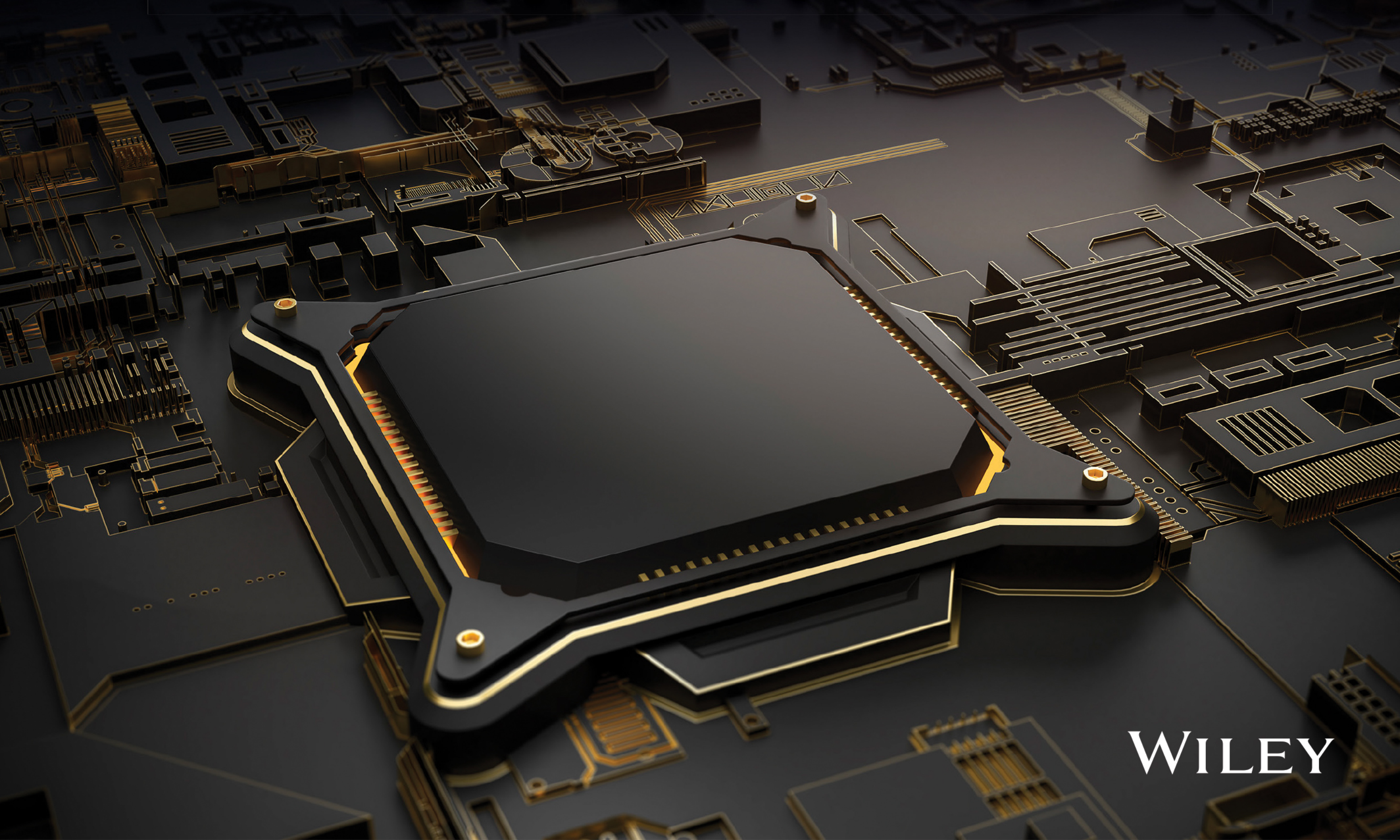
J. DAVID IRWIN • R. MARK NELMS

BASIC ENGINEERING CIRCUIT ANALYSIS

TWELFTH EDITION



WILEY

Basic Engineering Circuit Analysis

12th Edition

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In Loving Memory of My Grandson

Ryan Watson Frazier (1995-2015)

To my parents:

Robert and Elizabeth Nelms

Brief Contents

Contents

Preface

To the Student

Circuit analysis is not only fundamental to the entire breadth of electrical and computer engineering—the concepts studied here extend far beyond those boundaries. For this reason, it remains the starting point for many future engineers who wish to work in this field. The text and all the supplementary materials associated with it will aid you in reaching this goal. We strongly recommend while you are here to read the Preface closely and view all the resources available to you as a learner. One last piece of advice: Learning to analyze electric circuits is like learning to play a musical instrument. Most people take music lessons as a starting point. Then, they become proficient through practice, practice, and more practice. Lessons on circuit analysis are provided by your instructor and this textbook. Proficiency in circuit analysis can only be obtained through practice. Take advantage of the many opportunities throughout this textbook to practice, practice, and practice. In the end, you'll be thankful you did.

To the instructor

The Twelfth Edition has been prepared based on a careful examination of feedback received from instructors and students. The revisions and changes made should appeal to a wide variety of instructors. We are aware of significant changes taking place in the way this material is being taught and learned. Consequently, the authors and the publisher have created a formidable array of traditional and nontraditional learning resources to meet the needs of students and teachers of modern circuit analysis.

By design, this book contains a bank of reserve assessment problems not available to students. As a time-saving measure, the instructor can use this bank of problems for exam questions, term after term, without repetition. These problems are organized by chapter section and categorized as easy, medium, and hard. The end-of-chapter problems have been arranged in the same manner. Dedicated students will find these problems an excellent resource for testing their understanding on a range of problems.

Flipping the classroom has risen recently as an alternative mode of instruction, which attempts to help the student grasp the material quicker. Studies to date have shown that this mode also tends to minimize instructor office time. This book, with its combination of Learning Assessments and problem-solving videos, is an ideal vehicle for teaching in this format. These resources provide the instructor with the tools necessary to modify the format of the presentation in the hope of enhancing the student's rapid understanding of the material.

In accordance with the earlier editions, the book contains a plethora of examples that are designed to help the student grasp the salient features of the material quickly. New examples have been introduced, and MATLAB® has been employed, where appropriate, to provide a quick and easy software solution as a means of comparison, as well as to check other solution techniques. Application and Design Examples have returned to this edition. These help students answer questions such as, "Why is this important" or "How am I going to use what I learn from this course?"

Highlights of the Twelfth Edition

- A crisp, clean new interior design aimed at enhancing, clarifying, and unifying the text as well as increasing accessibility for all.
- End-of-chapter homework problems have been substantially revised and augmented. There are now approximately 2400 problems in the Twelfth Edition, of which over 800 are new! Multiple-choice Fundamentals of Engineering (FE) Exam problems also appear at the end of each chapter.
- This course offers Reserve Assessment Bank Problems, which are not available to students unless they are assigned. This allows you, as the instructor, to regulate who sees these questions and the solutions. We now publish new Reserve Assessment Bank Problems on an ongoing basis so you'll frequently have fresh material to work with without needing to move to a whole new edition, and you'll have a constant source of problems to use for higher-stakes assessment. Recognizing that third-party solutions become readily available online as soon as a new edition is published, we now offer whole sets of problems for every chapter that are visible only to the instructor, hindering cheating and allowing instructors to choose which problems to assign as homework, and when and whether to provide solutions.
- Problem-Solving Videos (PSVs) have been created, showing students step-by-step how to solve all Learning Assessment problems within each chapter. This is a special feature that should significantly enhance the learning experience for each subsection in a chapter.
- In order to provide maximum flexibility, online supplements contain solutions to examples in the book using MATLAB, PSpice®, or MultiSim®. The worked examples can be supplied to students as digital files, or one or more of them can be incorporated into custom print editions of the text, depending on the instructor's preference.

- Problem-Solving Strategies have been retained in the Twelfth Edition. They are utilized as a guide for the solutions contained in the PSVs.

Organization

This text is suitable for a one-semester, a two-semester, or a three-quarter course sequence. The first seven chapters are concerned with the analysis of dc circuits. An introduction to operational amplifiers is presented in Chapter 4. This chapter may be omitted without any loss of continuity; a few examples and homework problems in later chapters must be skipped. Chapters 8–12 are focused on the analysis of ac circuits beginning with the analysis of single-frequency circuits (single-phase and three-phase) and ending with variable-frequency circuit operation. Calculation of power in single-phase and three-phase ac circuits is also presented. The important topics of the Laplace transform, Fourier transform, and two-port networks are covered in Chapters 13–16.

The organization of the text provides instructors maximum flexibility in designing their courses. One instructor may choose to cover the first seven chapters in a single semester, while another may omit Chapter 4 and cover Chapters 1–3 and 5–8. Other instructors have chosen to cover Chapters 1–3, 5–6, and sections 7.1 and 7.2 and then cover Chapters 8 and 9. The remaining chapters can be covered in a second semester course.

Text Pedagogy

The pedagogy of this text is rich and varied. It includes print and media, and much thought has been put into integrating its use. To gain the most from this pedagogy, please review the following elements commonly available in most chapters of this book.

Learning Objectives are provided at the outset of each chapter. This tabular list tells the reader what is important and what will be gained from studying the material in the chapter.

Examples are the mainstay of any circuit analysis text, and numerous examples have always been a trademark of this textbook. These examples provide a more graduated level of presentation with simple, medium, and challenging examples. Besides regular examples, numerous **Application Examples** and **Design Examples** are found throughout the text.

Hints can often be found in the page margins. They facilitate understanding and serve as reminders of key issues.

Learning Assessments are a critical learning tool in this text. These exercises test the cumulative concepts to that point in a given section or sections. Not only is the answer provided, but a problem-solving video accompanies each of these exercises, demonstrating the solution in step-by-step detail. The student who masters these is ready to move forward.

Problem-Solving Strategies are step-by-step problem-solving techniques that many students find particularly useful. They answer the frequently asked question, “Where do I begin?” Nearly every chapter has one or more of these strategies, which are a kind of summation on problem solving for concepts presented.

Problems have been greatly revised for the Twelfth Edition. This edition has over 800 new problems of varying depth and level. Any instructor will find numerous problems appropriate for any level class. There are approximately 2400 problems in the Twelfth Edition! Included with the Problems are FE Exam Problems for each chapter. If you plan on taking the FE Exam, these problems closely match problems you will typically find on the FE Exam. Reserve Assessment Bank Problems, not available to students, have been added to this edition for the instructor to utilize as exam questions.

Circuit Simulation and Analysis Software represents a fundamental part of engineering circuit design today. Software such as **PSpice**, **MultiSim**, and **MATLAB** allow engineers to design and simulate circuits quickly and efficiently. As an enhancement with enormous flexibility, all three of these software packages can be employed in the Twelfth Edition. In each case, online supplements are available that contain the solutions to numerous examples in each of these software programs. Instructors can opt to make this material available online or as part of a customized print edition, making this software an integral and effective part of the presentation of course material.

The rich collection of material that is provided for this edition offers a distinctive and helpful way for exploring the book’s examples and exercises from a variety of simulation techniques.

Supplements

The supplements list is extensive and provides instructors and students with a wealth of traditional and modern resources to match different learning needs.

Problem-Solving Videos are offered again in the Twelfth Edition in an iOS-compatible format. The videos provide step-by-step solutions to Learning Assessments. Videos for Learning Assessments will directly follow a chapter feature called *Problem-Solving Strategy*. Students who have used these videos with past editions have found them to be very helpful.

The **Solutions Manual** for the Twelfth Edition has been completely redone, checked, and double-checked for accuracy. It is the most accurate solutions manual ever created for this textbook.

Acknowledgments

Over the more than three decades that this text has been in existence, we estimate that more than one thousand instructors have used our book in teaching circuit analysis to hundreds of thousands of students. As authors, there is no greater reward than having your work used by so many. We are grateful for the confidence shown in our text and for the numerous evaluations and suggestions from professors and their students over the years. This feedback has helped us continuously improve the presentation. For this Twelfth Edition, we especially thank Elizabeth Devore, Tanner Grider, Markus Kreitzer, and Austin Taylor with Auburn University for their assistance with the solutions manual.

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Senior Content Manager, *Valerie Zaborski*
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Each member of this team played a vital role in preparing the package that is the Twelfth Edition of *Basic Engineering Circuit Analysis*. We are most appreciative of their many contributions.

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J. DAVID IRWIN AND R. MARK NELMS

CHAPTER **1**

Basic Concepts

LEARNING OBJECTIVES

The learning goals for this chapter are that students should be able to:

- Use appropriate SI units and standard prefixes when calculating voltages, currents, resistances, and powers.
- Explain the relationships between basic electrical quantities: voltage, current, and power.
- Use the appropriate symbols for independent and dependent voltage and current sources.
- Calculate the value of the dependent sources when analyzing a circuit that contain independent and dependent sources.
- Calculate the power absorbed by a circuit element using the passive sign convention.

1.1 System of Units

The system of units we employ is the international system of units, the *Système international d'unités*, which is normally referred to as the SI standard system. This system, which is composed of the basic units meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), and candela (cd), is defined in all modern physics texts and therefore will not be defined here. However, we will discuss the units in some detail as we encounter them in our subsequent analyses.

The standard prefixes that are employed in SI are shown in **Fig. 1.1**. Note the decimal relationship between these prefixes. These standard prefixes are employed throughout our study of electric circuits.

Circuit technology has changed drastically over the years. For example, in the early 1960s, the space on a circuit board occupied by the base of a single vacuum tube was about the size of a quarter (25-cent coin). Today that same space could be occupied by an Intel Core i7 integrated circuit chip containing 1.75 billion transistors. These chips are the engine for a host of electronic equipment.

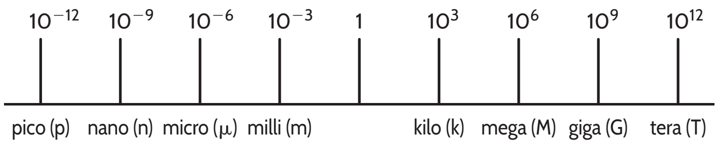

FIGURE 1.1 Standard SI prefixes.

1.2 Basic Quantities

Before we begin our analysis of electric circuits, we must define terms that we will employ. However, in this chapter and throughout the book, our definitions and explanations will be as simple as possible to foster an understanding of the use of the material. No attempt will be made to give complete definitions of many of the quantities because such definitions are not only unnecessary at this level but are often confusing. Although most of us have an intuitive concept of what is meant by a circuit, we will simply refer to an *electric circuit* as an interconnection of electrical components, each of which we will describe with a mathematical model.

The most elementary quantity in an analysis of electric circuits is the electric *charge*. Our interest in electric charge is centered around its motion, since charge in motion results in an energy transfer. Of particular interest to us are those situations in which the motion is confined to a definite closed path.

An electric circuit is essentially a pipeline that facilitates the transfer of charge from one point to another. The time rate of change of charge constitutes an electric *current*. Mathematically, the relationship is expressed as

$$i(t) = \frac{dq(t)}{dt} \qquad \text{or} \qquad q(t) = \int_{-\infty}^{t} i(x)\,dx \tag{1.1}$$

where i and q represent current and charge, respectively (lowercase letters represent time dependency, and capital letters are reserved for constant quantities). The basic unit of current is the ampere (A), and 1 ampere is 1 coulomb (C) per second.

Although we know that current flow in metallic conductors results from electron motion, the conventional current flow, which is universally adopted, represents the movement of positive charges. It is important that the reader think of current flow as the movement of positive charge regardless of the physical phenomena that take place. The symbolism that will be used to represent current flow is shown in **Fig. 1.2**. In **Fig. 1.2a**, $I_1 = 2$ A indicates that at any point in the wire shown, 2 C of charge pass from left to right each second. In **Fig. 1.2b,** $I_2 = -3$ A indicates that at any point in the wire shown, 3 C of charge pass from right to left each second. Therefore, it is important to specify not only the magnitude of the variable representing the current but also its direction.

The two types of current that we encounter often in our daily lives, alternating current (ac) and direct current (dc), are shown as a function of time in **Fig. 1.3**. *Alternating current* is the common current found in every household and is used to run the refrigerator, stove, washing machine, and so on. Batteries, which are used in automobiles and flashlights, are one source of *direct current*. In addition to these two types of currents, which have a wide variety of uses, we can generate many other types of currents. We will examine some of these other types later in the book. In the meantime, it is interesting to note that the magnitude of currents in elements familiar to us ranges from soup to nuts, as shown in **Fig. 1.4**.

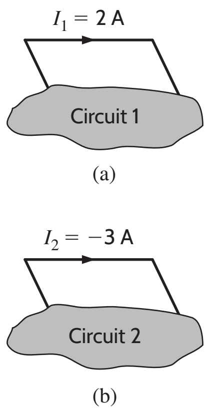

FIGURE 1.2 Conventional current flow: (a) positive current flow; (b) negative current flow.

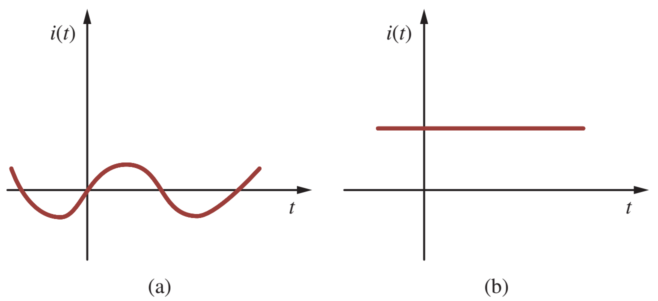

FIGURE 1.3 Two common types of current: (a) alternating current (ac); (b) direct current (dc).

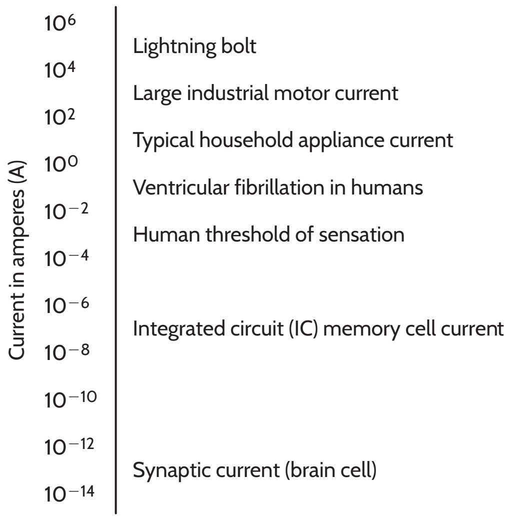

FIGURE 1.4 Typical current magnitudes.

We have indicated that charges in motion yield an energy transfer. Now we define the *voltage* (also called the *electromotive force*, or *potential*) between two points in a circuit as the difference in energy level of a unit charge located at each of the two points. Voltage is very similar to a gravitational force. Think about a bowling ball being dropped from a ladder into a tank of water. As soon as the ball is released, the force of gravity pulls it toward the bottom of the tank. The potential energy of the bowling ball decreases as it approaches the bottom. The gravitational force is pushing the bowling ball through the water. Think of the bowling ball as a charge and the voltage as the force pushing the charge through a circuit. Charges in motion represent a current, so the motion of the bowling ball could be thought of as a current. The water in the tank will resist the motion of the bowling ball. The motion of charges in an electric circuit will be impeded or resisted as well. We will introduce the concept of resistance in Chapter 2 to describe this effect.

Work or energy, $w(t)$ or W, is measured in joules (J); 1 joule is 1 newton meter (N·m). Hence, voltage [$v(t)$ or V] is measured in volts (V), and 1 volt is 1 joule per coulomb; that is, 1 volt = 1 joule per coulomb = 1 newton meter per coulomb. If a unit positive charge is moved between two points, the energy required to move it is the difference in energy level between the two points and is the defined voltage. It is extremely important that the variables used to represent voltage between two points be defined in such a way that the solution will let us interpret which point is at the higher potential with respect to the other.

In **Fig. 1.5a**, the variable that represents the voltage between points A and B has been defined as V_1, and it is assumed that point A is at a higher potential than point B, as indicated by the + and − signs associated with the variable and defined in the figure. The + and − signs define a reference direction for V_1. If $V_1 = 2$ V, then the difference in potential of points A and B is 2 V, and point A is at the higher potential. If a unit positive charge is moved from point A through the circuit to point B, it will give up energy to the circuit and have 2 J less energy when it reaches point B. If a unit positive charge is moved from point B to point A, extra energy must be added to the charge by the circuit, and hence the charge will end up with 2 J more energy at point A than it started with at point B.

For the circuit in **Fig. 1.5b**, $V_2 = -5$ V means that the potential between points A and B is 5 V and point B is at the higher potential. The voltage in Fig. 1.5b can be expressed as shown in **Fig. 1.5c**. In this equivalent case, the difference in potential between points A and B is $V_2 = 5$ V, and point B is at the higher potential.

Note that it is important to define a variable with a reference direction so that the answer can be interpreted to give the physical condition in the circuit. We will find that it is not possible in many cases to define the variable so that the answer is positive, and we will also find that it is not necessary to do so.

As demonstrated in Figs. 1.5b and 1.5c, a negative number for a given variable, for example, V_2 in Fig. 1.5b, gives exactly the same information as a positive number; that is, V_2 in

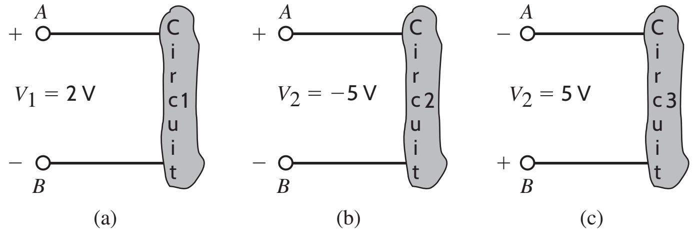

FIGURE 1.5 Voltage representations.

Fig. 1.5c, except that it has an opposite reference direction. Hence, when we define either current or voltage, it is absolutely necessary that we specify both magnitude and direction. Therefore, it is incomplete to say that the voltage between two points is 10 V or the current in a line is 2 A, since only the magnitude and not the direction for the variables has been defined.

The range of magnitudes for voltage, equivalent to that for currents in Fig. 1.4, is shown in **Fig. 1.6**. Once again, note that this range spans many orders of magnitude.

At this point, we have presented the conventions that we employ in our discussions of current and voltage. *Energy* is yet another important term of basic significance. Let's investigate the voltage–current relationships for energy transfer using the flashlight shown in **Fig. 1.7**. The basic elements of a flashlight are a battery, a switch, a light bulb, and connecting wires. Assuming a good battery, we all know that the light bulb will glow when the switch is closed. A current now flows in this closed circuit as charges flow out of the positive terminal of the battery through the switch and light bulb and back into the negative terminal of the battery. The current heats up the filament in the bulb, causing it to glow and emit light. The light bulb converts electrical energy to thermal energy; as a result, charges passing through the bulb lose energy. These charges acquire energy as they pass through the battery as chemical energy is converted to electrical energy. An energy conversion process is occurring in the flashlight as the chemical energy in the battery is converted to electrical energy, which is then converted to thermal energy in the light bulb.

Let's redraw the flashlight as shown in **Fig. 1.8**. There is a current *I* flowing in this diagram. Since we know that the light bulb uses energy, the charges coming out of the bulb have less energy than those entering the light bulb. In other words, the charges expend energy as they move through the bulb. This is indicated by the voltage shown across the bulb. The charges gain energy as they pass through the battery, which is indicated by the voltage across

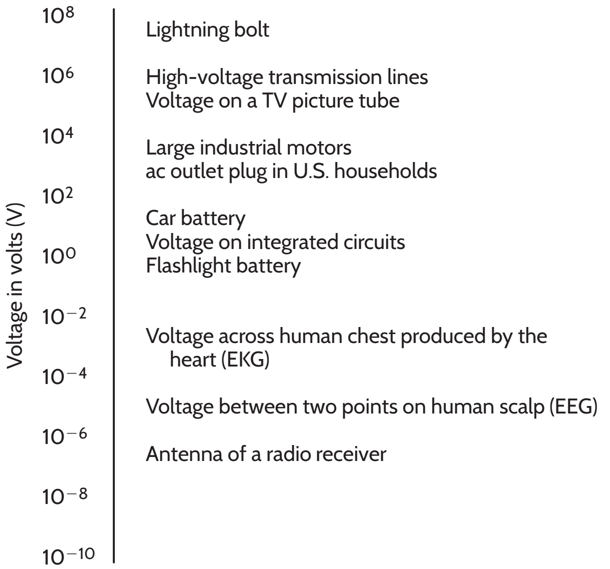

FIGURE 1.6 Typical voltage magnitudes.

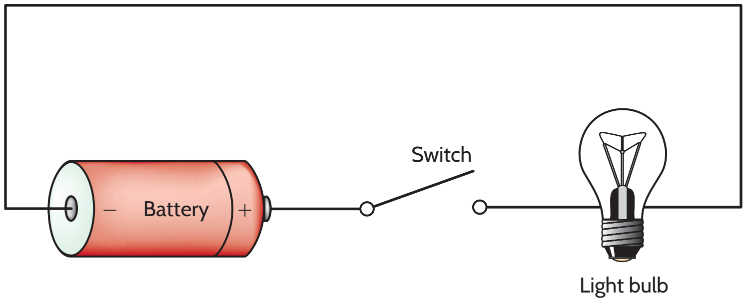

FIGURE 1.7 Flashlight circuit.

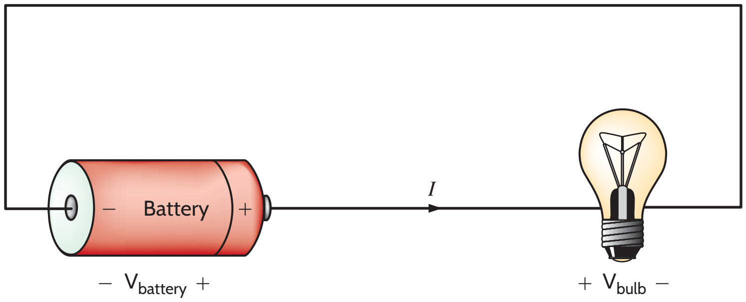

FIGURE 1.8 Flashlight circuit with voltages and current.

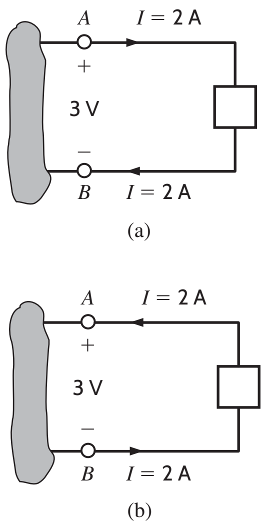

FIGURE 1.9 Voltage–current relationships for (a) energy absorbed and (b) energy supplied.

the battery. Note the voltage–current relationships for the battery and bulb. We know that the bulb is absorbing energy; the current is entering the positive terminal of the voltage. For the battery, the current is leaving the positive terminal, which indicates that energy is being supplied.

This is further illustrated in **Fig. 1.9**, where a circuit element has been extracted from a larger circuit for examination. In **Fig. 1.9a**, energy is being supplied *to* the element by whatever is attached to the terminals. Note that 2 A—that is, 2 C of charge—are moving from point *A* to point *B* through the element each second. Each coulomb loses 3 J of energy as it passes through the element from point *A* to point *B*. Therefore, the element is absorbing 6 J of energy per second. Note that when the element is *absorbing* energy, a positive current enters the positive terminal. In **Fig. 1.9b**, energy is being supplied *by* the element to whatever is connected to terminals *A*-*B*. In this case, note that when the element is *supplying* energy, a positive current enters the negative terminal and leaves via the positive terminal. In this convention, a negative current in one direction is equivalent to a positive current in the opposite direction, and vice versa. Similarly, a negative voltage in one direction is equivalent to a positive voltage in the opposite direction.

EXAMPLE 1.1

Suppose that your car will not start. To determine whether the battery is faulty, you turn on the light switch and find that the lights are very dim, indicating a weak battery. You borrow a friend's car and a set of jumper cables. However, how do you connect his car's battery to yours? What do you want his battery to do?

Solution Essentially, his car's battery must supply energy to yours, and therefore it should be connected in the manner shown in **Fig. 1.10**. Note that the positive current leaves the positive terminal of the good battery (supplying energy) and enters the positive terminal of the weak battery (absorbing energy). Note that the same connections are used when charging a battery.

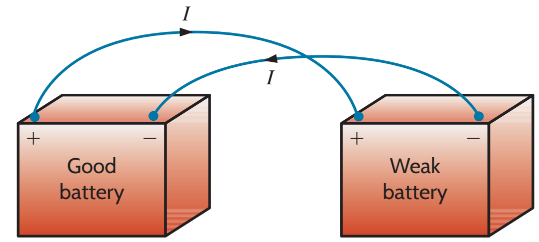

FIGURE 1.10 Diagram for Example 1.1.

In practical applications, there are often considerations other than simply the electrical relations (e.g., safety). Such is the case with jump-starting an automobile. Automobile batteries produce explosive gases that can be ignited accidentally, causing severe physical injury. Be safe—follow the procedure described in your auto owner's manual.

We have defined voltage in joules per coulomb as the energy required to move a positive charge of 1 C through an element. If we assume that we are dealing with a differential amount of charge and energy, then

$$v = \frac{dw}{dq} \tag{1.2}$$

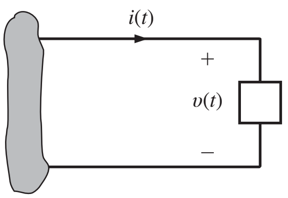

FIGURE 1.11 Sign convention for power.

Multiplying this quantity by the current in the element yields

$$vi = \frac{dw}{dq}\left(\frac{dq}{dt}\right) = \frac{dw}{dt} = p \tag{1.3}$$

which is the time rate of change of energy or power measured in joules per second, or watts (W). Since, in general, both v and i are functions of time, p is also a time-varying quantity. Therefore, the change in energy from time t_1 to time t_2 can be found by integrating Eq. (1.3); that is,

$$\Delta w = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} v\,i\,dt \tag{1.4}$$

HINT 1.1

The passive sign convention is used to determine whether power is being absorbed or supplied.

At this point, let us summarize our sign convention for power. To determine the sign of any of the quantities involved, the variables for the current and voltage should be arranged as shown in **Fig. 1.11**. The variable for the voltage $v(t)$ is defined as the voltage across the element with the positive reference at the same terminal that the current variable $i(t)$ is entering. This convention is called the *passive sign convention* and will be so noted in the remainder of this book. The product of v and i, with their attendant signs, will determine the magnitude and sign of the power (see **HINT 1.1**). If the sign of the power is positive, power is being absorbed by the element; if the sign is negative, power is being supplied by the element.

EXAMPLE 1.2

Given the two diagrams shown in **Fig. 1.12**, determine whether the element is absorbing or supplying power and how much.

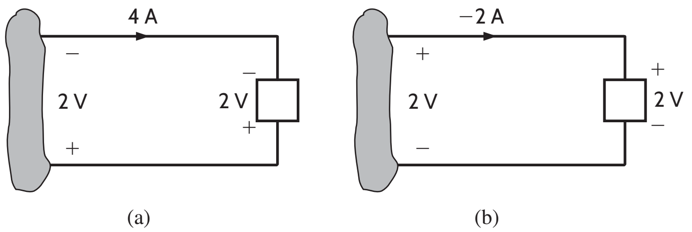

FIGURE 1.12 Elements for Example 1.2.

Solution In **Fig. 1.12a**, the power is $P = (2\text{ V})(-4\text{ A}) = -8\text{ W}$. Therefore, the element is supplying power. In **Fig. 1.12b**, the power is $P = (2\text{ V})(-2\text{ A}) = -4\text{ W}$. Therefore, the element is supplying power.

Learning Assessment

E1.1 Determine the amount of power absorbed or supplied by the elements in Fig. E1.1.

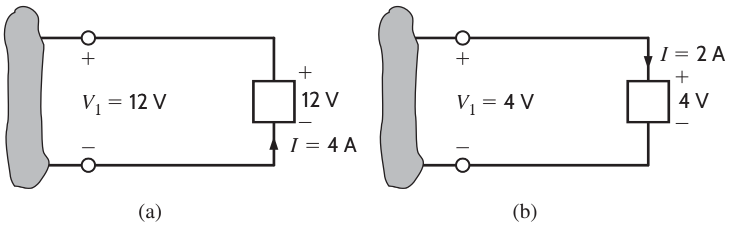

FIGURE E1.1

Answer:
(a) $P = -48$ W;
(b) $P = 8$ W.

EXAMPLE 1.3

We wish to determine the unknown voltage or current in **Fig. 1.13**.

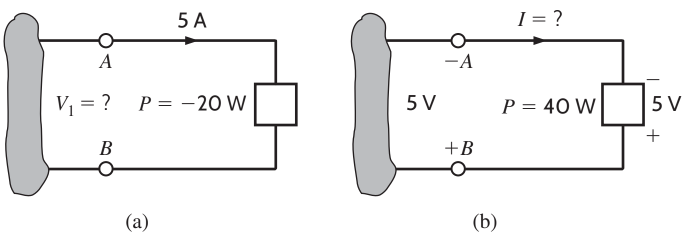

FIGURE 1.13 Elements for Example 1.3.

Solution In **Fig. 1.13a**, a power of −20 W indicates that the element is delivering power. Therefore, the current enters the negative terminal (terminal A), and from Eq. (1.3) the voltage is 4 V. Thus, B is the positive terminal, A is the negative terminal, and the voltage between them is 4 V.

In **Fig. 1.13b**, a power of +40 W indicates that the element is absorbing power and, therefore, the current should enter the positive terminal B. The current thus has a value of −8 A, as shown in the figure.

Learning Assessment

E1.2 Determine the unknown variables in Fig. E1.2.

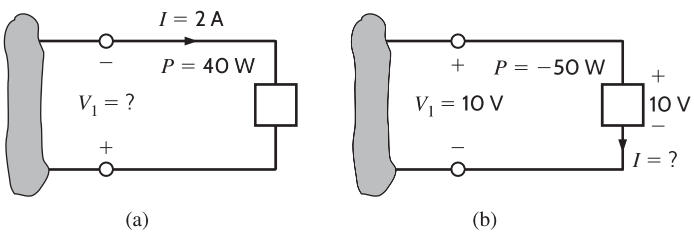

FIGURE E1.2

Answer:
(a) $V_1 = -20$ V;
(b) $I = -5$ A.

Finally, it is important to note that our electrical networks satisfy the principle of conservation of energy. Because of the relationship between energy and power, it can be implied that power is also conserved in an electrical network. This result was formally stated in 1952 by B. D. H. Tellegen and is known as Tellegen's theorem—the sum of the powers absorbed by all elements in an electrical network is zero. Another statement of this theorem is that the power supplied in a network is exactly equal to the power absorbed. Checking to verify that Tellegen's theorem is satisfied for a particular network is one way to check our calculations when analyzing electrical networks.

1.3 Circuit Elements

Thus far, we have defined voltage, current, and power. In the remainder of this chapter we will define both independent and dependent current and voltage sources. Although we will assume ideal elements, we will try to indicate the shortcomings of these assumptions as we proceed with the discussion.

In general, the elements we will define are terminal devices that are completely characterized by the current through the element and/or the voltage across it. These elements, which we will employ in constructing electric circuits, will be broadly classified as being either active or passive. The distinction between these two classifications depends essentially on one thing—whether they supply or absorb energy. As the words themselves imply, an *active* element is capable of generating energy and a *passive* element cannot generate energy.

However, later we will show that some passive elements are capable of storing energy. Typical active elements are batteries and generators. The three common passive elements are resistors, capacitors, and inductors.

In Chapter 2, we will launch an examination of passive elements by discussing the resistor in detail. Before proceeding with that element, we first present some very important active elements.

1. Independent voltage source
2. Independent current source
3. Two dependent voltage sources
4. Two dependent current sources

Independent Sources

An *independent voltage source* is a two-terminal element that maintains a specified voltage between its terminals *regardless of the current through it,* as shown by the v-i plot in **Fig. 1.14a**. The general symbol for an independent source, a circle, is also shown in Fig. 1.14a. As the figure indicates, terminal A is $v(t)$ volts positive with respect to terminal B.

In contrast to the independent voltage source, the *independent current source* is a two-terminal element that maintains a specified current *regardless of the voltage across its terminals*, as illustrated by the v-i plot in **Fig. 1.14b**. The general symbol for an independent current source is also shown in Fig. 1.14b, where $i(t)$ is the specified current and the arrow indicates the positive direction of current flow.

In their normal mode of operation, independent sources supply power to the remainder of the circuit. However, they may also be connected into a circuit in such a way that they absorb power. A simple example of this latter case is a battery-charging circuit, such as that shown in Example 1.1.

It is important that we pause here to interject a comment concerning a shortcoming of the models. In general, mathematical models approximate actual physical systems only under a certain range of conditions. Rarely does a model accurately represent a physical system under every set of conditions. To illustrate this point, consider the model for the voltage source in Fig. 1.14a. We assume that the voltage source delivers v volts regardless of what is connected to its terminals. Theoretically, we could adjust the external circuit so that an infinite amount of current would flow, and therefore the voltage source would deliver an infinite amount of power. This is, of course, physically impossible. A similar argument could be made for the independent current source. Hence, the reader is cautioned to keep in mind that models have limitations and thus are valid representations of physical systems only under certain conditions.

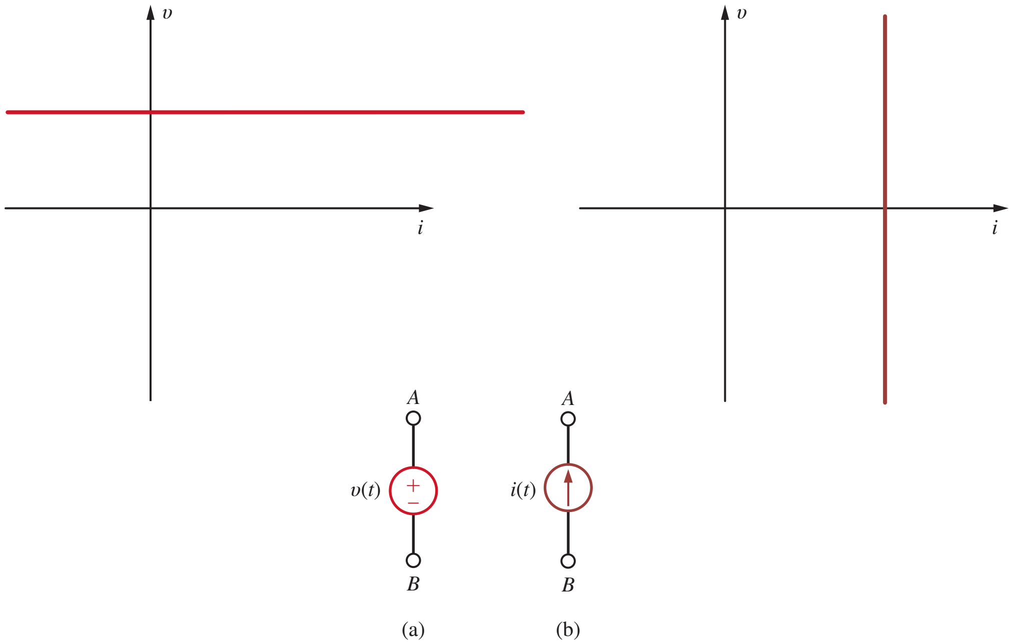

FIGURE 1.14 Symbols for (a) independent voltage source and (b) independent current source.

For example, can the independent voltage source be utilized to model the battery in an automobile under all operating conditions? With the headlights on, turn on the radio. Do the headlights dim with the radio on? They probably won't if the sound system in your automobile was installed at the factory. If you try to crank your car with the headlights on, you will notice that the lights dim. The starter in your car draws considerable current, thus causing the voltage at the battery terminals to drop and dimming the headlights. The independent voltage source is a good model for the battery with the radio turned on; however, an improved model is needed for your battery to predict its performance under cranking conditions.

EXAMPLE 1.4

Determine the power absorbed or supplied by the elements in the network in Fig. 1.15.

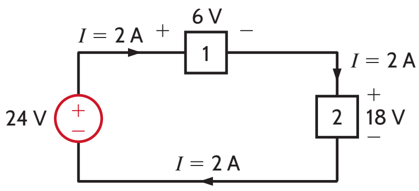

FIGURE 1.15 Network for Example 1.4.

Solution The current flow is out of the positive terminal of the 24-V source, and therefore this element is supplying (2)(24) = 48 W of power. The current is into the positive terminals of elements 1 and 2, and therefore elements 1 and 2 are absorbing (2)(6) = 12 W and (2)(18) = 36 W, respectively. Note that the power supplied is equal to the power absorbed. Also note that the same 2 A current flows through all elements in this circuit (see **HINT 1.2**).

HINT 1.2

Elements that are connected in series have the same current.

Learning Assessment

E1.3 Find the power that is absorbed or supplied by the elements in Fig. E1.3.

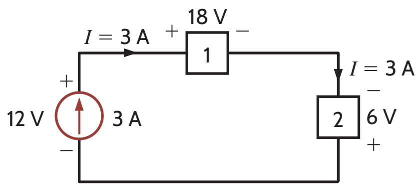

FIGURE E1.3

Answer:

Current source supplies 36 W, element 1 absorbs 54 W, and element 2 supplies 18 W.

Dependent Sources

In contrast to the independent sources, which produce a particular voltage or current completely unaffected by what is happening in the remainder of the circuit, dependent sources generate a voltage or current that is determined by a voltage or current at a specified location in the circuit. These sources are very important because they are an integral part of the mathematical models used to describe the behavior of many electronic circuit elements.

For example, metal-oxide-semiconductor field-effect transistors (MOSFETs) and bipolar transistors, both of which are commonly found in a host of electronic equipment, are modeled with dependent sources, and therefore the analysis of electronic circuits involves the use of these controlled elements.

In contrast to the circle used to represent independent sources, a diamond is used to represent a dependent or controlled source. **Fig. 1.16** illustrates the four types of dependent sources. The input terminals on the left represent the voltage or current that controls the dependent source, and the output terminals on the right represent the output current or voltage of the controlled source. Note that in **Figs. 1.16a** and **d**, the quantities μ and β are dimensionless constants because we are transforming voltage to voltage and current to current. This is not the case in **Figs. 1.16b** and **c**; hence, when we employ these elements a short time later, we must describe the units of the factors r and g.

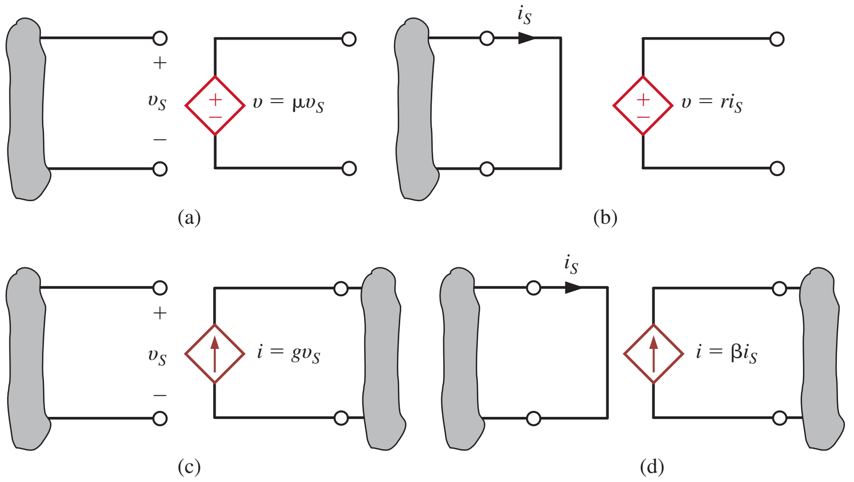

FIGURE 1.16 Four different types of dependent sources.

EXAMPLE 1.5

Given the two networks shown in Fig. 1.17, we wish to determine the outputs.

Solution In Fig. 1.17a, the output voltage is $V_o = \mu V_S$ or $V_o = 20\ V_S = (20)(2\text{ V}) = 40$ V. Note that the output voltage has been amplified from 2 V at the input terminals to 40 V at the output terminals; that is, the circuit is a voltage amplifier with an amplification factor of 20.

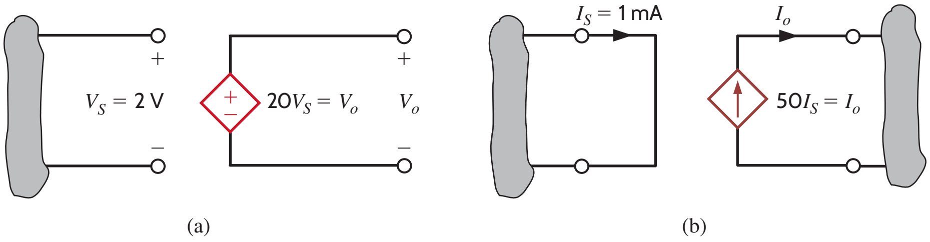

FIGURE 1.17 Circuits for Example 1.5.

In Fig. 1.17b, the output current is $I_o = \beta I_S = (50)(1\text{ mA}) = 50$ mA; that is, the circuit has a current gain of 50, meaning that the output current is 50 times greater than the input current.

Learning Assessment

E1.4 Determine the power supplied by the dependent sources in Fig. E1.4.

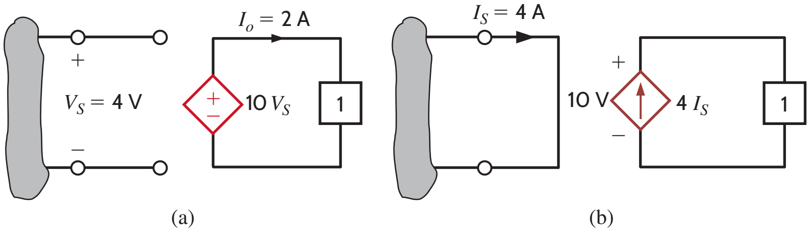

FIGURE E1.4

Answer:

(a) Power supplied = 80 W;

(b) Power supplied = 160 W.

EXAMPLE 1.6

Calculate the power absorbed by each element in the network of Fig. 1.18. Also verify that Tellegen's theorem is satisfied by this network.

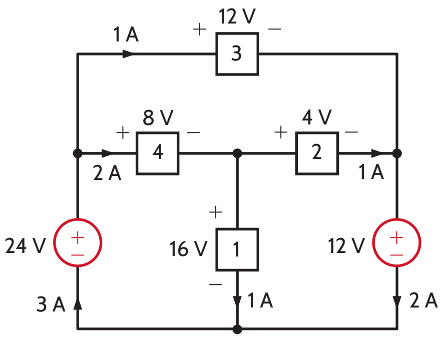

FIGURE 1.18 Circuit used in Example 1.6.

Solution Let's calculate the power absorbed by each element using the sign convention for power.

$$P_1 = (16)(1) = 16\text{ W}$$
$$P_2 = (4)(1) = 4\text{ W}$$
$$P_3 = (12)(1) = 12\text{ W}$$
$$P_4 = (8)(2) = 16\text{ W}$$
$$P_{12\text{ V}} = (12)(2) = 24\text{ W}$$
$$P_{24\text{ V}} = (24)(-3) = -72\text{ W}$$

Note that to calculate the power absorbed by the 24-V source, the current of 3 A flowing up through the source was changed to a current −3 A flowing down through the 24-V source.

Let's sum up the power absorbed by all elements: $16 + 4 + 12 + 16 + 24 - 72 = 0$

This sum is zero, which verifies that Tellegen's theorem is satisfied.

EXAMPLE 1.7

Use Tellegen's theorem to find the current I_o in the network in **Fig. 1.19**.

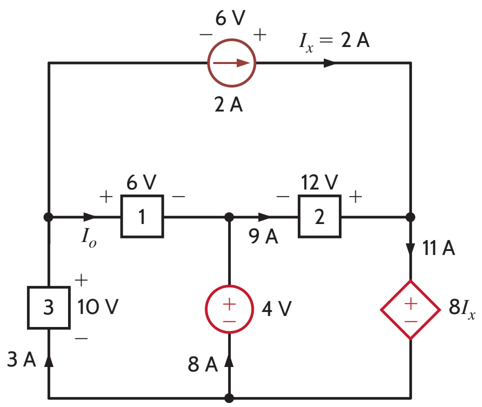

FIGURE 1.19 Circuit used in Example 1.7.

Solution First, we must determine the power absorbed by each element in the network. Using the sign convention for power, we find

$$P_{2\text{ A}} = (6)(-2) = -12\text{ W}$$
$$P_1 = (6)(I_o) = 6I_o\text{ W}$$
$$P_2 = (12)(-9) = -108\text{ W}$$
$$P_3 = (10)(-3) = -30\text{ W}$$
$$P_{4\text{ V}} = (4)(-8) = -32\text{ W}$$
$$P_{DS} = (8I_x)(11) = (16)(11) = 176\text{ W}$$

Applying Tellegen's theorem yields

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

or

$$6I_o + 176 = 12 + 108 + 30 + 32$$

Hence,

$$I_o = 1\text{ A}$$

Learning Assessments

E1.5 Find the power that is absorbed or supplied by the circuit elements in the network in Fig. E1.5.

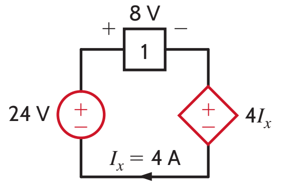

FIGURE E1.5

Answer:

$P_{24\text{ V}} = 96$ W supplied;

$P_1 = 32$ W absorbed;

$P_{4I_x} = 64$ W absorbed.

E1.6 Find the power that is absorbed or supplied by the network elements in Fig. E1.6.

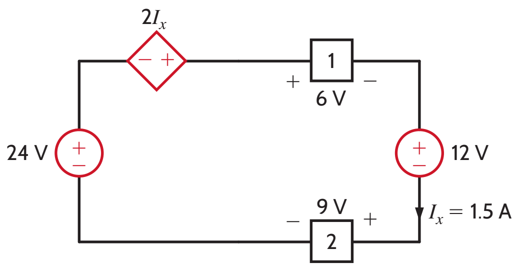

FIGURE E1.6

Answer:

$P_{24\text{ V}} = 36$ W supplied;

$P_{12\text{ V}} = 18$ W absorbed;

$P_{2I_x} = 4.5$ W supplied;

$P_1 = 9$ W absorbed;

$P_2 = 13.5$ W absorbed.

E1.7 Find I_x in Fig. E1.7 using Tellegen's theorem.

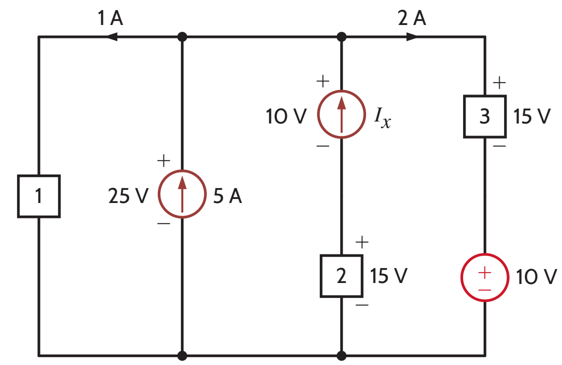

FIGURE E1.7

Answer:

$I_x = -2$ A.

EXAMPLE 1.8

The charge that enters the BOX is shown in **Fig. 1.20**. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.

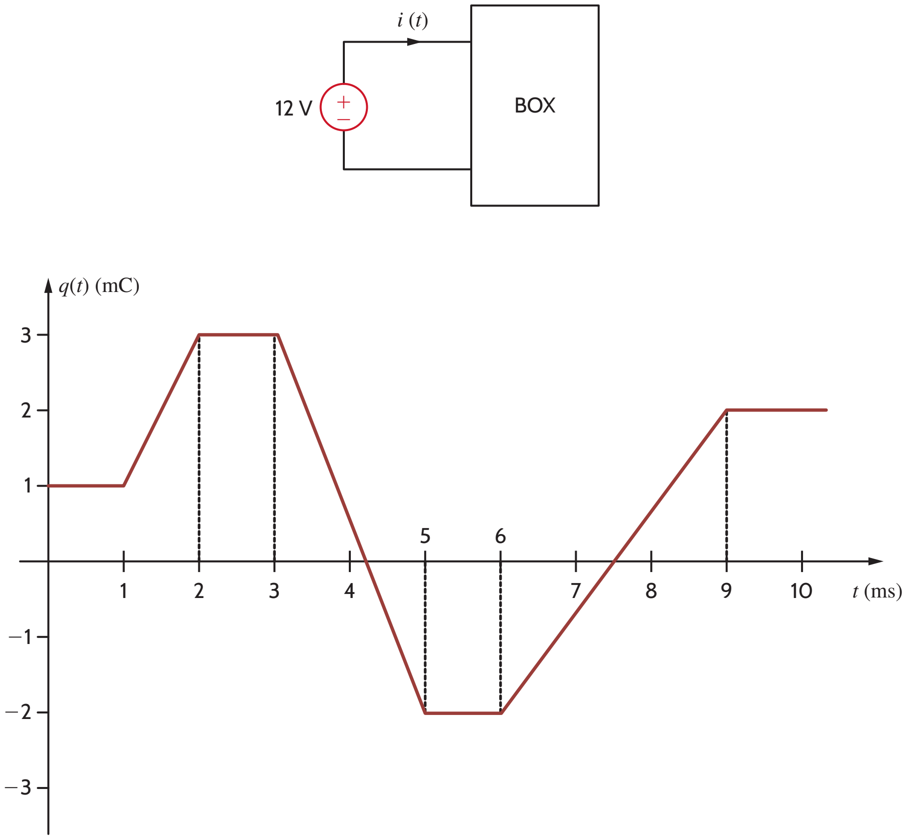

FIGURE 1.20 Diagrams for Example 1.8.

Solution Recall that current is related to charge by $i(t) = \dfrac{dq(t)}{dt}$. The current is equal to the slope of the charge waveform.

$$i(t) = 0 \qquad 0 \le t \le 1 \text{ ms}$$

$$i(t) = \frac{3\times10^{-3} - 1\times10^{-3}}{2\times10^{-3} - 1\times10^{-3}} = 2 \text{ A} \qquad 1 \le t \le 2 \text{ ms}$$

$$i(t) = 0 \qquad 2 \le t \le 3 \text{ ms}$$

$$i(t) = \frac{-2\times10^{-3} - 3\times10^{-3}}{5\times10^{-3} - 3\times10^{-3}} = -2.5 \text{ A} \qquad 3 \le t \le 5 \text{ ms}$$

$$i(t) = 0 \qquad 5 \le t \le 6 \text{ ms}$$

$$i(t) = \frac{2\times10^{-3} - (-2\times10^{-3})}{9\times10^{-3} - 6\times10^{-3}} = 1.33 \text{ A} \qquad 6 \le t \le 9 \text{ ms}$$

$$i(t) = 0 \qquad t \ge 9 \text{ ms}$$

The current is plotted with the charge waveform in **Fig. 1.21**. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

The power absorbed by the BOX is $12 \times i(t)$.

$$p(t) = 12(0) = 0 \qquad 0 \le t \le 1 \text{ ms}$$

$$p(t) = 12(2) = 24 \text{ W} \qquad 1 \le t \le 2 \text{ ms}$$

$$p(t) = 12(0) = 0 \qquad 2 \le t \le 3 \text{ ms}$$

$$p(t) = 12(-2.5) = -30 \text{ W} \qquad 3 \le t \le 5 \text{ ms}$$

$$p(t) = 12(0) = 0 \qquad 5 \le t \le 6 \text{ ms}$$
$$p(t) = 12(1.33) = 16 \text{ W} \qquad 6 \le t \le 9 \text{ ms}$$
$$p(t) = 12(0) = 0 \qquad t \ge 9 \text{ ms}$$

The power absorbed by the BOX is plotted in **Fig. 1.22**. For the time intervals, $1 \le t \le 2$ ms and $6 \le t \le 9$ ms, the BOX is absorbing power. During the time interval $3 \le t \le 5$ ms, the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.

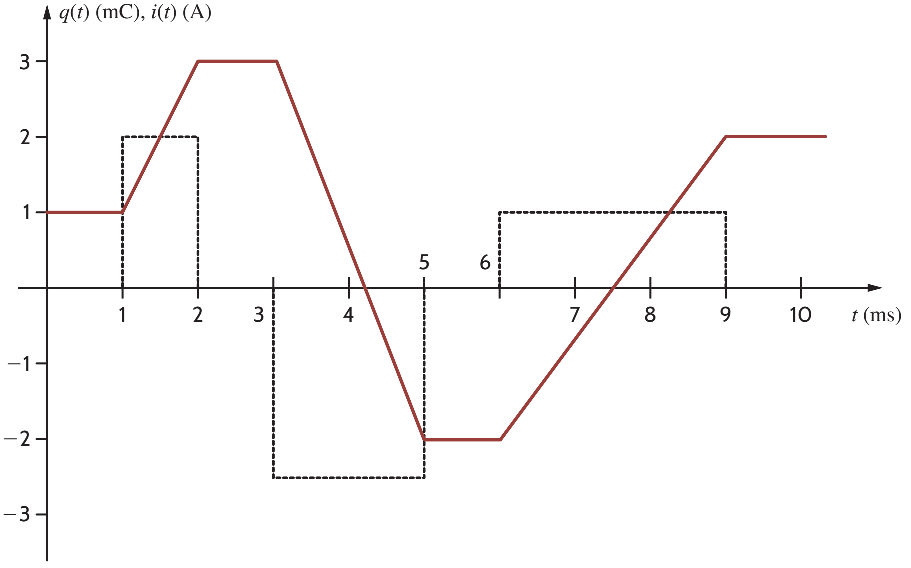

FIGURE 1.21 Charge and current waveforms for Example 1.8.

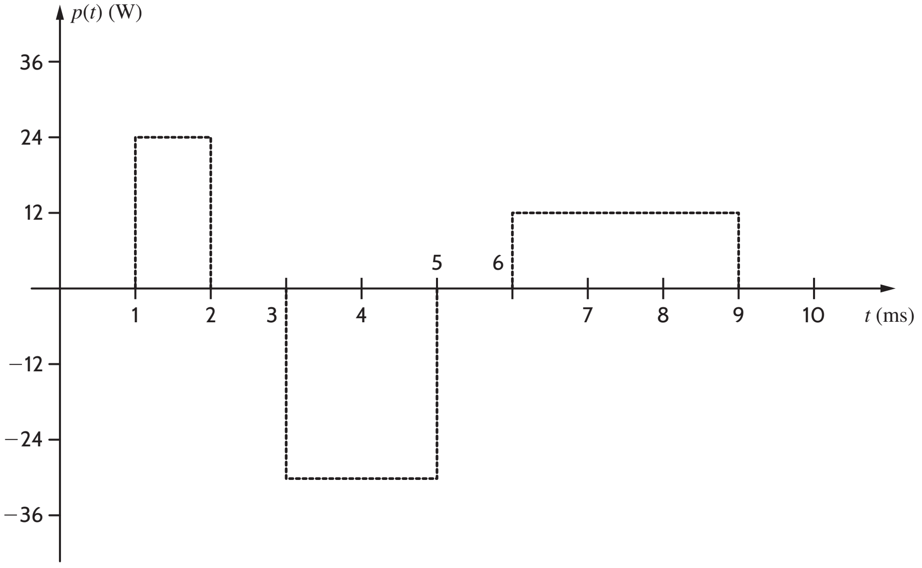

FIGURE 1.22 Power waveform for Example 1.8.

Learning Assessments

E1.8 The power absorbed by the BOX in Fig. E1.8 is $p(t) = 2.5e^{-4t}$ W. Compute the energy and charge delivered to the BOX in the time interval $0 < t < 250$ ms.

Answer:

395.1 mJ; 8.8 mC.

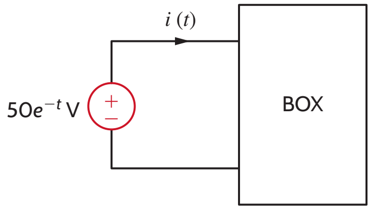

FIGURE E1.8

E1.9 The energy absorbed by the BOX is shown in Fig. E1.9. Calculate and sketch the current flowing into the BOX. Also calculate the charge that enters the BOX between 0 and 12 seconds.

Answer:

$Q = 0$.

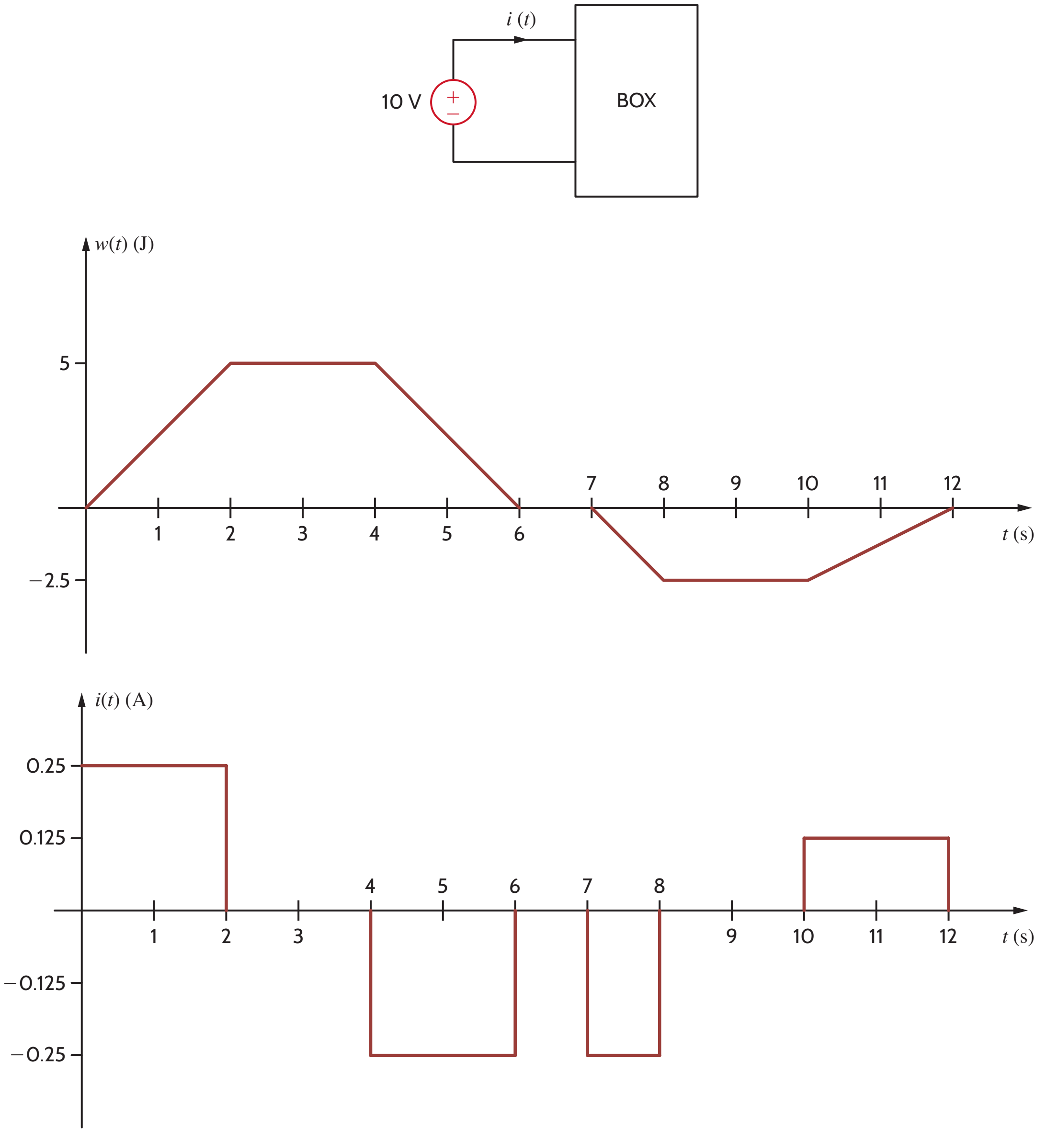

FIGURE E1.9

EXAMPLE 1.9

The ubiquitous universal serial bus (USB) port is commonly utilized to charge smartphones, as shown in **Fig. 1.23**. Technical details for USB specifications can be found at www.usb.org. The amount of current that can be provided over a USB port is defined in the USB specifications.

According to the USB 2.0 standard, a device is classified as low power if it draws 100 mA or less and high power if it draws between 100 and 500 mA.

1. A 1000 mAh lithium-ion battery has been fully discharged (i.e., 0 mAh). How long will it take to recharge it from a USB port supplying a constant current of 250 mA? How much charge is stored in the battery when it is fully charged?
2. A fully charged 1000 mAh lithium-ion battery supplies a load, which draws a constant current of 200 mA for 4 hours. How much charge is left in the battery at the end of the 4 hours? Assuming that the load remains constant at 3.6 V, how much energy is absorbed by the load in joules?

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FIGURE 1.23 Charging an Apple iPhone® using a USB port.

Solution

1. With a constant current of 250 mA, the time required to recharge the battery is 1000 mAh/250 mA = 4 h. The battery has a capacity of 1000 mAh. The charge stored in the battery when fully charged is 1000 mAh × 1 A/1000 mA × 3600 s/h = 3600 As = 3600 C.
2. A constant current of 200 mA is drawn from the battery for 4 hours, so 800 mAh × 1 A/1000 mA × 3600 s/h = 2880 C removed from the battery. The charge left in the battery is 3600 – 2880 = 720 C. The power absorbed by the load is 3.6 V × 0.2 A = 0.72 W. The energy absorbed by the load is 0.72 W × 4 h × 3600 s/h = 10,368 J.

Summary

- ***The standard prefixes employed***

$$\begin{array}{ll} p = 10^{-12} & k = 10^{3} \\ n = 10^{-9} & M = 10^{6} \\ \mu = 10^{-6} & G = 10^{9} \\ m = 10^{-3} & T = 10^{12} \end{array}$$

- ***The relationships between current and charge***

$$i(t) = \frac{dq(t)}{dt} \qquad \text{or} \qquad q(t) = \int_{-\infty}^{t} i(x)\,dx$$

- ***The relationships among power, energy, current, and voltage***

$$p = \frac{dw}{dt} = vi$$

$$\Delta w = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} vi\,dt$$

- ***The passive sign convention*** The passive sign convention states that if the voltage and current associated with an element are as shown in Fig. 1.11, the product of v and i, with their attendant signs, determines the magnitude and sign of the power. If the sign is positive, power is being absorbed by the element, and if the sign is negative, the element is supplying power.
- ***Independent and dependent sources*** An ideal independent voltage (current) source is a two-terminal element that maintains a specified voltage (current) between its terminals, regardless of the current (voltage) through (across) the element. Dependent or controlled sources generate a voltage or current that is determined by a voltage or current at a specified location in the circuit.
- ***Conservation of energy*** The electric circuits under investigation satisfy the conservation of energy.
- ***Tellegen's theorem*** The sum of the powers absorbed by all elements in an electrical network is zero.

Resistive Circuits

LEARNING OBJECTIVES

The learning goals for this chapter are that students should be able to:

- Use Ohm's law to calculate the voltages and currents in electric circuits.
- Apply Kirchhoff's current law and Kirchhoff's voltage law to determine the voltages and currents in an electric circuit.
- Analyze single-loop and single-node-pair circuits to calculate the voltages and currents in an electric circuit.
- Determine the equivalent resistance of a resistor network where the resistors are in series and parallel.
- Calculate the voltages and currents in a simple electric circuit using voltage and current division.
- Transform the basic wye resistor network to a delta resistor network, and visa versa.
- Analyze electric circuits to determine the voltages and currents in electric circuits that contain dependent sources.

2.1 Ohm's Law

Ohm's law is named for the German physicist Georg Simon Ohm, who is credited with establishing the voltage–current relationship for resistance. As a result of his pioneering work, the unit of resistance bears his name.

Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it. The resistance, measured in ohms, is the constant of proportionality between the voltage and current.

A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in **Fig. 2.1a**. A resistor is a physical device that can be purchased in certain standard values in an electronic parts store. These resistors, which find use in a variety of electrical applications, are normally carbon composition or wirewound. In addition, resistors can be fabricated using thick oxide or thin metal films for use in hybrid circuits, or they can be diffused in semiconductor integrated circuits. Some typical discrete resistors are shown in **Fig. 2.1b**.

The mathematical relationship of Ohm's law is illustrated by the equation

$$v(t) = Ri(t), \text{ where } R \geqq 0 \qquad \textbf{2.1}$$

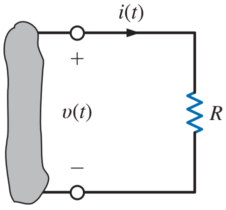
or, equivalently, by the voltage–current characteristic shown in **Fig. 2.2a**. Note carefully the relationship between the polarity of the voltage and the direction of the current (see **HINT 2.1**). In addition, note that we have tacitly assumed that the resistor has a constant value and therefore that the voltage–current characteristic is linear.

HINT 2.1

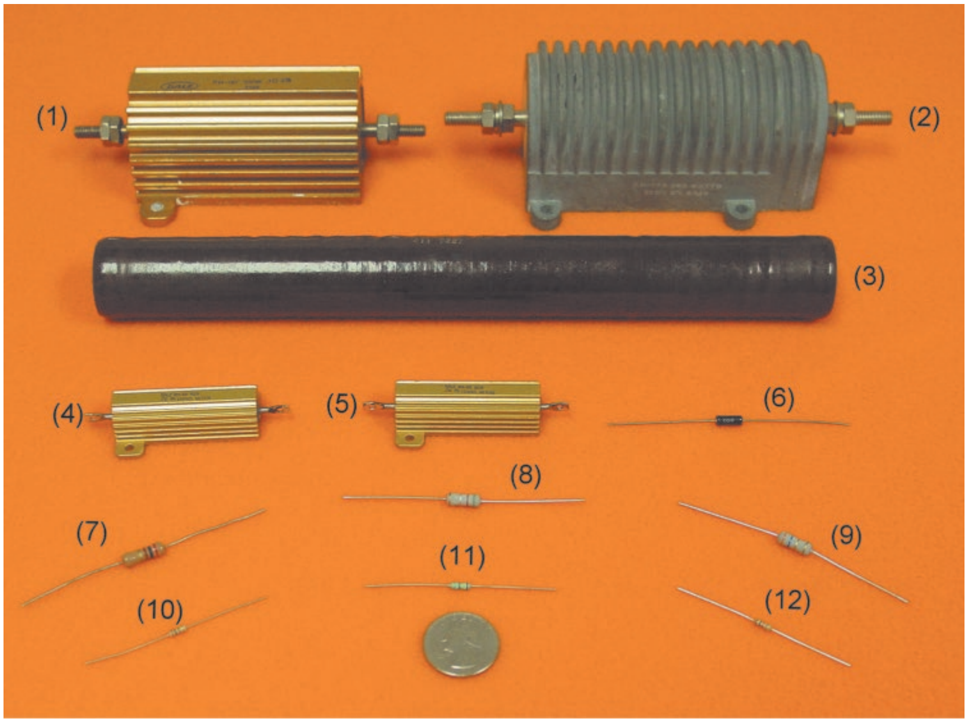
The passive sign convention will be employed in conjunction with Ohm's law.

The symbol Ω is used to represent ohms, and therefore,

$$1\ \Omega = 1\ \text{V/A}$$



(a)



(b)

FIGURE 2.1 (a) Symbol for a resistor; (b) some practical devices. (1), (2), and (3) are high-power resistors. (4) and (5) are high-wattage fixed resistors. (6) is a high-precision resistor. (7)–(12) are fixed resistors with different power ratings. (Photo courtesy of Mark Nelms and Jo Ann Loden)

Although, in our analysis, we will always assume that the resistors are *linear* and are thus described by a straight-line characteristic that passes through the origin, it is important that readers realize that some very useful and practical elements do exist that exhibit a *nonlinear* resistance characteristic; that is, the voltage–current relationship is not a straight line.

The light bulb from the flashlight in Chapter 1 is an example of an element that exhibits a nonlinear characteristic. A typical characteristic for a light bulb is shown in **Fig. 2.2b**.

Since a resistor is a passive element, the proper current–voltage relationship is illustrated in Fig. 2.1a. The power supplied to the terminals is absorbed by the resistor. Note that the charge moves from the higher to the lower potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat. As indicated in Chapter 1, the rate of energy dissipation is the instantaneous power, and therefore

$$p(t) = v(t)i(t) \tag{2.2}$$

which, using Eq. (2.1), can be written as

$$p(t) = Ri^2(t) = \frac{v^2(t)}{R} \tag{2.3}$$

This equation illustrates that the power is a nonlinear function of either current or voltage and that it is always a positive quantity.

Conductance, represented by the symbol G, is another quantity with wide application in circuit analysis. By definition, conductance is the reciprocal of resistance; that is,

$$G = \frac{1}{R} \tag{2.4}$$

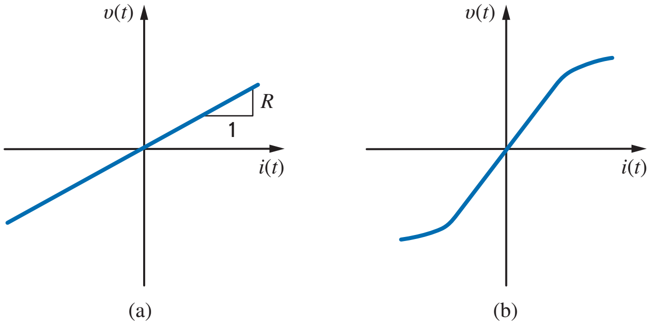


FIGURE 2.2 Graphical representation of the voltage–current relationship for (a) a linear resistor and (b) a light bulb.

The unit of conductance is the siemens, and the relationship between units is

$$1\ \text{S} = 1\ \text{A/V}$$

Using Eq. (2.4), we can write two additional expressions,

$$i(t) = Gv(t) \tag{2.5}$$

and

$$p(t) = \frac{i^2(t)}{G} = Gv^2(t) \tag{2.6}$$

Eq. (2.5) is another expression of Ohm's law.

Two specific values of resistance, and therefore conductance, are very important: $R = 0$ and $R = \infty$.

In examining the two cases, consider the network in **Fig. 2.3a**. The variable resistance symbol is used to describe a resistor such as the volume control on a radio or television set. As the resistance is decreased and becomes smaller and smaller, we finally reach a point where the resistance is zero and the circuit is reduced to that shown in **Fig. 2.3b**; that is, the resistance can be replaced by a short circuit. On the other hand, if the resistance is increased and becomes larger and larger, we finally reach a point where it is essentially infinite and the resistance can be replaced by an open circuit, as shown in **Fig. 2.3c**. Note that in the case of a short circuit where $R = 0$,

$$\begin{aligned} v(t) &= Ri(t) \\ &= 0 \end{aligned}$$

Therefore, $v(t) = 0$, although the current could theoretically be any value. In the open-circuit case where $R = \infty$,

$$\begin{aligned} i(t) &= v(t)/R \\ &= 0 \end{aligned}$$

Therefore, the current is zero regardless of the value of the voltage across the open terminals.

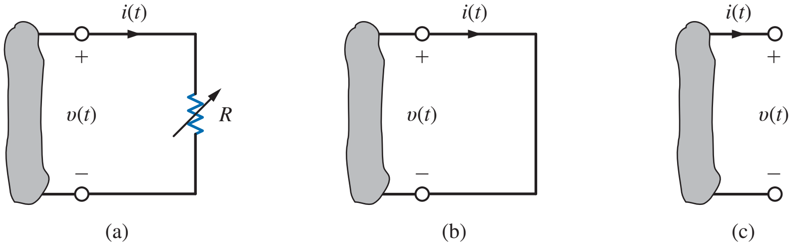


FIGURE 2.3 Short-circuit and open-circuit descriptions.

EXAMPLE 2.1

In the circuit in **Fig. 2.4a**, determine the current and the power absorbed by the resistor.

Solution Using Eq. (2.1), we find the current to be

$$I = V/R = 12/2\text{k} = 6 \text{ mA}$$

Note that because many of the resistors employed in our analysis are in kΩ, we will use k in the equations in place of 1000. The power absorbed by the resistor is given by Eq. (2.2) or (2.3) as

$$\begin{aligned} P &= VI = (12)(6 \times 10^{-3}) = 0.072 \text{ W} \\ &= I^2R = (6 \times 10^{-3})^2(2\text{k}) = 0.072 \text{ W} \\ &= V^2/R = (12)^2/2\text{k} = 0.072 \text{ W} \end{aligned}$$

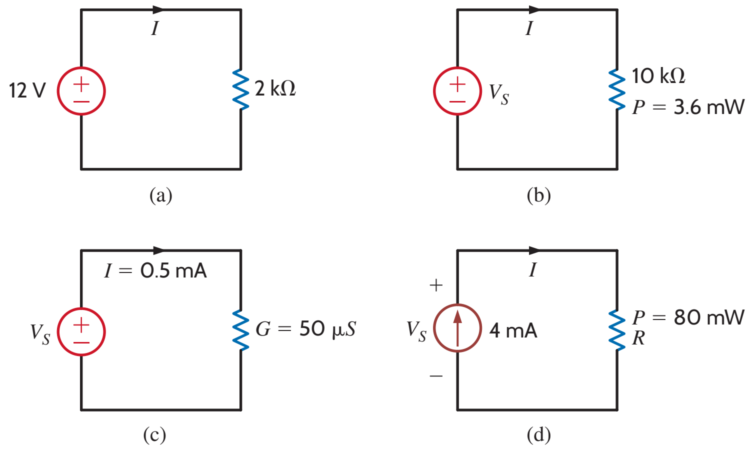


FIGURE 2.4 Circuits for Examples 2.1 to 2.4.

EXAMPLE 2.2

The power absorbed by the 10-kΩ resistor in **Fig. 2.4b** is 3.6 mW. Determine the voltage and the current in the circuit.

Solution Using the power relationship, we can determine either of the unknowns:

$$\begin{aligned} V_S^2/R &= P \\ V_S^2 &= (3.6 \times 10^{-3})(10\text{k}) \\ V_S &= 6 \text{ V} \end{aligned}$$

and

$$\begin{aligned} I^2R &= P \\ I^2 &= (3.6 \times 10^{-3})/10\text{k} \\ I &= 0.6 \text{ mA} \end{aligned}$$

Furthermore, once V_S is determined, I could be obtained by Ohm's law, and likewise once I is known, then Ohm's law could be used to derive the value of V_S. Note carefully that the equations for power involve the terms I^2 and V_S^2. Therefore, $I = -0.6$ mA and $V_S = -6$ V also satisfy the mathematical equations and, in this case, the direction of *both* the voltage and current is reversed.

EXAMPLE 2.3

Given the circuit in Fig. 2.4c, we wish to find the value of the voltage source and the power absorbed by the resistance.

Solution The voltage is

$$V_S = I/G = (0.5 \times 10^{-3})/(50 \times 10^{-6}) = 10 \text{ V}$$

The power absorbed is then

$$P = I^2/\text{G} = (0.5 \times 10^{-3})^2/(50 \times 10^{-6}) = 5 \text{ mW}$$

Or we could simply note that

$$R = 1/G = 20 \text{ k}\Omega$$

and therefore

$$V_S = IR = (0.5 \times 10^{-3})(20\text{k}) = 10 \text{ V}$$

and the power could be determined using $P = I^2R = V_S^2/R = V_SI$.

EXAMPLE 2.4

Given the network in Fig. 2.4d, we wish to find R and V_S.

Solution Using the power relationship, we find that

$$R = P/I^2 = (80 \times 10^{-3})/(4 \times 10^{-3})^2 = 5 \text{ k}\Omega$$

The voltage can now be derived using Ohm's law as

$$V_S = IR = (4 \times 10^{-3})(5\text{k}) = 20 \text{ V}$$

The voltage could also be obtained from the remaining power relationships in Eqs. (2.2) and (2.3).

Before leaving this initial discussion of circuits containing sources and a single resistor, it is important to note a phenomenon that we will find to be true in circuits containing many sources and resistors. The presence of a voltage source between a pair of terminals tells us precisely what the voltage is between the two terminals regardless of what is happening in the balance of the network. What we do not know is the current in the voltage source. We must apply circuit analysis to the entire network to determine this current. Likewise, the presence of a current source connected between two terminals specifies the exact value of the current through the source between the terminals. What we do not know is the value of the voltage across the current source. This value must be calculated by applying circuit analysis to the entire network. Furthermore, it is worth emphasizing that when applying Ohm's law, the relationship $V = IR$ specifies a relationship between the voltage *directly across* a resistor R and the current that is *present* in this resistor. Ohm's law does not apply when the voltage is present in one part of the network and the current exists in another. This is a common mistake made by students who try to apply $V = IR$ to a resistor R in the middle of the network while using a V at some other location in the network.

Learning Assessments

E2.1 Given the circuits in Fig. E2.1, find (a) the current I and the power absorbed by the resistor in Fig. E2.1a, and (b) the voltage across the current source and the power supplied by the source in Fig. E2.1b.

Answer:
(a) $I = 0.3$ mA, $P = 3.6$ mW;
(b) $V_S = 3.6$ V, $P = 2.16$ mW.

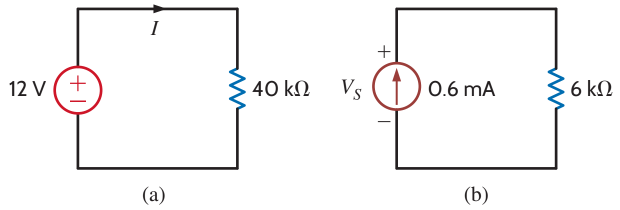


FIGURE E2.1

E2.2 Given the circuits in Fig. E2.2, find (a) R and V_S in the circuit in Fig. E2.2a, and (b) find I and R in the circuit in Fig. E2.2b.

Answer:
(a) $R = 10$ kΩ, $V_S = 4$ V;
(b) $I = 20.8$ mA, $R = 576$ Ω.

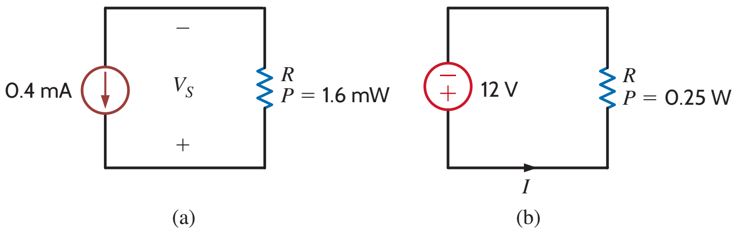


FIGURE E2.2

E2.3 The power absorbed by G_x in Fig. E2.3 is 50 mW. Find G_x.

Answer:
$G_x = 500$ µS.

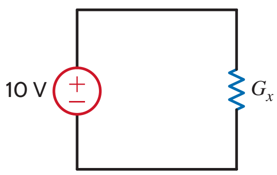


FIGURE E2.3

2.2 Kirchhoff's Laws

The circuits we have considered previously have all contained a single resistor, and we have analyzed them using Ohm's law. At this point we begin to expand our capabilities to handle more complicated networks that result from an interconnection of two or more of these simple elements. We will assume that the interconnection is performed by electrical conductors (wires) that have zero resistance—that is, perfect conductors. Because the wires have zero resistance, the energy in the circuit is in essence lumped in each element, and we employ the term *lumped-parameter circuit* to describe the network.

To aid us in our discussion, we will define a number of terms that will be employed throughout our analysis. As will be our approach throughout this text, we will use examples to illustrate the concepts and define the appropriate terms. For example, the circuit shown in **Fig. 2.5a** will be used to describe the terms *node, loop,* and *branch*. A node is simply a point of connection of two or more circuit elements. The reader is cautioned to note that, although one node can be spread out with perfect conductors, it is still only one node. This is illustrated in **Fig. 2.5b**, where the circuit has been redrawn. Node 5 consists of the entire bottom connector of the circuit.

If we start at some point in the circuit and move along perfect conductors in any direction until we encounter a circuit element, the total path we cover represents a single node.

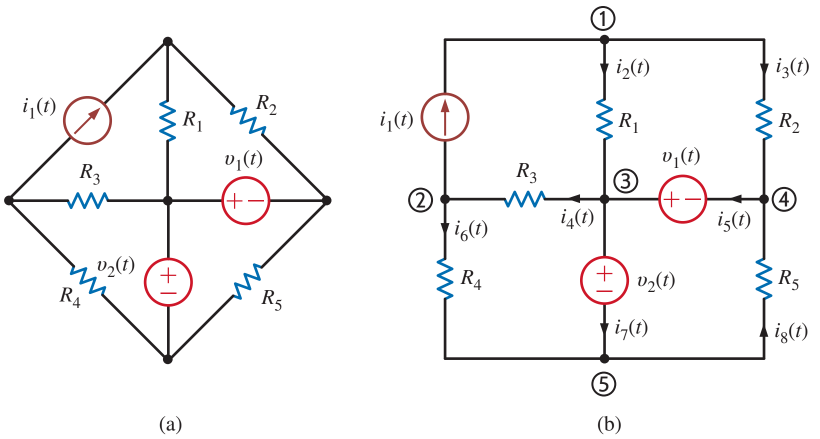


FIGURE 2.5 Circuit used to illustrate KCL.

Therefore, we can assume that a node is one end of a circuit element together with all the perfect conductors that are attached to it. Examining the circuit, we note that there are numerous paths through it. A *loop* is simply any *closed path* through the circuit in which no node is encountered more than once. For example, starting from node 1, one loop would contain the elements R_1, v_2, R_4, and i_1; another loop would contain R_2, v_1, v_2, R_4, and i_1; and so on. However, the path R_1, v_1, R_5, v_2, R_3, and i_1 is not a loop because we have encountered node 3 twice. Finally, a *branch* is a portion of a circuit containing only a single element and the nodes at each end of the element. The circuit in Fig. 2.5 contains eight branches.

Given the previous definitions, we are now in a position to consider Kirchhoff's laws, named after German scientist Gustav Robert Kirchhoff. These two laws are quite simple but extremely important. We will not attempt to prove them because the proofs are beyond our current level of understanding. However, we will demonstrate their usefulness and attempt to make the reader proficient in their use. The first law is *Kirchhoff's current law* (KCL), which states that *the algebraic sum of the currents entering any node is zero* (see **HINT 2.2**). In mathematical form the law appears as

HINT 2.2
KCL is an extremely important and useful law.

$$\sum_{j=1}^{N} i_j(t) = 0 \qquad 2.7$$

where $i_j(t)$ is the jth current entering the node through branch j and N is the number of branches connected to the node. To understand the use of this law, consider node 3 shown in Fig. 2.5. Applying Kirchhoff's current law to this node yields

$$i_2(t) - i_4(t) + i_5(t) - i_7(t) = 0$$

We have assumed that the algebraic signs of the currents entering the node are positive and, therefore, that the signs of the currents leaving the node are negative.

If we multiply the foregoing equation by -1, we obtain the expression

$$-i_2(t) + i_4(t) - i_5(t) + i_7(t) = 0$$

which simply states that *the algebraic sum of the currents leaving a node is zero*. Alternatively, we can write the equation as

$$i_2(t) + i_5(t) = i_4(t) + i_7(t)$$

which states that *the sum of the currents entering a node is equal to the sum of the currents leaving the node*. Both of these italicized expressions are alternative forms of Kirchhoff's current law.

Once again it must be emphasized that the latter statement means that the sum of the *variables* that have been defined entering the node is equal to the sum of the *variables* that have been defined leaving the node, not the actual currents. For example, $i_j(t)$ may be defined entering the node, but if its actual value is negative, there will be positive charge leaving the node.

Note carefully that Kirchhoff's current law states that the *algebraic* sum of the currents either entering or leaving a node must be zero. We now begin to see why we stated in Chapter 1 that it is critically important to specify both the magnitude and the direction of a current. Recall that current is charge in motion. Based on our background in physics, charges cannot be stored at a node. In other words, if we have a number of charges entering a node, then an equal number must be leaving that same node. Kirchhoff's current law is based on this principle of conservation of charge.

Finally, it is possible to generalize Kirchhoff's current law to include a closed surface. By a closed surface we mean some set of elements completely contained within the surface that are interconnected. Since the current entering each element within the surface is equal to that leaving the element (i.e., the element stores no net charge), it follows that the current entering an interconnection of elements is equal to that leaving the interconnection. Therefore, Kirchhoff's current law can also be stated as follows: *The algebraic sum of the currents entering any closed surface is zero.*

EXAMPLE 2.5

Let us write KCL for every node in the network in Fig. 2.5, assuming that the currents leaving the node are positive.

Solution The KCL equations for nodes 1 through 5 are

$$
\begin{aligned}
-i_1(t) + i_2(t) + i_3(t) &= 0 \\
i_1(t) - i_4(t) + i_6(t) &= 0 \\
-i_2(t) + i_4(t) - i_5(t) + i_7(t) &= 0 \\
-i_3(t) + i_5(t) - i_8(t) &= 0 \\
-i_6(t) - i_7(t) + i_8(t) &= 0
\end{aligned}
$$

Note carefully that if we add the first four equations, we obtain the fifth equation. What does this tell us? Recall that this means that this set of equations is not linearly independent. We can show that the first four equations are, however, linearly independent. Store this idea in memory because it will become very important when we learn how to write the equations necessary to solve for all the currents and voltages in a network in the following chapter.

EXAMPLE 2.6

The network in Fig. 2.5 is represented by the topological diagram shown in **Fig. 2.6**. We wish to find the unknown currents in the network.

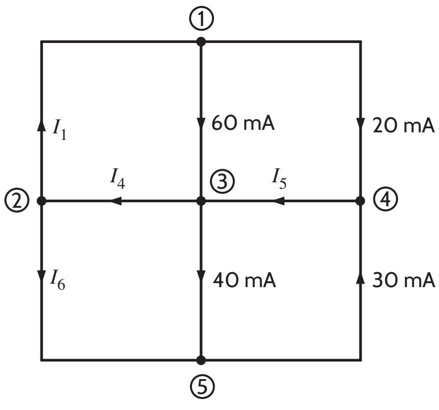


FIGURE 2.6 Topological diagram for the circuit in Fig. 2.5.

Solution Assuming the currents leaving the node are positive, the KCL equations for nodes 1 through 4 are

$$-I_1 + 0.06 + 0.02 = 0$$
$$I_1 - I_4 + I_6 = 0$$
$$-0.06 + I_4 - I_5 + 0.04 = 0$$
$$-0.02 + I_5 - 0.03 = 0$$

The first equation yields I_1, and the last equation yields I_5. Knowing I_5, we can immediately obtain I_4 from the third equation. Then the values of I_1 and I_4 yield the value of I_6 from the second equation. The results are $I_1 = 80$ mA, $I_4 = 70$ mA, $I_5 = 50$ mA, and $I_6 = -10$ mA.

As indicated earlier, dependent or controlled sources are very important because we encounter them when analyzing circuits containing active elements such as transistors. The following example presents a circuit containing a current-controlled current source.

EXAMPLE 2.7

Let us write the KCL equations for the circuit shown in Fig. 2.7.

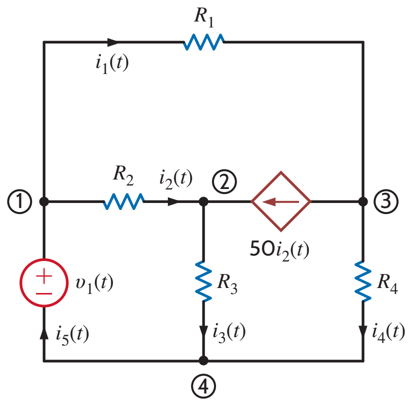


FIGURE 2.7 Circuit containing a dependent current source.

Solution The KCL equations for nodes 1 through 4 follow:

$$i_1(t) + i_2(t) - i_5(t) = 0$$
$$-i_2(t) + i_3(t) - 50i_2(t) = 0$$
$$-i_1(t) + 50i_2(t) + i_4(t) = 0$$
$$i_5(t) - i_3(t) - i_4(t) = 0$$

If we added the first three equations, we would obtain the negative of the fourth. What does this tell us about the set of equations?

Kirchhoff's second law, called *Kirchhoff's voltage law* (KVL), states that the *algebraic sum of the voltages around any loop is zero*. As was the case with Kirchhoff's current law, we will defer the proof of this law and concentrate on understanding how to apply it. Once again the reader is cautioned to remember that we are dealing only with lumped-parameter circuits. These circuits are conservative, meaning that the work required to move a unit charge around any loop is zero.

In Chapter 1, we related voltage to the difference in energy levels within a circuit and talked about the energy conversion process in a flashlight. Because of this relationship between voltage and energy, Kirchhoff's voltage law is based on the conservation of energy.

Recall that in Kirchhoff's current law, the algebraic sign was required to keep track of whether the currents were entering or leaving a node. In Kirchhoff's voltage law, the algebraic sign is used to keep track of the voltage polarity. In other words, as we traverse the circuit, it is necessary to sum to zero the increases and decreases in energy level. Therefore, it is important we keep track of whether the energy level is increasing or decreasing as we go through each element.

EXAMPLE 2.8

Let us find I_4 and I_1 in the network represented by the topological diagram in Fig. 2.6.

Solution This diagram is redrawn in **Fig. 2.8**; node 1 is enclosed in surface 1, and nodes 3 and 4 are enclosed in surface 2. A quick review of the previous example indicates that we derived a value for I_4 from the value of I_5. However, I_5 is now completely enclosed in surface 2. If we apply KCL to surface 2, assuming the currents out of the surface are positive, we obtain

$$I_4 - 0.06 - 0.02 - 0.03 + 0.04 = 0$$

or

$$I_4 = 70 \text{ mA}$$

which we obtained without any knowledge of I_5. Likewise for surface 1, what goes in must come out, and, therefore, $I_1 = 80$ mA. The reader is encouraged to cut the network in Fig. 2.6 into two pieces in any fashion and show that KCL is always satisfied at the boundaries.

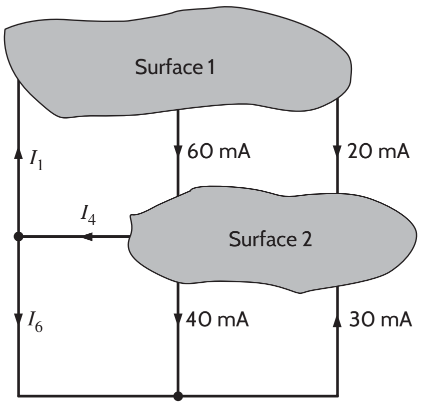


FIGURE 2.8 Diagram used to demonstrate KCL for a surface.

Learning Assessments

E2.4 Given the networks in Fig. E2.3, find (a) I_1 in Fig. E2.4a and (b) I_T in Fig. E2.4b.

Answer:

(a) $I_1 = -50$ mA;

(b) $I_T = 70$ mA.

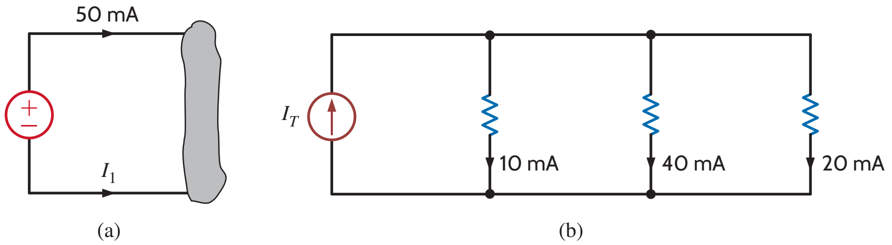


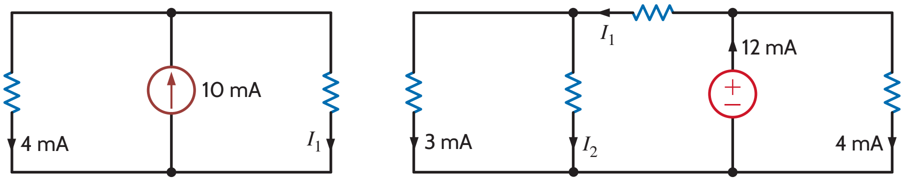
FIGURE E2.4

E2.5 Find (a) I_1 in the network in Fig. E2.5a and (b) I_1 and I_2 in the circuit in Fig. E2.5b.

Answer:

(a) $I_1 = 6$ mA;

(b) $I_1 = 8$ mA and $I_2 = 5$ mA.



(a) (b)

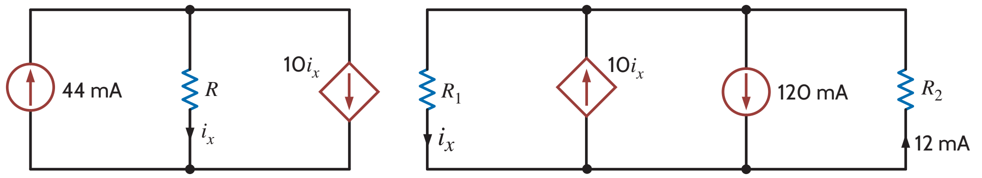
FIGURE E2.5

E2.6 Find the current i_x in the circuits in Fig. E2.6.

Answer:

(a) $i_x = 4$ mA;

(b) $i_x = 12$ mA.



(a) (b)

FIGURE E2.6

In applying KVL, we must traverse any loop in the circuit and sum to zero the increases and decreases in energy level. At this point, we have a decision to make. Do we want to consider a decrease in energy level as positive or negative? We will adopt a policy of considering a decrease in energy level as positive and an increase in energy level as negative. As we move around a loop, we encounter the plus sign first for a decrease in energy level and a negative sign first for an increase in energy level.

Finally, we employ the convention V_{ab} to indicate the voltage of point a with respect to point b; that is, the variable for the voltage between point a and point b, with point a considered positive relative to point b. Since the potential is measured between two points, it is convenient to use an arrow between the two points, with the head of the arrow located

EXAMPLE 2.9

Consider the circuit shown in Fig. 2.9. If V_{R_1} and V_{R_2} are known quantities, let us find V_{R_3}.

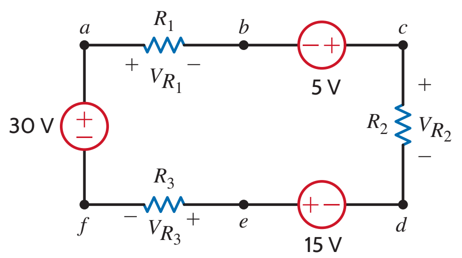


FIGURE 2.9 Circuit used to illustrate KVL.

Solution Starting at point a in the network and traversing it in a clockwise direction, we obtain the equation

$$+V_{R_1} - 5 + V_{R_2} - 15 + V_{R_3} - 30 = 0$$

which can be written as

$$\begin{aligned} +V_{R_1} + V_{R_2} + V_{R_3} &= 5 + 15 + 30 \\ &= 50 \end{aligned}$$

Now suppose that V_{R_1} and V_{R_2} are known to be 18 V and 12 V, respectively. Then $V_{R_3} = 20$ V.

EXAMPLE 2.10

Consider the network in **Fig. 2.10**.

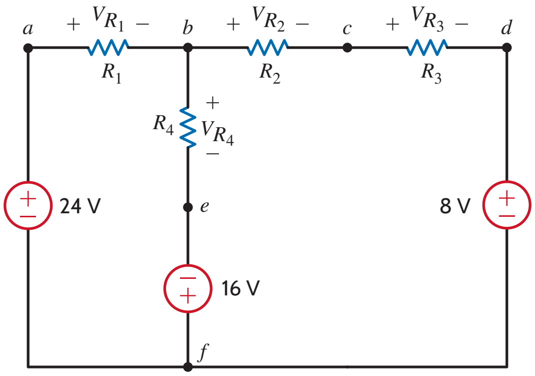


FIGURE 2.10 Circuit used to explain KVL.

Let us demonstrate that only two of the three possible loop equations are linearly independent.

Solution Note that this network has three closed paths: the left loop, right loop, and outer loop. Applying our policy for writing KVL equations and traversing the left loop starting at point *a*, we obtain

$$V_{R_1} + V_{R_4} - 16 - 24 = 0$$

The corresponding equation for the right loop starting at point *b* is

$$V_{R_2} + V_{R_3} + 8 + 16 - V_{R_4} = 0$$

The equation for the outer loop starting at point *a* is

$$V_{R_1} + V_{R_2} + V_{R_3} + 8 - 24 = 0$$

Note that if we add the first two equations, we obtain the third equation. Therefore, as we indicated in Example 2.5, the three equations are not linearly independent. Once again, we will address this issue in the next chapter and demonstrate that we need only the first two equations to solve for the voltages in the circuit.

at the positive node. Note that the double-subscript notation, the + and − notation, and the single-headed arrow notation are all the same if the head of the arrow is pointing toward the positive terminal and the first subscript in the double-subscript notation. All of these equivalent forms for labeling voltages are shown in **Fig. 2.11**. The usefulness of the arrow notation stems from the fact that we may want to label the voltage between two points that are far apart in a network. In this case, the other notations are often confusing.

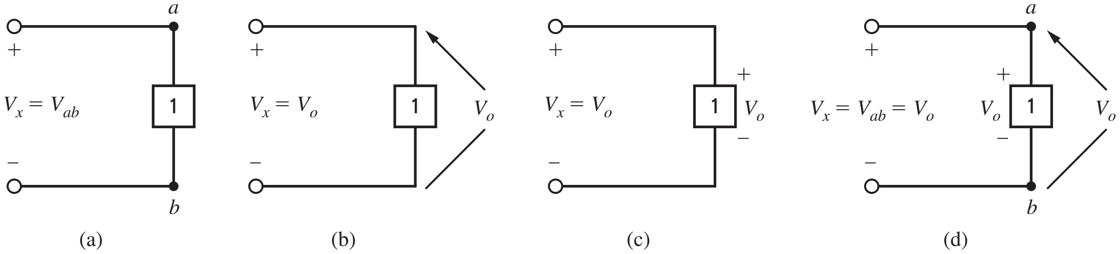


FIGURE 2.11 Equivalent forms for labeling voltage.

EXAMPLE 2.11

Consider the network in **Fig. 2.12a**. Let us apply KVL to determine the voltage between two points. Specifically, in terms of the double-subscript notation, let us find V_{ae} and V_{ec}.

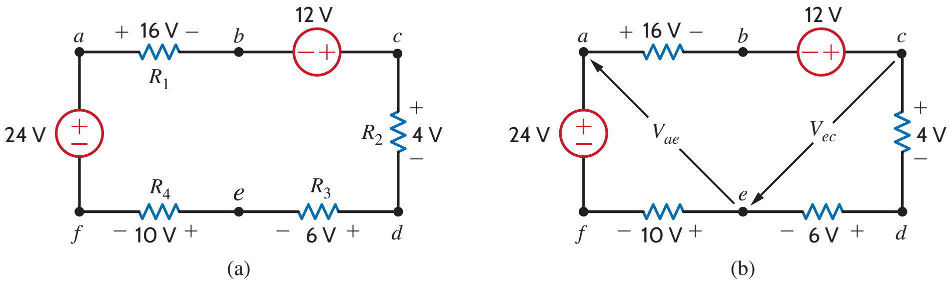


FIGURE 2.12 Network used in Example 2.11.

Solution The circuit is redrawn in **Fig. 2.12b**. Since points *a* and *e* as well as *e* and *c* are not physically close, the arrow notation is very useful. Our approach to determining the unknown voltage is to apply KVL with the unknown voltage in the closed path. Therefore, to determine V_{ae}, we can use the path *aefa* or *abcdea*. The equations for the two paths in which V_{ae} is the only unknown are

$$V_{ae} + 10 - 24 = 0$$

and

$$16 - 12 + 4 + 6 - V_{ae} = 0$$

Note that both equations yield $V_{ae} = 14$ V. Even before calculating V_{ae}, we could calculate V_{ec} using the path *cdec* or *cefabc*. However, since V_{ae} is now known, we can also use the path *ceabc*. KVL for each of these paths is

$$4 + 6 + V_{ec} = 0$$
$$-V_{ec} + 10 - 24 + 16 - 12 = 0$$

and

$$-V_{ec} - V_a + 16 - 12 = 0$$

Each of these equations yields $V_{ec} = -10$ V.

In general, the mathematical representation of Kirchhoff's voltage law is

$$\sum_{j=1}^{N} v_j(t) = 0 \qquad \textbf{2.8}$$

where $v_j(t)$ is the voltage across the jth branch (with the proper reference direction) in a loop containing N voltages (see **HINT 2.3**). This expression is analogous to Eq. (2.7) for Kirchhoff's current law.

HINT 2.3

KVL is an extremely important and useful law.

EXAMPLE 2.12

Given the network in Fig. 2.13 containing a dependent source, let us write the KVL equations for the two closed paths: *abda* and *bcdb*.

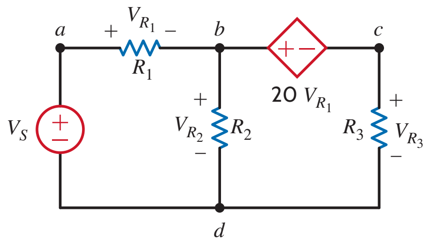


FIGURE 2.13 Network containing a dependent source.

Solution The two KVL equations are

$$V_{R_1} + V_{R_2} - V_S = 0$$
$$20V_{R_1} + V_{R_3} - V_{R_2} = 0$$

Learning Assessments

E2.7 Find I_x and I_1 in Fig. E2.7.

Answer:
$I_x = 2$ mA;
$I_1 = 4$ mA.

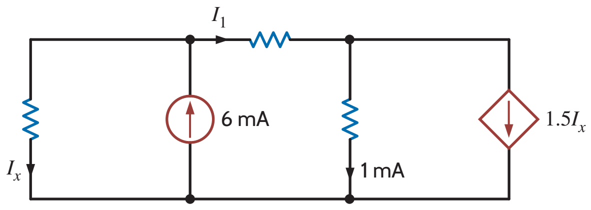


FIGURE E2.7

E2.8 Find V_{ad} and V_{eb} in the network in Fig. E2.8.

Answer:
$V_{ad} = 26$ V;
$V_{eb} = 10$ V.

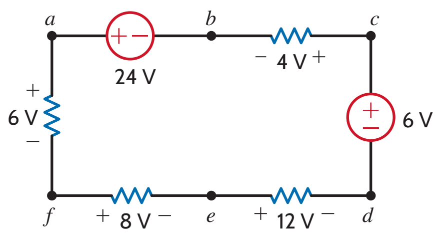


FIGURE E2.8

E2.9 Find V_{bd} in the circuit in Fig. E2.9.

Answer:
$V_{bd} = 11$ V.

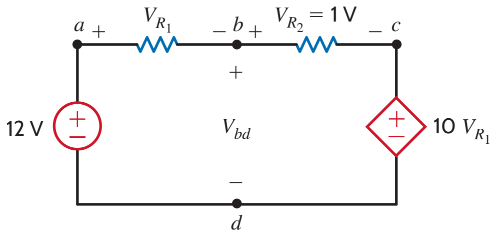


FIGURE E2.9

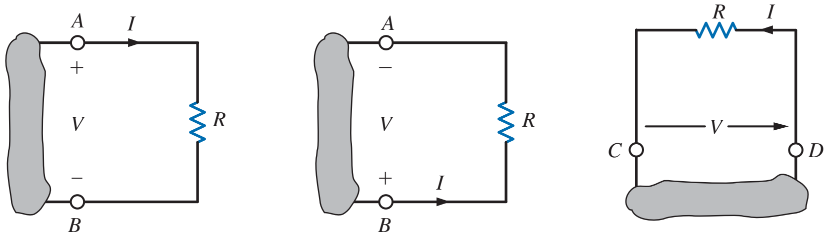


FIGURE 2.14 Circuits used to explain Ohm's law.

Before proceeding with the analysis of simple circuits, it is extremely important that we emphasize a subtle but very critical point (see **HINT 2.4**). Ohm's law as defined by the equation $V = IR$ refers to the relationship between the voltage and current as defined in **Fig. 2.14a**. If the direction of either the current or the voltage, but not both, is reversed, the relationship between the current and the voltage would be $V = -IR$. In a similar manner, given the circuit in **Fig. 2.14b**, if the polarity of the voltage between the terminals A and B is specified as shown, then the direction of the current I is from point B through R to point A. Likewise, in **Fig. 2.14c**, if the direction of the current is specified as shown, then the polarity of the voltage must be such that point D is at a higher potential than point C and, therefore, the arrow representing the voltage V is from point C to point D.

HINT 2.4

The subtleties associated with Ohm's law, as described here, are important and must be adhered to in order to ensure that the variables have the proper sign.

2.3 Single-Loop Circuits

Voltage Division

At this point, we can begin to apply the laws presented earlier to the analysis of simple circuits. To begin, we examine what is perhaps the simplest circuit—a single closed path, or loop, of elements.

Applying KCL to every node in a single-loop circuit reveals that the same current flows through all elements. We say that these elements are connected in series because they carry the same current. We will apply Kirchhoff's voltage law and Ohm's law to the circuit to determine various quantities in the circuit.

Our approach will be to begin with a simple circuit and then generalize the analysis to more complicated ones. The circuit shown in **Fig. 2.15** will serve as a basis for discussion. This circuit consists of an independent voltage source that is in series with two resistors. We have assumed that the current flows in a clockwise direction. If this assumption is correct, the solution of the equations that yields the current will produce a positive value. If the current is actually flowing in the opposite direction, the value of the current variable will simply be negative, indicating that the current is flowing in a direction opposite to that assumed. We have also made voltage polarity assignments for v_{R_1} and v_{R_2}. These assignments have been made using the convention employed in our discussion of Ohm's law and our choice for the direction of $i(t)$—that is, the convention shown in Fig. 2.14a.

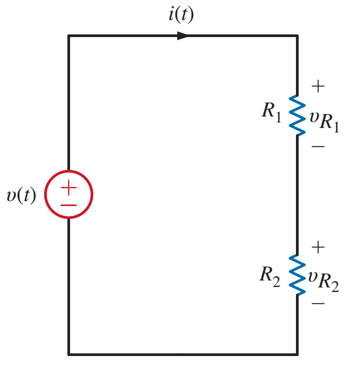


FIGURE 2.15 Single-loop circuit.

Applying Kirchhoff's voltage law to this circuit yields

$$-v(t) + v_{R_1} + v_{R_2} = 0$$

or

$$v(t) = v_{R_1} + v_{R_2}$$

However, from Ohm's law we know that

$$v_{R_1} = R_1 i(t)$$
$$v_{R_2} = R_2 i(t)$$

Therefore,

$$v(t) = R_1 i(t) + R_2 i(t)$$

Solving the equation for $i(t)$ yields

$$i(t) = \frac{v(t)}{R_1 + R_2} \tag{2.9}$$

Knowing the current, we can now apply Ohm's law to determine the voltage across each resistor:

$$\begin{aligned} v_{R_1} &= R_1 i(t) \\ &= R_1\left[\frac{v(t)}{R_1 + R_2}\right] \\ &= \frac{R_1}{R_1 + R_2} v(t) \end{aligned} \tag{2.10}$$

Similarly,

$$v_{R_2} = \frac{R_2}{R_1 + R_2} v(t) \tag{2.11}$$

Though simple, Eqs. (2.10) and (2.11) are very important because they describe the operation of what is called a *voltage divider.* In other words, the source voltage $v(t)$ *is divided between the resistors* R_1 and R_2 *in direct proportion to their resistances* (see **HINT 2.5**).

HINT 2.5

The manner in which voltage divides between two series resistors.

In essence, if we are interested in the voltage across the resistor R_1, we bypass the calculation of the current $i(t)$ and simply multiply the input voltage $v(t)$ by the ratio

$$\frac{R_1}{R_1 + R_2}$$

As illustrated in Eq. (2.10), we are using the current in the calculation, but not explicitly.

Note that the equations satisfy Kirchhoff's voltage law, since

$$-v(t) + \frac{R_1}{R_1 + R_2} v(t) + \frac{R_2}{R_1 + R_2} v(t) = 0$$

EXAMPLE 2.13

Consider the circuit shown in **Fig. 2.16**. The circuit is identical to Fig. 2.15, except that R_1 is a variable resistor such as the volume control for a radio or television set. Suppose that $V_S = 9$ V, $R_1 = 90$ kΩ, and $R_2 = 30$ kΩ.

Let us examine the change in both the voltage across R_2 and the power absorbed in this resistor as R_1 is changed from 90 kΩ to 15 kΩ.

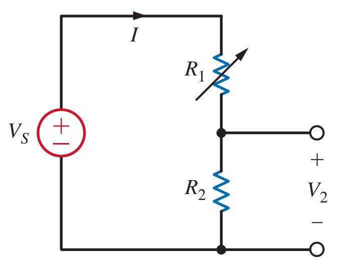


FIGURE 2.16 Voltage-divider circuit.

Solution Since this is a voltage-divider circuit, the voltage V_2 can be obtained directly as

$$V_2 = \left[\frac{R_2}{R_1 + R_2}\right] V_S$$
$$= \left[\frac{30\text{k}}{90\text{k} + 30\text{k}}\right](9)$$
$$= 2.25 \text{ V}$$

Now suppose that the variable resistor is changed from 90 kΩ to 15 kΩ. Then

$$V_2 = \left[\frac{30\text{k}}{30\text{k} + 15\text{k}}\right](9)$$
$$= 6 \text{ V}$$

The direct voltage-divider calculation is equivalent to determining the current I and then using Ohm's law to find V_2. Note that the larger voltage is across the larger resistance. This voltage-divider concept and the simple circuit we have employed to describe it are very useful because, as will be shown later, more complicated circuits can be reduced to this form.

Finally, let us determine the instantaneous power absorbed by the resistor R_2 under the two conditions $R_1 = 90$ kΩ and $R_1 = 15$ kΩ. For the case $R_1 = 90$ kΩ, the power absorbed by R_2 is

$$P_2 = I^2 R_2 = \left(\frac{9}{120\text{k}}\right)^2 (30\text{k})$$
$$= 0.169 \text{ mW}$$

In the second case

$$P_2 = \left(\frac{9}{45\text{k}}\right)^2 (30\text{k})$$
$$= 1.2 \text{ mW}$$

The current in the first case is 75 μA, and in the second case, it is 200 μA. Since the power absorbed is a function of the square of the current, the power absorbed in the two cases is quite different.

Let us now demonstrate the practical utility of this simple voltage-divider network.

EXAMPLE 2.14

Consider the circuit in **Fig. 2.17a**, which is an approximation of a high-voltage dc transmission facility. We have assumed that the bottom portion of the transmission line is a perfect conductor and will justify this assumption in the next chapter. The load can be represented by a resistor of value 183.5 Ω. Therefore, the equivalent circuit of this network is shown in **Fig. 2.17b**.

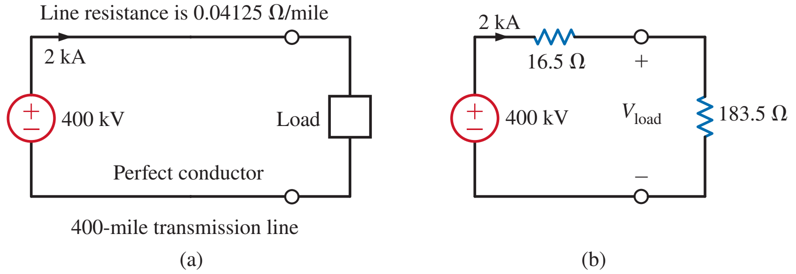


FIGURE 2.17 A high-voltage dc transmission facility.

Let us determine both the power delivered to the load and the power losses in the line.

Solution Using voltage division, the load voltage is

$$V_{\text{load}} = \left[\frac{183.5}{183.5 + 16.5}\right](400\text{k})$$
$$= 367 \text{ kV}$$

The input power is 800 MW, and the power transmitted to the load is

$$P_{\text{load}} = I^2 R_{\text{load}}$$
$$= 734 \text{ MW}$$

Therefore, the power loss in the transmission line is

$$P_{\text{line}} = P_{\text{in}} - P_{\text{load}} = I^2 R_{\text{line}}$$
$$= 66 \text{ MW}$$

Since $P = VI$, suppose now that the utility company supplied power at 200 kV and 4 kA. What effect would this have on our transmission network? Without making a single calculation, we know that because power is proportional to the square of the current, there would be a large increase in the power loss in the line and, therefore, the efficiency of the facility would decrease substantially. That is why, in general, we transmit power at high voltage and low current.

Multiple-Source/Resistor Networks

At this point, we wish to extend our analysis to include a multiplicity of voltage sources and resistors. For example, consider the circuit shown in **Fig. 2.18a**. Here we have assumed that the current flows in a clockwise direction, and we have defined the variable $i(t)$ accordingly. This may or may not be the case, depending on the value of the various voltage sources. Kirchhoff's voltage law for this circuit is

$$+v_{R_1} + v_2(t) - v_3(t) + v_{R_2} + v_4(t) + v_5(t) - v_1(t) = 0$$

or, using Ohm's law,

$$(R_1 + R_2)i(t) = v_1(t) - v_2(t) + v_3(t) - v_4(t) - v_5(t)$$

which can be written as

$$(R_1 + R_2)i(t) = v(t)$$

where

$$v(t) = v_1(t) + v_3(t) - [v_2(t) + v_4(t) + v_5(t)]$$

so that under the preceding definitions, Fig. 2.18a is equivalent to **Fig. 2.18b**. In other words, the sum of several voltage sources in series can be replaced by one source whose value is the algebraic sum of the individual sources. This analysis can, of course, be generalized to a circuit with N series sources.

Now consider the circuit with N resistors in series, as shown in **Fig. 2.19a**. Applying Kirchhoff's voltage law to this circuit yields

$$v(t) = v_{R_1} + v_{R_2} + \cdots + v_{R_N}$$
$$= R_1 i(t) + R_2 i(t) + \cdots + R_N i(t)$$