivalent fraction of $1/20$.

Example 1-14b Using Example 1-14 as a guide, reduce the fraction $2/6$ to its lowest equivalent fraction.

The fraction $2/6$ can be reduced to its equivalent fraction of $1/3$.

Example 1-14c Using Example 1-14 as a guide, reduce the fraction $4/100$ to its lowest equivalent fraction.

The fraction $4/100$ can be reduced to its equivalent fraction of $1/25$

WHAT Ratio and Proportion With Fractions

From the pizza discussion in Example 1-13 and Figures 1-4 through 1-6, the fractions can also be thought of as ratios of the number of pizza slices to the amount of pizza slices that were cut. For example, eating one slice of pizza out of a pizza cut into four slices is a 1:4 ratio. Ratios are written with a colon separating the part from the whole. This same ratio can be written as the fraction $1/4$.

Many calculations in the laboratory use the concept of ratio and proportion. This concept will also be covered in Chapter 4. Whenever a different quantity, but NOT different concentration of a substance, is required, ratio and proportion calculations are performed. A ratio represents the relationship of one value to another. A 1:1 ratio represents an equal relationship as shown in Example 1-13b, where all four slices were eaten from a pizza cut into four slices. The fraction $4/4$ is equal to a 1:1 ratio whereas a 1:4 ratio means that one value (the pizza cut into four slices) is four times that of the other (an individual slice).

Ratio and proportion are used when a new quantity of a substance is required, and are based on an existing ratio. Examples 1-14 and 1-14a through 1-14c demonstrated how to reduce a fraction to its lowest equivalent form. When performing additions and subtractions of fractions,

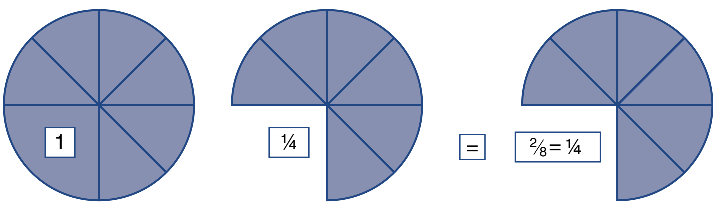

FIGURE 1-6 (A) One whole, (B) ¼, (C) $2/8 = 1/4$.

the denominators of each fraction must be the same value. By using ratio and proportion an equivalent fraction can be determined.

Example 1-15

Calculate the sum of the following fractions:

$$\frac{1}{8}+\frac{2}{8}+\frac{2}{4}=X$$

Because the denominators all need to be of the same value, $2/4$ must be converted to its equivalent with a denominator of 8 by the following equation:

$$\text{(Old fraction)}\frac{2}{4}=\frac{X}{8}\text{(New fraction)}$$

By crossmultiplying the numerator of $2/4$ with the denominator of $X/8$ and the denominator of $2/4$ with the numerator of $X/8$, the equation can be solved:

$$(2)(8) = (X)(4)$$
$$16 = (4)(X)$$

By multiplying both sides of the equation by $1/4$, the equation becomes:

$$\frac{16}{4}=X$$
$$4 = X$$

Thus, $2/4$ is equivalent to $4/8$.

Another way to solve this equation is to multiply both sides of the initial equation by 8:

$$\frac{8}{1}\times\frac{2}{4}=\frac{X}{8}\times\frac{8}{1}$$

The equation then becomes:

$$\frac{16}{4}=\frac{8X}{8}$$

and, by reducing the fractions, the answer is found:

$$4=X$$

for the equivalent fraction.

Next, complete the addition of all three fractions. Once a set of fractions to be added has the same denominator, simply add the numerators together:

$$\frac{1}{8}+\frac{2}{8}+\frac{4}{8}=X$$
$$\frac{7}{8}=X$$

HOW **Example 1-15a** Using ratio and proportion and Example 1-15 as a guide, what is the fraction $1/5$ equivalent to with a denominator of 15?

$$\frac{1}{5} = \frac{X}{15}$$

$$(1)(15) = (X)(5)$$

$$15 = (5)(X)$$

$$\frac{15}{5} = X$$

$$X = 3$$

Therefore, the fraction $1/5$ **is equivalent** to $3/15$.

Example 1-15b Using Example 1-15 as a guide, what is the fraction $1/10$ equivalent to with a denominator of 1000?

The fraction $1/10$ **is equivalent** to $100/1000$.

Example 1-15c Using Example 1-15 as a guide, what is the fraction $1/25$ equivalent to with a denominator of 125?

The fraction $1/25$ **is equivalent** to $5/125$.

Example 1-15d Using Example 1-15 as a guide, what is the fraction $1/3$ equivalent to with a denominator of 24?

The fraction $1/3$ **is equivalent** to $8/24$.

WHAT *Multiplication and Division of Fractions*

Multiplication of Fractions. When multiplying two or more fractions, the numerators are multiplied together and the denominators are multiplied together.

Example 1-16

What is the product of $2/5 \times 1/10$?

The product is determined by multiplying the numerators of each fraction ($2 \times 1 = 2$) followed by multiplying the denominators of each fraction ($5 \times 10 = 50$) to form the product $2/50$, or its reduced form $1/25$.

Example 1-16a What is the product of $1/3 \times 1\,5/8$?

To solve this problem, first convert the mixed fraction of $1\,5/8$ into its fraction form. This is done by multiplying the denominator 8 by the whole number 1 and adding that to the numerator to form the equivalent fraction of $13/8$. Next multiply $13/8$ and $1/3$. The product is $13/24$.

Example 1-16b Using Example 1-16 as a guide, what is the product of $7/8 \times 4/6$?

The product is $28/48$, reduced to $7/12$.

Example 1-16c Using Example 1-16a as a guide, what is the product of $1\,^3\!/_4 \times 1\,^1\!/_4$?

The product is $^{35}\!/_4$, reduced to $8\,^3\!/_4$.

Division of Fractions. Division problems have three terms: the dividend (which is the number that is being divided), the divisor (the number that is being used to divide the dividend), and the answer or quotient. When dividing fractions, the divisor fraction is simply flipped so that the numerator is now the denominator and the denominator is now the numerator. The flipped fraction is then multipled by the dividend fraction to calculate the quotient.

Example 1-17

Divide $^5\!/_8$ by $^1\!/_4$. The $^5\!/_8$ is the dividend and the $^1\!/_4$ is the divisor. Therefore, the problem becomes $^5\!/_8 \times {}^4\!/_1 = X$.

$$X = 20/8 \text{ or } 2\,^4\!/_8 \text{ or } 2\,^1\!/_2$$

Example 1-17a Using Example 1-17 as a guide, divide $^6\!/_{32}$ by $^4\!/_8$.

$$X = 48/128 = 3/8$$

Example 1-17b Using Example 1-17 as a guide, divide $^1\!/_5$ by $^3\!/_4$.

$$X = 4/15$$

Example 1-17c Using Example 1-17 as a guide, divide $^2\!/_5$ by $^1\!/_4$.

$$X = 1\,^3\!/_5 \text{ or } 2\,^3\!/_5$$

WHAT Percents

The use of percents and percentages in the laboratory is very common. A percent is a fraction with a constant denominator of 100 and is the ratio of the quantity of a substance "per 100" total parts.

Numbers may be written three different ways that all mean the same "per 100" relationship. One way is with the percentage (%) sign. For example, bleach used for decontamination is at a concentration of 10%, or ethanol (EtOH) may be at a concentration of 75%. The 10% bleach is a ratio of 10 parts bleach in 100 total parts. A 70% EtOH solution may be prepared by adding 70 mL absolute (100%) EtOH in a 100-mL volumetric flask and then adding 30 mL of water for the remaining volume. Chapter 5 also has more information on percent calculations.

In the second way, percents can be written as a fraction: 10% bleach could be written as $^{10}\!/_{100}$.

Some common fractions and their percent forms are:

$$\frac{1}{2} = 50\%$$

$$\frac{1}{4} = 25\%$$

$$\frac{1}{10} = 10\%$$

$$\frac{3}{4} = 75\%$$

The third way is in decimal form: 10% can be written as 0.10, 50% can be written as 0.50, and so forth.

WHAT

ROUNDING NUMBERS

In the laboratory, the preciseness of a measurement is determined by the rules that guide the use of significant figures and rounding numbers. Significant figures will be discussed in detail later in this chapter. "Rounding off" is the process of removing excess digits from a number so that number has its correct quantity of significant figures. For example, when using a calculator, a result shown on the display may contain many digits, far more than necessary for the equation that was solved. By using the rules for rounding numbers, the digits can be reduced to the correct number of significant figures based on the accuracy of a measurement.

WHAT

Rules for Rounding Numbers

1. If the First Digit to Be Dropped Is Less Than 5

If the first digit to be dropped is less than 5, then the last remaining digit stays the same.

HOW

Example 1-18

The number 15.683 needs to be rounded from five significant figures to four significant figures. Because the digit to be dropped (3) is less than 5, the last remaining digit, 8, stays the same and the number becomes 15.68.

Example 1-18a Using Example 1-18 as a guide, round the number 3.654 to three significant figures.

Because the number 4 is less than 5, the last remaining digit stays the same, and the number becomes 3.65.

Example 1-18b Using Example 1-18 as a guide, round the number 6.43 to two significant figures.

The number becomes 6.4.

Example 1-18c Using Example 1-18 as a guide, round the number 1.421 to three significant figures.

The number becomes 1.42.

WHAT

2. If the First Digit to Be Dropped Is Greater Than 5

If the first digit to be dropped is greater than 5, then the last remaining digit is rounded to the **next highest number.**

HOW

Example 1-19

The number 9.68 needs to be rounded from three significant figures to two significant figures. The number 8 is dropped, and because it is higher than 5, the number 6 is rounded to 7. Thus the final number becomes 9.7.

Example 1-19a Using Example 1-19 as a guide, round the number 3.469 to three significant figures.

The number becomes 3.47.

Example 1-19b Using Example 1-19 as a guide, round the number 5.37 to two significant figures.

The number becomes 5.4.

Example 1-19c Using Example 1-19 as a guide, round the number 13.369 to four significant figures.

The number becomes 13.37.

WHAT

3. If the First Digit to Be Dropped Is the Number 5

If the first digit to be dropped is the number 5, **the last remaining digit is rounded to the next higher number if it is an ODD number**. If the last remaining digit is an EVEN number, it is unchanged because statistically, to do so reduces positive bias in rounding. **For this rule, remember the last remaining digit always is or becomes an even number**.

HOW

Example 1-20

The number 6.35 needs to be rounded from three significant figures to two significant figures. As the last digit is 5, and the preceding digit is an odd number (3), the odd number is rounded to the next even number. Thus the final number becomes 6.4.

Example 1-20a Using Example 1-20 as a guide, round the number 41.675 to four significant figures.

The number 5 is dropped and the 7, an odd number, becomes an 8. The new number is 41.68.

Example 1-20b Using Example 1-20 as a guide, round the number 7.435 to three significant figures.

The new number is 7.44.

Example 1-20c Using Example 1-20 as a guide, round the number 5.35 to two significant figures.

The new number is 5.4.

Example 1-20d The number 8916.5 has to be rounded from five significant figures to four significant figures. The 5 is dropped, but because the preceding digit, 6, is an even number, it is unchanged. Thus the number becomes 8916, not 8917.

Example 1-20e Using Example 1-20d as a guide, round the number 12.45 to three significant figures.

The new number becomes 12.4.

Example 1-20f Using Example 1-20d as a guide, round the number 75.865 to four significant figures.

The new number becomes 75.86.

Example 1-20g Using Example 1-20d as a guide, round the number 7.125 to three significant figures.

The new number becomes 7.12.

WHAT SIGNIFICANT FIGURES

You've just seen how significant figures impact rounding numbers. But what are significant figures? Significant figures are used in the laboratory to determine the precision of measurements. If your supervisor told you to measure out 30 grams of sodium chloride, you would have to know if you needed to measure 30.0 grams or 30.1 grams or 29.9 grams. With any measurement there is an implied degree of uncertainty to the precision of the measurement. If the length of an instrument was measured to be 23.4 inches long by using a ruler that was only precise to the tenths, then the actual measurement could be anywhere between 23.35 and 23.45 inches long. This concept is called implied uncertainty. By using significant figures, the degree of implied uncertainty of measurements can be established.

In general, the exactness of a group of measurements is dependent on the least exact measurement. For example, if you were given the following volumes of 10.0 mL, 15.85 mL, and 7.775 mL for three different solutions that needed to be added together, the *sum* of these three solutions would not be 33.625 but 33.6. This is because of the three measurements taken, 10.0 mL has the least number of significant figures. You do not know if 10.05 mL instead of 10.0 mL was actually measured. The 10.0 mL measurement is precise only to the first decimal place. Because this measurement is limited in its precision, all other measurements must be limited in their precision as well.

However, in other measurements taken, the measurements may need to be much more precise. In many areas of the laboratory, measurements of solids, such as salts, are commonly taken to the second or third decimal place. Liquids may be measured by use of a graduated cylinder or by weighing. In either case, it is common to have the precision of the measurement determined by use of a decimal point and digits to the right of the decimal point. For example, if 5.75 grams of calcium carbonate ($CaCO_3$) has to be weighed, then the medical laboratory scientist (MLS; also referred to as a medical technologist) or medical laboratory technician (MLT) would know to measure to the hundredth gram.

Another common way to express the precision of a particular number is to use scientific notation. For example, 1000 could be written as 1.0×10^3, or 154,388 could be written as

1.54388×10^5. A number less than 1, such as 0.00465, could be written as 4.65×10^{-3}. In most scientific measurements, scientific notation is used most often because it is the most error-free method to denote precision. You will learn more information about scientific notation in Chapter 2.

WHAT

Working With Significant Figures

WHAT

Significant Figures and Zeroes

There are three rules to remember that apply to numbers that contain zeroes when working with significant figures.

WHAT 1. **Numbers That Contain Zeroes Within Them.** When a number contains a zero or zeroes as part of that number, the zero or zeroes are considered to be significant to that number and are never dropped out of the number.

HOW

Example 1-21

The number 1350723 has seven significant figures. The zero within the number is a significant number because it designates the thousands position. The decimal point after the last 3 (1350723) signifies that the 3 in the ones position is the least significant number and that the precision of this number is to the nearest whole number.

Example 1-21a Using Example 1-21 as a guide, how many significant figures are in the number 857.0042?

The number 857.0042 has seven significant figures. The two zeroes to the right of the decimal point are significant because they are within the entire number.

Example 1-21b Using Example 1-21 as a guide, how many significant figures are in the number 501.03?

The number 501.03 contains five significant figures.

Example 1-21c Using Example 1-21 as a guide, how many significant figures are in the number 1.006?

The number 1.006 contains four significant figures.

WHAT 2. **Numbers That Contain Zeroes at the End of the Number.** When a number greater than 1 contains a zero or zeroes at the end of the number, or to the right of the decimal place, the zero or zeroes are considered to be significant.

HOW

Example 1-22

The number 6300.0 has five significant figures.

Example 1-22a Using Example 1-22 as a guide, how many significant figures are in the number 310.00?

The number 310.00 contains five significant figures.

Example 1-22b Using Example 1-22 as a guide, how many significant figures are in the number 47.00?

The number 47.00 contains four significant figures.

Example 1-22c Using Example 1-22 as a guide, how many significant figures are in the number 34.00?

The number 34.00 contains four significant figures.

WHAT **3. Numbers Less Than 1 That Contain Zeroes to the Right of the Decimal Point.** When a **number is less than 1, any zeros that are before or after a decimal point but before the first nonzero number within the number are NOT considered to be significant**.

HOW ***Example 1-23***

The number 0.00663 has three, not five, significant figures. The two zeroes to the right of the decimal point are not considered significant.

Example 1-23a Using Example 1-19 as a guide, how many significant figures are in the number 0.037?

The number 0.037 has two significant figures, the 3 and 7.

Example 1-23b Using Example 1-19 as a guide, how many significant figures are in the number 0.006?

The number 0.006 has one significant figure.

Example 1-23c Using Example 1-19 as a guide, how many significant figures are in the number 0.0333?

The number 0.0333 has three significant figures.

WHAT ***Significant Figures in Addition and Subtraction***

There are rules to use when working with significant figures and performing calculations. For calculations involving addition or subtraction, the sum or difference of a group of numbers cannot be any more precise than the least precise number in the group of numbers that were added or subtracted. The final result of an addition or subtraction problem can only have as many digits to the right of the decimal point as the least precise number involved in the problem.

HOW ***Example 1-24***

Solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 11.5 \\ 14.411 \\ +13.65 \\ \hline 39.561 \end{array} = 39.6$$

The answer, 39.6, can only be precise to one digit to the right of the decimal point, which is what is found in the least precise number 11.5.

Example 1-24a Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 4.05 \\ +\,4.8 \\ \hline 8.85 \end{array} = 8.8$$

The sum is 8.8. The 5 of 8.85 is dropped and the 8 stays the same as it is an even number.

Example 1-24b Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 2.77 \\ 4.489 \\ +\,15.44 \\ \hline 22.699 \end{array} = 22.70$$

The sum is 22.70.

Example 1-24c Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 34.654 \\ 12.895 \\ +\,4.66 \\ \hline 52.209 \end{array} = 52.21$$

The sum is 52.21.

Example 1-24d Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 10.233 \\ -\,3.45 \\ \hline 6.783 \end{array} = 6.78$$

The result of this equation becomes 6.78, not 6.783, because the least precise number, 3.45, has two digits to the right of the decimal point.

Example 1-24e Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 168.314 \\ -104.4 \\ \hline 63.914 \end{array} = 63.9$$

The result is 63.9.

Example 1-24f Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures:

$$\begin{array}{r} 71.27 \\ -\ 3.4 \\ \hline 67.87 \end{array} = 67.9$$

The result is 67.9.

Example 1-24g Using Example 1-24 as a guide, solve the following equation with the correct amount of significant figures.

$$\begin{array}{r} 450.48 \\ -\ 10.4 \\ \hline 440.08 \end{array} = 440.1$$

The result is 440.1.

WHAT ***Significant Figures in Multiplication and Division***

When performing multiplication and division calculations, the **final result cannot contain more significant figures than the number in the calculation with the least number of significant figures**.

HOW ***Example 1-25***

$$\begin{array}{r} 4.977 \\ \times 1.83 \\ \hline 9.10791 \end{array} = 9.11$$

The final answer is 9.11 because 1.83, with three significant figures, is the limiting number.

Example 1-25a Using Example 1-25 as a guide, determine the product of the following equation:

$$\begin{array}{r} 61.45 \\ \times\ 2.70 \\ \hline 165.915 \end{array} = 166$$

The product is 166, with three significant figures.

Example 1-25b Using Example 1-25 as a guide, determine the product of the following equation:

$$\begin{array}{r} 15.1 \\ \times\ 11.5 \\ \hline 173.65 \end{array} = 174$$

The product is 174, with three significant figures.

Example 1-25c Using Example 1-25 as a guide, determine the product of the following equation:

$$\begin{array}{r} 245.981 \\ \times \quad 3.456 \\ \hline 850.11033 = 850.1 \end{array}$$

The product is 850.1, with four significant figures.

Example 1-25d Determine the correct amount of significant figures for this division problem:

$$45.14 \div 8.6 = 5.2488 = 5.2$$

The answer becomes 5.2 because 8.6 has the least number of significant figures.

Example 1-25e Using Example 1-25d as a guide, solve the following equation:

$$6.28 \div 1.1 = 5.709 = 5.7$$

5.709 becomes 5.7 because 1.1 has two significant figures.

Example 1-25f Using Example 1-25d as a guide, solve the following equation:

$$8.96 \div 2.1 = 4.267$$

4.267 becomes 4.3 because 2.1 has two significant figures.

Example 1-25g Using Example 1-25d as a guide, solve the following equation:

$$935.44 \div 15.7 = 59.582$$

59.582 becomes 59.6 because 15.7 has three significant figures.

HOW EXAMPLE PROBLEMS

This section is designed to be useful to both the student and the laboratory professional. Students can use the additional problems to master the material. The laboratory professional can use the examples as templates for solving laboratory calculations. By finding an example similar to the problem that you need to solve, you can substitute into the equation the numbers appropriate to your calculation.

1. **Q.** What is the sum of +2, +9, and +7?
   A. The sum is +18. The problem is solved using simple addition.
2. **Q.** What is the sum of +14, +32, +53, and +47?
   A. The sum is +146. The problem is solved using simple addition.
3. **Q.** What is the sum of −3, −7, and −4?
   A. The sum is −14. The problem is solved using simple addition. The sum of two or more negative numbers will also be a negative number.
4. **Q.** What is the sum of −61, −25, −35, −83, and −98?
   A. The sum is −302. This calculation is solved similar to Example Problem 3.

5. **Q.** What is the sum of -71 and $+14$?
   A. The sum is -57. Remember to place the positive number first in the equation, followed by the negative number. Remember also that in an addition equation with a positive and a negative number, the final result will be the sign of the larger number as it "pulls" the smaller number across the number line.

6. **Q.** What is the sum of -11 and $+34$?
   A. The sum is $+23$. Refer to Example Problem 5 for details.

7. **Q.** What is the sum of -5, $+16$, -11, and $+13$?
   A. The sum is 13. Remember to group all of the positive numbers together first, and then group the negative numbers together. Add the positive numbers, then add the negative numbers together, and finally add the sum of the positive numbers with the sum of the negative numbers to give you the final sum.

$$(+16) + (+13) = +29$$
$$(-5) + (-11) = (-16)$$
$$29 - 16 = 13$$

8. **Q.** What is the difference of $(+6) - (+4)$?
   A. The difference is $+2$. This is a simple subtraction calculation with positive numbers. $6 - 4 = 2$.

9. **Q.** What is the difference of $(-9) - (+4)$?
   A. The difference is -13. When a positive number is subtracted from a negative number, it is actually added to the negative number.

10. **Q.** What is the difference of $(+6) - (+2)$?
    A. The difference is $+4$. This is a simple subtraction problem of two positive numbers.

11. **Q.** What is the answer if -20 is subtracted from -45?
    A. The answer is -25. The -20 is actually added to the -45 using the Two Negatives Rule where the negative sign becomes a positive sign.

12. **Q.** What is the answer to the following problem?

$$(65.0) \times (5.00) = X$$

    A. The answer is 325.This is solved as a simple multiplication calculation.

13. **Q.** Solve for X.

$$55 \div 5.0 = X$$

    A. The answer is 11. This is a simple division calculation.

14. **Q.** Solve for X.

$$(-60) \div (-5.0) = X$$

    A. The answer is $+12$. Remember that when two or more negative numbers are multiplied or divided, the result will be a positive number.

15. **Q.** Solve for X.

$$7.0 \times (15 \div 3.0) = X$$

A. When peforming a calculation that includes an internal calculation, always peform the calculation within the parentheses first, followed by the rest of the equation. The answer is 35.

16. **Q.** Solve for X.

$$316 \div (50 + 9.0) = X$$

A. The answer is 5.4. This calculation is solved similar to Example Problem 15.

17. **Q.** What is the name of the number that is in the top part of a fraction?
A. The numerator is the top number in a fraction.

18. **Q.** What is the name of the number that is in the bottom part of a fraction?
A. The denominator is the bottom number in a fraction.

19. **Q.** Calculate the lowest equivalent fraction for $^{5}/_{15}$.
A. The lowest equivalent fraction is $^{1}/_{3}$.

20. **Q.** Calculate the lowest equivalent fraction for $^{10}/_{70}$.
A. The lowest equivalent fraction is $^{1}/_{7}$.

21. **Q.** Solve for X.

$$\frac{4}{5} + \frac{1}{2} = X$$

A. Remember that the denominators for addition and subtraction calculations must be the same value. The equivalent fractions become $^{8}/_{10} + {}^{5}/_{10} = {}^{13}/_{10}$ or 1 $^{3}/_{10}$.

22. **Q.** Solve for X. Use Example Problem 21 as a guide.

$$\frac{2}{3} + \frac{1}{4} = X$$

A. $X = {}^{11}/_{12}$.

23. **Q.** Solve for X. Use Example Problem 21 as a guide.

$$\frac{3}{4} - \frac{4}{8} = X$$

A. $X = {}^{2}/_{8}$ or ¼.

24. **Q.** Solve for X.

$$\frac{4}{5} \times \frac{1}{3} = X$$

A. The answer is $^{4}/_{15}$. Remember that when multiplying fractions, the numerators are multiplied together and the denominators are multiplied together.

25. Q. Solve for X. Use Example Problem 24 as a guide.

$$\frac{2}{5} \times \frac{3}{4} = X$$

A. $X = 3/10$.

26. Q. Solve for X.

$$\frac{6}{7} \div \frac{1}{2} = X$$

A. Remember that in division calculations, the divisor fraction is inverted and the calculation becomes a multiplication problem.

$$\frac{6}{7} \times \frac{2}{1} = X$$

$$X = \frac{12}{7} \text{ or } 1\frac{5}{7}$$

27. Q. If 4.863 is to be rounded to only two significant figures, what would the final number become?

A. The final number would be 4.9. The 3 would be dropped and the 6 would round the number 8 up to 9.

28. Q. If 0.95482 is to be rounded to three significant figures, what would the final number become?

A. The final number would be 0.955. The 2 and 8 would be dropped, and because 8 is more than 5, the 4 is rounded up to 5.

29. Q. Given that 7.36 is to be rounded to two significant figures, what would the final number become?

A. 7.36 would be rounded to 7.4. The 6 is dropped, and as 6 is more than 5, the 3 is changed to the number 4.

30. Q. The number 0.928 must be changed to contain only two significant figures. What would the final number become?

A. The final number would become 0.93. The 8 is dropped, and the 2 rounded to the number 3.

31. Q. Given that the number 6.97 is to be rounded to two significant figures, what would that number be?

A. 6.97 would be rounded to 7.0. The 7 would be dropped, and the 9 rounded to 10. This moves the 6 to become 7.

32. Q. If the number 7.35 is to be rounded to two significant figures, what would the final number become?

A. 7.35 would be rounded to 7.4. Remember that when the digit to be dropped is 5, the preceding digit is rounded up if it is an odd number, but unchanged if it is an even number.

33. Q. If the number 0.4845 needs to be rounded to three significant figures, what would the number become?

A. 0.4845 would become 0.484. The 5 is dropped because only three, not four, significant numbers are needed. However, as the preceding digit is an even number, it remains unchanged.

34. Q. The number 3.57 has how many significant figures?

A. This number has three significant figures and is precise to the $1/100$ place.

35. Q. The number 14.067 has how many significant figures?

A. This number has five significant figures and is precise to the $1/1000$ place.

36. Q. How many significant figures are contained in the number 0.00352?

A. This number has three significant figures. Remember that zeroes that are before or after a decimal point but precede any nonzero numbers are not significant.

37. Q. How many significant figures are contained in the number 0.05092?

A. This number has four significant figures. The zeroes on either side of the decimal point are *not* significant, whereas the zero contained within the number is significant.

38. Q. How many significant figures are found in the number 5370.00?

A. 5370.00 contains six significant figures. The zeroes before and after the decimal point are considered to be significant because they describe the preciseness of this number.

39. Q. How many significant figures are in the sum of $15.7 + 12.9$?

A. The sum, 28.6, has three significant figures. Remember: For calculations involving addition or subtraction, the sum or difference of a group of numbers cannot be any more precise than the least precise number in the group of numbers that were added or subtracted. The final result of an addition or subtraction problem can have as many **digits** to the right of the decimal point as the least precise number involved in the problem.

40. Q. The sum of $45.29 + 8.4$ has how many significant figures?

A. The sum, 53.7, has three significant figures. Remember that the sum of a group of numbers must have the number of digits to the right of the decimal point of the least precise number and cannot be more precise than the least precise number (8.4) in the problem.

41. Q. $45.2 - 3.6$ results in an answer with how many significant figures?

A. The answer, 41.6, contains three significant figures.

42. Q. The product of 4.55×8.77 contains how many significant figures?

A. The product, 39.9, contains three significant figures. Remember that in multiplication, the final product cannot contain more significant figures than the least precise number in the problem.

43. Q. How many significant figures does the product of 0.382×0.415 contain?

A. The product, 0.158, contains three significant figures.

44. Q. $592.1 \div 8.7$ results in an answer with how many significant figures?

A. The answer, 68, contains two significant figures. You may have calculated the answer to be 68.06. Remember that, in division, the final answer cannot contain more significant figures than the least significant number in the problem. Because 8.7 contains two significant figures, the final answer is limited to two significant figures.

HOW

PRACTICE PROBLEMS

Solve the following practice problems to further master the material. Answers and explanations to some problems can be found in the Answer Key.

Round the following list of numbers to three significant figures.

1. 0.743
2. 17.33
3. 5.687
4. 5.889
5. 5.558
6. 7.735
7. 2.325

Given the following list of numbers, determine the quantity of significant figures in each number.

8. 751.50
9. 5080
10. 987.441
11. 0.91
12. 0.98260
13. 294.00

Solve the following equations. Correct answers must contain the required quantity of significant figures.

14. $18.2 + 19.8 + 15.97 + 14.1 = ?$
15. $42.5 + 15.99 = ?$
16. $(-5) + (-7) = ?$
17. $(-21.0) + (-32.1) + (-15.0) =$
18. $(-32) + (+25) = ?$
19. $(-14) + (+5) = ?$
20. $(-6) + (+12) = ?$
21. $(-5) + (+14) = ?$
22. $(+32) + (-8) + (+15) + (-7) = ?$
23. $215.947 - 32.22 = ?$
24. $0.066 - 0.003 = ?$
25. $(+47) - (+22) = ?$

26. $(+2) - (+9) = ?$

27. $(-16) - (-12) = ?$

28. $(-5) - (+1) = ?$

29. $(-19) - (+14) = ?$

30. $(-8) - (-5) = ?$

31. $8.555 \times 0.11 = ?$

32. $2.824 \times 8.44 = ?$

33. $(-41) \times (-2.0) = ?$

34. $(-4.20) \times (-11.1) = ?$

35. $(-3) \times (+5) = ?$

36. $(-2.3) \times (+1.1) = ?$

37. $(-3.93) \div (-1.44) = ?$

38. $(-25) \div (-5) = ?$

39. $4.77 \div 0.63 = ?$

40. $5.2 \div 4.1 = ?$

41. $4.1 \times (12.0 \div 8.0) = ?$

42. $15.2 + (41.0 - 23.5) = ?$

Reduce the following fractions to their lowest equivalent forms.

43. $\frac{15}{20}$

44. $\frac{12}{60}$

45. $\frac{4}{8}$

46. $\frac{3}{9}$

Solve the following calculations.

47. $\frac{2}{9} \times \frac{1}{4} = X$

48. $1\frac{1}{7} \times \frac{1}{2} = X$

49. $\frac{3}{8} + \frac{3}{5} = X$

50. $\frac{5}{8} + \frac{1}{4} = X$

51. $\frac{3}{8} - \frac{1}{5} = X$

52. $\frac{5}{7} - \frac{1}{2} = X$

53. $\frac{1}{2} \div \frac{1}{3} = X$

54. $\frac{5}{8} \div \frac{1}{4} = X$

QUICK NOTES

- The sum of two or more positive numbers will be a **positive number.**
- The sum of two or more negative numbers will be a **negative number.**
- The sum of an addition of both positive and negative numbers will be the sign of the **larger number** involved in the addition. When a negative number is added to a positive number, it is actually **subtracted** from the positive number.
- When subtracting two or more positive numbers, if the difference is greater than zero, it remains a positive number. However, if a larger positive number is subtracted from a smaller positive number, the difference will have a value less than zero and will be a negative number.
- When two negative numbers are subtracted from each other, the remainder remains negative as long as it is less than zero. However, as with positive numbers, if a larger negative number is subtracted from a smaller negative number, the **difference** will have a value greater than zero and be a positive number. Remember the Two Negatives Rule: Two negatives make a positive.
- When a positive number is subtracted from a negative number, it is actually **ADDED** to the negative number.
- When a negative number is subtracted from a positive number, it is actually **ADDED** to the positive number because of the Two Negatives Rule.
- When two or more positive numbers are multiplied or divided, the result will be a **positive number.**
- When two or more negative numbers are multiplied or divided, the result will be a **positive number.**
- When there is a mixture of positive and negative numbers in both multiplication and/or division calculations, the results will always be a **negative number.**
- When a calculation contains numbers within parentheses, always perform the calculation within the parentheses first.
- When rounding numbers, the final number cannot be more precise than the least precise number within the calculation.
- When rounding numbers, if the first digit to be dropped is less than 5, then the last remaining digit stays the same.
- When rounding numbers, if the first digit to be dropped is greater than 5, then the last remaining digit is rounded up to the next highest number.
- When rounding numbers, if the first digit to be dropped is equal to 5, then the last remaining digit is rounded up to the next highest number if it is an ODD number but left the same if it is an EVEN number.
- A fraction is a mathematical way to represent the amount of "parts" within a number or substance. The top number, called the numerator, represents the "part"; the bottom number of the fraction, called the denominator, represents the entire amount of parts.

- When performing additions and subtractions of fractions, the denominators of each fraction must be the same value. By using ratio and proportion, an equivalent fraction can be determined.
- When multiplying two or more fractions, the numerators are multiplied together and the denominators are multiplied together.
- Division problems with fractions have three terms: the dividend (which is the number that is being divided), the divisor (the number that is being used to divide the dividend), and the answer or quotient. When dividing fractions, the divisor fraction is simply flipped so that the numerator is now the denominator and the denominator is now the numerator. The flipped fraction is then multipled by the dividend fraction to calculate the quotient.
- To determine significant figures, when a number has zeroes within the number, those zeroes are considered significant and are not removed from the number.
- To determine significant figures, when a number has zeroes at the end of the number, those zeroes are considered significant to the number and not removed.
- To determine significant figures, when a number is less than 1.0, any zeroes that are before or after the decimal point but before the first nonzero number within the number are not considered significant.
- When performing addition or subtraction calculations, the final result can only have as many digits to the right of the decimal point as the least precise number involved in the problem.
- When performing multiplication or division calculations, the final result cannot contain more significant figures than the number in the calculation with the least number of significant figures.

CHAPTER 2

Scientific Notation and Logarithms

OUTLINE

OBJECTIVES

At the end of this chapter, the reader should be able to do the following:

1. State the rules for multiplication, division, subtraction, and addition using scientific notation.
2. Convert scientific notation numbers to whole numbers and vice versa.
3. Perform multiplication, division, subtraction, and addition calculations involving scientific notation.
4. State the rules for multiplication, division, subtraction, and addition using logarithms.
5. Perform multiplication, division, subtraction, and addition calculations using logarithms.
6. Define the terms *mantissa* and *characteristic* and explain how they relate to logarithms.
7. Define the terms *mantissa, base,* and *exponents* for scientific notation.

WHAT EXPONENTS AND SCIENTIFIC NOTATION

In mathematics, multiplication is used as a faster method than simple addition when performing multiple addition problems. For example, $6 \times 7 = 42$. The same answer could be found by adding 6 seven times: $6 + 6 + 6 + 6 + 6 + 6 + 6 = 42$.

Exponents are a way to simplify complex multiplication problems.

Exponents are written as x^a, where x is called the base and a is the exponent. The base is the number that is to be multiplied by itself, and the exponent determines how many times it will be multiplied.

For example, 2^4 is equal to $2 \times 2 \times 2 \times 2$ or 16 and 3^3 is equal to $3 \times 3 \times 3$ or 27. In complicated calculations involving many exponents, it is easy to see how the use of exponents can reduce the chance of errors in performing the arithmetic of the calculation.

In the clinical laboratory, scientific notation is often used in calculations to simplify the calculation and is used when expressing standard concentrations of some analytes. For example, a white blood cell count is expressed in units of number of cells counted per

10^9/L (e.g., 4.00 × 10^9/L). Scientific notation is also sometimes used when performing calculations with logarithms, which will be covered later in this chapter. In scientific notation, a number is written in such a way that it is larger than 1 but less than 10 and an integral power of 10. For example, the number 2820000.0 can be expressed as 2.82 × 10^6 because the decimal point can be moved six places to the left. The 2.82 is referred to as the **mantissa number** by some mathematicians. This book will also refer to it as the mantissa number.

$$2\underbrace{8}_{6}\underbrace{2}_{5}\underbrace{0}_{4}\underbrace{0}_{3}\underbrace{0}_{2}\underbrace{0}_{1}.0 = 2.82 \times 10^6$$

There are seven rules (Rules 1–7) for using scientific notation and three rules (Rules 8–10) for determining logarithms or performing calculations with logarithms. If these rules are understood and followed, errors in calculations using scientific notation will be reduced.

WHAT Rules 1 to 7 for Using Scientific Notation

Rule 1

Exponents used in scientific notation can be positive or negative numbers. Exponents that are negative usually indicate a number that is less than 1. A negative sign is placed to the left of the exponent to indicate that it is a negative exponent. Exponents that are positive generally do not have any sign associated with them. It is assumed that if a negative sign is not present, then the exponent is positive.

Positive Exponents. Remember that in scientific notation, the mantissa number is written to be between 1 and 10 with a power of 10. Sometimes in the clinical laboratory there will be exceptions in keeping the mantissa number between 1 and 10. You may see some examples of this in the clinical hematology and the clinical chemistry chapters later in this text.

HOW ***Example 2-1*** $6 \times 10^2 = (6) \times (10) \times (10) = 6 \times 100 = 600$

Example 2-1a $5 \times 10^4 = (5) \times (10) \times (10) \times (10) \times (10) = 5 \times 10000 = 50{,}000$

Example 2-1b $6 \times 10^1 = (6) \times (10) = 6 \times 10 = 60$

Example 2-1c $7 \times 10^5 = (7) \times (10) \times (10) \times (10) \times (10) \times (10) = 7 \times 100000 = 700{,}000$

WHAT **Negative Exponents.** Numbers with negative exponents are expressed with the following formula:

$$b^{-a} = \frac{1}{b^a}$$