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An Introduction to Management Science

Quantitative Aproaches to Decision Making

An Introduction to Management Science 16e

Quantitative Approaches to Decision Making

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An Introduction to Management Science: Quantitative Approaches to Decision Making, 16e

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Dedication

To My Parents
Ray and Ilene Anderson
DRA

To My Parents
James and Gladys Sweeney
DJS

To My Parents
Phil and Ann Williams
TAW

To My Parents
Randall and Jeannine Camm
JDC

To My Wife
Teresa
JJC

To My Parents
Mike and Cynthia Fry
MJF

To My Parents
Willis and Phyllis Ohlmann
JWO

Brief Contents

Contents

Preface

We are very excited to publish the sixteenth edition of a text that has been a leader in the field for over 30 years. The purpose of this sixteenth edition, as with previous editions, is to provide undergraduate and graduate students with a sound conceptual understanding of the role that management science plays in the decision-making process. The text describes many of the applications where management science is used successfully. Former users of this text have told us that the applications we describe have led them to find new ways to use management science in their organizations.

An Introduction to Management Science: Quantiative Approaches to Decision Making, 16e is applications oriented and continues to use the problem-scenario approach that is a hallmark of every edition of the text. Using the problem-scenario approach, we describe a problem in conjunction with the management science model being introduced. The model is then solved to generate a solution and recommendation to management. We have found that this approach helps to motivate the student by demonstrating not only how the procedure works, but also how it contributes to the decision-making process.

From the first edition, we have been committed to the challenge of writing a textbook that would help make the mathematical and technical concepts of management science understandable and useful to students of business and economics. Judging from the responses from our teaching colleagues and thousands of students, we have successfully met the challenge. Indeed, it is the helpful comments and suggestions of many loyal users that have been a major reason why the text is so successful.

Throughout the text we have utilized generally accepted notation for the topic being covered, so those students who pursue study beyond the level of this text should be comfortable reading more advanced material. To assist in further study, a references and bibliography section is included at the back of the book.

Changes in the Sixteenth Edition

The sixteenth edition of this text has moved to a full-color internal design for improved aesthetics. Further, we’re excited about the following changes:

Content Changes

We have responded to readers’ feedback and provided more elaborate discussions on several concepts throughout the book. One example of our responsiveness is the introduction of a new section on sensitivity analysis for nonlinear optimization.

We maintain a software-agnostic approach when presenting concepts within the chapters. However, we have removed appendices covering LINGO and Analytic Solver from the textbook. Instead of these two software packages (which require paid licensing), we strengthen our coverage of Excel Solver (which is included with Microsoft Excel).

Management Science in Action

The Management Science in Action vignettes describe how the material covered in a chapter is used in practice. Some of these are provided by practitioners. Others are based on articles from practice-oriented research journals such as the *INFORMS Journal on Applied Analytics* (formerly known as *Interfaces*). We updated the text with six new Management Science in Action vignettes in this edition.

Cases and Problems

The quality of the problems and case problems is an important feature of the text. In this edition we have added over 15 new problems and updated numerous others.

Learning Objectives

At the beginning of each chapter, we have added a list of learning objectives that reflects Bloom's taxonomy. For each end-of-chapter problem, a descriptive problem title has been added and the learning objectives addressed by the problem are listed.

Computer Software Integration

To make it easy for new users of Excel Solver, we provide Excel files with the model formulation for every optimization problem that appears in the body of the text. The model files are well-documented and should make it easy for the user to understand the model formulation. The text is updated for current versions of Microsoft Excel, but Excel 2010 and later versions allow all problems to be solved using the standard version of Excel Solver.

WebAssign

Prepare for class with confidence using WebAssign from Cengage. This online learning platform fuels practice, so students can truly absorb what you learn – and are better prepared come test time. Videos, Problem Walk-Throughs, and End-of-Chapter problems with instant feedback help them understand the important concepts, while instant grading allows you and them to see where they stand in class. Class Insights allows students to see what topics they have mastered and which they are struggling with, helping them identify where to spend extra time. Study Smarter with WebAssign.

Features and Pedagogy

We have continued many of the features that appeared in previous editions. Some of the important ones are noted here.

Annotations

Annotations that highlight key points and provide additional insights for the student are a continuing feature of this edition. These annotations, which appear in the margins, are designed to provide emphasis and enhance understanding of the terms and concepts being presented in the text.

Notes and Comments

At the end of many sections, we provide Notes and Comments designed to give the student additional insights about the methodology and its application. Notes and Comments include warnings about or limitations of the methodology, recommendations for application, brief descriptions of additional technical considerations, and other matters.

Instructor & Student Resources:

Additional instructor and student resources for this product are available online. Instructor assets include an Instructor's Manual, Educator's Guide, PowerPoint® slides, and a test bank powered by Cognero®. Students will find a download for all data sets. Sign up or sign in at www.cengage.com to search for and access this product and its online resources.

Acknowledgments

We owe a debt to many of our colleagues and friends whose names appear below for their helpful comments and suggestions during the development of this and previous editions. Our associates from organizations who supplied several of the Management Science in Action vignettes make a major contribution to the text. These individuals are cited in a credit line associated with each vignette.

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Kansas State University

John K. Fielding
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Texas A&M University

Dan Matthews
Trine University

Aravind Narasipur
Chennai Business School

Nicholas W. Twigg
Coastal Carolina University

Julie Ann Stuart Williams
University of West Florida

We are also indebted to the entire team at Cengage who worked on this title: our Senior Product Manager, Aaron Arnsparger; our Senior Content Manager, Conor Allen; Senior Learning Designer, Brandon Foltz; Digital Delivery Quality Partner, Steven McMillian; Associate Subject-Matter Expert Nancy Marchant; Content Program Manager Jessica Galloway; Content Quality Assurance Engineer Douglas Marks; and our Project Manager at MPS Limited, Shreya Tiwari, for their editorial counsel and support during the preparation of this text.

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About the Authors

Jeffrey D. Camm. Jeffrey D. Camm is the Inmar Presidential Chair and Associate Dean of Analytics in the School of Business at Wake Forest University. Born in Cincinnati, Ohio, he holds a B.S. from Xavier University (Ohio) and a Ph.D. from Clemson University. Prior to joining the faculty at Wake Forest, he was on the faculty of the University of Cincinnati. He has also been a visiting scholar at Stanford University and a visiting professor of business administration at the Tuck School of Business at Dartmouth College.

Dr. Camm has published over 45 papers in the general area of optimization applied to problems in operations management and marketing. He has published his research in *Science*, *Management Science*, *Operations Research*, and other professional journals. Dr. Camm was named the Dornoff Fellow of Teaching Excellence at the University of Cincinnati and he was the 2006 recipient of the INFORMS Prize for the Teaching of Operations Research Practice. A firm believer in practicing what he preaches, he has served as an operations research consultant to numerous companies and government agencies. From 2005 to 2010 he served as editor-in-chief of *Interfaces.* In 2016, Professor Camm received the George E. Kimball Medal for service to the operations research profession, and in 2017 he was named an INFORMS Fellow.

James J. Cochran. James J. Cochran is Professor of Applied Statistics and the Rogers-Spivey Faculty Fellow in the Department of Information Systems, Statistics, and Management Science at The University of Alabama. Born in Dayton, Ohio, he holds a B.S., an M.S., and an M.B.A. from Wright State University and a Ph.D. from the University of Cincinnati. He has been a visiting scholar at Stanford University, Universidad de Talca, the University of South Africa, and Pole Universitaire Leonard de Vinci.

Professor Cochran has published over 50 papers in the development and application of operations research and statistical methods. He has published his research in *Management Science*, *The American Statistician*, *Communications in Statistics - Theory and Methods*, *Annals of Operations Research, Statistics and Probability Letters, European Journal of Operational Research*, *Journal of Combinatorial Optimization*, and other professional journals. He was the 2008 recipient of the INFORMS Prize for the Teaching of Operations Research Practice, the 2010 recipient of the Mu Sigma Rho Statistical Education Award, and the 2016 recipient of the Waller Distinguised Teaching Career Award from the American Statistical Association. Professor Cochran was elected to the International Statistics Institute in 2005, named a Fellow of the American Statistical Association in 2011, and named a Fellow of INFORMS in 2017. He received the Founders Award from the American Statistical Association in 2014, the Karl E. Peace Award for Outstanding Statistical Contributions for the Betterment of Society from the American Statistical Association in 2015, and the INFORMS President's Award in 2019.

A strong advocate for effective operations research and statistics education as a means of improving the quality of applications to real problems, Professor Cochran has organized and chaired teaching effectiveness workshops in Montevideo, Uruguay; Cape Town, South Africa; Cartagena, Colombia; Jaipur, India; Buenos Aires, Argentina; Nairobi, Kenya; Buea, Cameroon; Osijek, Croatia; Kathmandu, Nepal; Havana, Cuba; Ulaanbaatar, Mongolia; Chișinău Moldova; Sozopol, Bulgaria; and Saint George's, Grenada. He has served as an operations research or statistical consultant to numerous companies and not-for-profit organizations. From 2007 to 2012 Professor Cochran served as editor-in-chief of *INFORMS Transactions on Education*. He is on the editorial board of several journals including *INFORMS Journal on Applied Analytics*, *Significance*, and *International Transactions in Operational Research*.

Michael J. Fry. Michael J. Fry is Professor and Interim Head of the Department of Operations, Business Analytics, and Information Systems in the Carl H. Lindner College of

Business at the University of Cincinnati. Born in Killeen, Texas, he earned a B.S. from Texas A&M University, and M.S.E. and Ph.D. degrees from the University of Michigan. He has been at the University of Cincinnati since 2002, where he previously served as Department Head from 2014 to 2018 and as Academic Director of the Center for Business Analytics from 2018 to 2021. He has also been a visiting professor at the Samuel Curtis Johnson Graduate School of Management at Cornell University and the Sauder School of Business at the University of British Columbia.

Professor Fry has published over 25 research papers in journals such as *Operations Research, Manufacturing and Service Operations Management, Transportation Science, Naval Research Logistics, IIE Transactions, Critical Care Medicine,* and *Interfaces*. He serves on editorial boards for journals such as *Production and Operations Management, INFORMS Journal on Applied Analytics,* and *Journal of Quantitative Analysis in Sports*. His research interests are in applying quantitative management methods to the areas of supply chain analytics, sports analytics, and public-policy operations. He has worked with many different organizations for his research, including Dell, Inc., Starbucks Coffee Company, Great American Insurance Group, the Cincinnati Fire Department, the State of Ohio Election Commission, the Cincinnati Bengals, and the Cincinnati Zoo and Botanical Gardens. In 2008, he was named a finalist for the Daniel H. Wagner Prize for Excellence in Operations Research Practice, and he has been recognized for both his research and teaching excellence at the University of Cincinnati. In 2019, he led the team that was awarded the INFORMS UPS George D. Smith Prize on behalf of the OBAIS Department at the University of Cincinnati.

Jeffrey W. Ohlmann. Jeffrey W. Ohlmann is Associate Professor of Management Sciences and Huneke Research Fellow in the Tippie College of Business at the University of Iowa. Born in Valentine, Nebraska, he earned a B.S. from the University of Nebraska, and M.S. and Ph.D. degrees from the University of Michigan. He has been at the University of Iowa since 2003.

Professor Ohlmann's research on the modeling and solution of decision-making problems has produced over two dozen research papers in journals such as *Operations Research, Mathematics of Operations Research, INFORMS Journal on Computing, Transportation Science, European Journal of Operational Research, Decision Analysis, Decision Support Systems*, and *Big Data*. He has collaborated with companies such as Transfreight, LeanCor, Cargill, the Hamilton County Board of Elections, and three National Football League franchises. Due to the relevance of his work to industry, he was bestowed the George B. Dantzig Dissertation Award and was recognized as a finalist for the Daniel H. Wagner Prize for Excellence in Operations Research Practice.

David R. Anderson. David R. Anderson is Professor Emeritus of Quantitative Analysis at the University of Cincinnati. Born in Grand Forks, North Dakota, he earned his B.S., M.S., and Ph.D. degrees from Purdue University. Professor Anderson has served as Head of the Department of Quantitative Analysis and Operations Management and as Associate Dean of the College of Business Administration. In addition, he was the coordinator of the College's first Executive Program.

At the University of Cincinnati, Professor Anderson has taught introductory statistics for business students as well as graduate-level courses in regression analysis, multivariate analysis, and management science. He has also taught statistical courses at the Department of Labor in Washington, D.C. He has been honored with nominations and awards for excellence in teaching and excellence in service to student organizations.

Professor Anderson has coauthored 10 textbooks in the areas of statistics, management science, linear programming, and production and operations management. He is an active consultant in the field of sampling and statistical methods.

Dennis J. Sweeney. Dennis J. Sweeney is Professor Emeritus of Quantitative Analysis at the University of Cincinnati. Born in Des Moines, Iowa, he earned a B.S.B.A. degree from Drake University and his M.B.A. and D.B.A. degrees from Indiana University, where he was an NDEA Fellow. During 1978–79, Professor Sweeney worked in the management science group at Procter & Gamble; during 1981–82, he was a visiting professor at Duke University. Professor Sweeney served as Head of the Department of Quantitative Analysis and as Associate Dean of the College of Business Administration at the University of Cincinnati.

Professor Sweeney has published more than 30 articles and monographs in the area of management science and statistics. The National Science Foundation, IBM, Procter & Gamble, Federated Department Stores, Kroger, and Cincinnati Gas & Electric have funded his research, which has been published in *Management Science, Operations Research, Mathematical Programming, Decision Sciences,* and other journals.

Professor Sweeney has coauthored 10 textbooks in the areas of statistics, management science, linear programming, and production and operations management.

Thomas A. Williams. Thomas A. Williams is Professor Emeritus of Management Science at Rochester Institute of Technology. Born in Elmira, New York, he earned his B.S. degree at Clarkson University. He did his graduate work at Rensselaer Polytechnic Institute, where he received his M.S. and Ph.D. degrees.

Before joining the College of Business at RIT, Professor Williams served for seven years as a faculty member at the University of Cincinnati, where he developed the undergraduate program in Information Systems and then served as its coordinator. At RIT he was the first chairman of the Decision Sciences Department. He teaches courses in management science and statistics, as well as graduate courses in regression and decision analysis.

Professor Williams is the coauthor of 11 textbooks in the areas of management science, statistics, production and operations management, and mathematics. He has been a consultant for numerous *Fortune* 500 companies and has worked on projects ranging from the use of data analysis to the development of large-scale regression models.

An Introduction to Management Science 16e

Quantitative Approaches to Decision Making

Chapter 1

Introduction

Contents

Learning Objectives

After completing this chapter, you will be able to

LO 1 Define the terms *management science* and *operations research*.

LO 2 List the steps in the decision-making process and explain the roles of qualitative and quantitative approaches to managerial decision making.

LO 3 Explain the modelling process and its benefits to analyzing real situations.

LO 4 Formulate basic mathematical models of cost, revenue, and profit and compute the breakeven point.

Management science, an approach to decision making based on the scientific method, makes extensive use of quantitative analysis. A variety of names exist for the body of knowledge involving quantitative approaches to decision making; in addition to management science, two other widely known and accepted names are operations research and decision science. Today, many use the terms *management science, operations research,* and *decision science* interchangeably.

The scientific management revolution of the early 1900s, initiated by Frederic W. Taylor, provided the foundation for the use of quantitative methods in management. But modern management science research is generally considered to have originated during the World War II period, when teams were formed to deal with strategic and tactical problems faced by the military. These teams, which often consisted of people with diverse specialties (e.g., mathematicians, engineers, and behavioral scientists), were joined together to solve a common problem by utilizing the scientific method. After the war, many of these team members continued their research in the field of management science.

Two developments that occurred during the post–World War II period led to the growth and use of management science in nonmilitary applications. First, continued research resulted in numerous methodological developments. Probably the most significant development was the discovery by George Dantzig, in 1947, of the simplex method for solving linear programming problems. At the same time these methodological developments were taking place, digital computers prompted a virtual explosion in computing power. Computers enabled practitioners to use the methodological advances to solve a large variety of problems. The computer technology explosion continues; smartphones, tablets, and other mobile-computing devices can now be used to solve problems larger than those solved on mainframe computers in the 1990s.

More recently, the explosive growth of data from sources such as smartphones and other personal-electronic devices provide access to much more data today than ever before. Additionally, the Internet allows for easy sharing and storage of data, providing extensive access to a variety of users to the necessary inputs to management-science models.

As stated in the Preface, the purpose of the text is to provide students with a sound conceptual understanding of the role that management science plays in the decision-making process. We also said that the text is application oriented. To reinforce the applications nature of the text and provide a better understanding of the variety of applications in which management science has been used successfully, Management Science in Action articles are presented throughout the text. Each Management Science in Action article summarizes an application of management science in practice. The first Management Science in Action in this chapter, Revenue Management at AT&T Park, describes one of the most important applications of management science in the sports and entertainment industry.

Management Science in Action

Revenue Management at AT&T Park*

Imagine the difficult position Russ Stanley, Vice President of Ticket Services for the San Francisco Giants, found himself facing late in the 2010 baseball season. Prior to the season, his organization adopted a dynamic approach to pricing its tickets similar to the model successfully pioneered by Thomas M. Cook and his operations research group at American Airlines. Stanley desperately wanted the Giants to clinch a playoff berth but he didn't want the team to do so *too quickly*.

When dynamically pricing a good or service, an organization regularly reviews supply and demand of the product and uses operations research to determine if the price should be changed to reflect these conditions. As the scheduled takeoff date for a flight nears, the cost of a ticket increases if seats for the flight are relatively scarce. On the other hand, the airline discounts tickets

*Based on Peter Horner, "The Sabre Story," *OR/MS Today* (June 2000); Ken Belson, "Baseball Tickets Too Much? Check Back Tomorrow," *NewYorkTimes.com* (May 18, 2009); and Rob Gloster, "Giants Quadruple Price of Cheap Seats as Playoffs Drive Demand," *Bloomberg Business-week* (September 30, 2010).

for an approaching flight with relatively few ticketed passengers. Through the use of optimization to dynamically set ticket prices, American Airlines generates nearly $1 billion annually in incremental revenue.

The management team of the San Francisco Giants recognized similarities between their primary product (tickets to home games) and the primary product sold by airlines (tickets for flights) and adopted a similar revenue management system. If a particular Giants' game is appealing to fans, tickets sell quickly and demand far exceeds supply as the date of the game approaches; under these conditions fans will be willing to pay more and the Giants charge a premium for the ticket. Similarly, tickets for less attractive games are discounted to reflect relatively low demand by fans. This is why Stanley found himself in a quandary at the end of the 2010 baseball season. The Giants were in the middle of a tight pennant race with the San Diego Padres that effectively increased demand for tickets to Giants' games, and the team was actually scheduled to play the Padres in San Francisco for the last three games of the season. While Stanley certainly wanted his club to win its division and reach the Major League Baseball playoffs, he also recognized that his team's revenues would be greatly enhanced if it didn't qualify for the playoffs until the last day of the season. "I guess financially it is better to go all the way down to the last game," Stanley said in a late season interview. "Our hearts are in our stomachs; we're pacing watching these games."

Does revenue management and operations research work? Today, virtually every airline uses some sort of revenue-management system, and the cruise, hotel, and car rental industries also now apply revenue-management methods. As for the Giants, Stanley said dynamic pricing provided a 7% to 8% increase in revenue per seat for Giants' home games during the 2010 season. Coincidentally, the Giants did win the National League West division on the last day of the season and ultimately won the World Series. Several professional sports franchises are now looking to the Giants' example and considering implementation of similar dynamic ticket-pricing systems.

1.1 Problem Solving and Decision Making

Problem solving can be defined as the process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve the difference. For problems important enough to justify the time and effort of careful analysis, the problem-solving process involves the following seven steps:

1. Identify and define the problem.
2. Determine the set of alternative solutions.
3. Determine the criterion or criteria that will be used to evaluate the alternatives.
4. Evaluate the alternatives.
5. Choose an alternative.
6. Implement the selected alternative.
7. Evaluate the results to determine whether a satisfactory solution has been obtained.

Decision making is the term generally associated with the first five steps of the problem-solving process. Thus, the first step of decision making is to identify and define the problem. Decision making ends with the choosing of an alternative, which is the act of making the decision.

Let us consider the following example of the decision-making process. For the moment assume that you are currently unemployed and that you would like a position that will lead to a satisfying career. Suppose that your job search has resulted in offers from companies in Rochester, New York; Dallas, Texas; Greensboro, North Carolina; and Pittsburgh, Pennsylvania. Thus, the alternatives for your decision problem can be stated as follows:

1. Accept the position in Rochester.
2. Accept the position in Dallas.
3. Accept the position in Greensboro.
4. Accept the position in Pittsburgh.

The next step of the problem-solving process involves determining the criteria that will be used to evaluate the four alternatives. Obviously, the starting salary is a factor of some

Table 1.1 Data for the Job Evaluation Decision-Making Problem

Alternative	Starting Salary	Potential for Advancement	Job Location
1. Rochester	$58,500	Average	Average
2. Dallas	$56,000	Excellent	Good
3. Greensboro	$56,000	Good	Excellent
4. Pittsburgh	$57,000	Average	Good

importance. If salary were the only criterion of importance to you, the alternative selected as "best" would be the one with the highest starting salary. Problems in which the objective is to find the best solution with respect to one criterion are referred to as **single-criterion decision problems**.

Suppose that you also conclude that the potential for advancement and the location of the job are two other criteria of major importance. Thus, the three criteria in your decision problem are starting salary, potential for advancement, and location. Problems that involve more than one criterion are referred to as **multicriteria decision problems**.

The next step of the decision-making process is to evaluate each of the alternatives with respect to each criterion. For example, evaluating each alternative relative to the starting salary criterion is done simply by recording the starting salary for each job alternative. Evaluating each alternative with respect to the potential for advancement and the location of the job is more difficult to do, however, because these evaluations are based primarily on subjective factors that are often difficult to quantify. Suppose for now that you decide to measure potential for advancement and job location by rating each of these criteria as poor, fair, average, good, or excellent. The data that you compile are shown in Table 1.1.

You are now ready to make a choice from the available alternatives. What makes this choice phase so difficult is that the criteria are probably not all equally important, and no one alternative is "best" with regard to all criteria. Although we will present a method for dealing with situations like this one later in the text, for now let us suppose that after a careful evaluation of the data in Table 1.1, you decide to select alternative 3; alternative 3 is thus referred to as the **decision**.

At this point in time, the decision-making process is complete. In summary, we see that this process involves five steps:

1. Define the problem.
2. Identify the alternatives.
3. Determine the criteria.
4. Evaluate the alternatives.
5. Choose an alternative.

Note that missing from this list are the last two steps in the problem-solving process: implementing the selected alternative and evaluating the results to determine whether a satisfactory solution has been obtained. This omission is not meant to diminish the importance of each of these activities, but to emphasize the more limited scope of the term *decision making* as compared to the term *problem solving*. Figure 1.1 summarizes the relationship between these two concepts.

1.2 Quantitative Analysis and Decision Making

Consider the flowchart presented in Figure 1.2. Note that it combines the first three steps of the decision-making process under the heading "Structuring the Problem" and the latter two steps under the heading "Analyzing the Problem." Let us now consider in greater detail how to carry out the set of activities that make up the decision-making process.

Figure 1.1 The Relationship Between Problem Solving and Decision Making

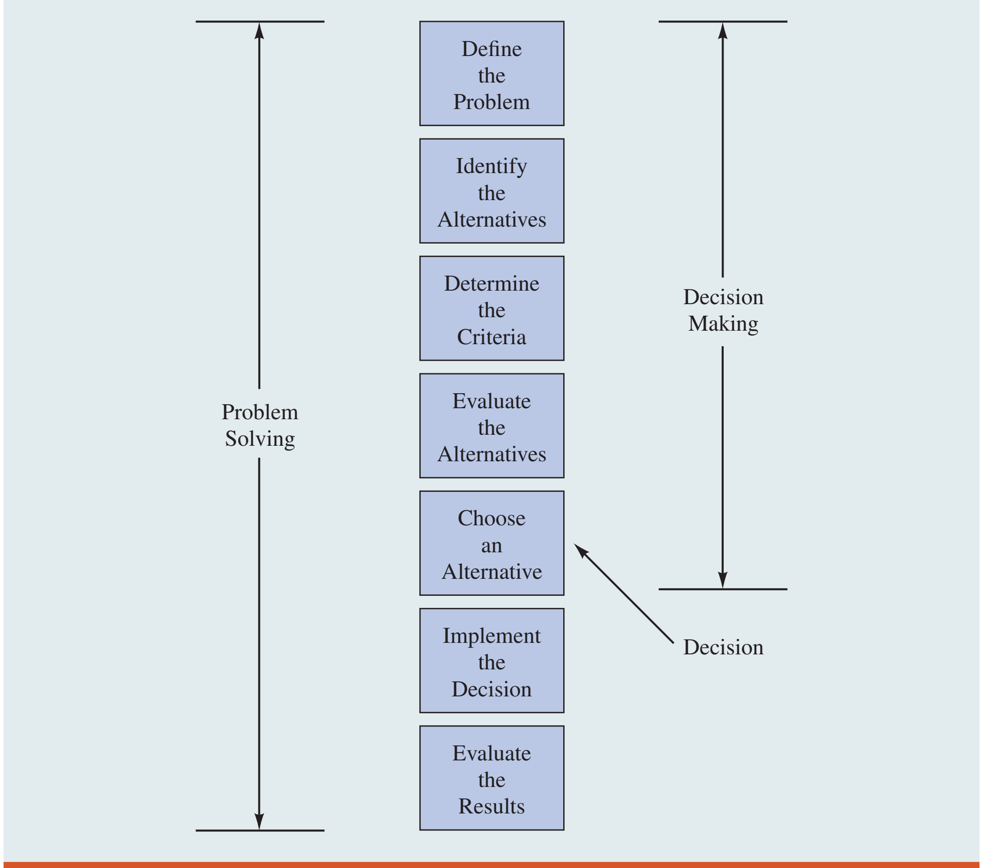


Figure 1.2 An Alternate Classification of the Decision-Making Process

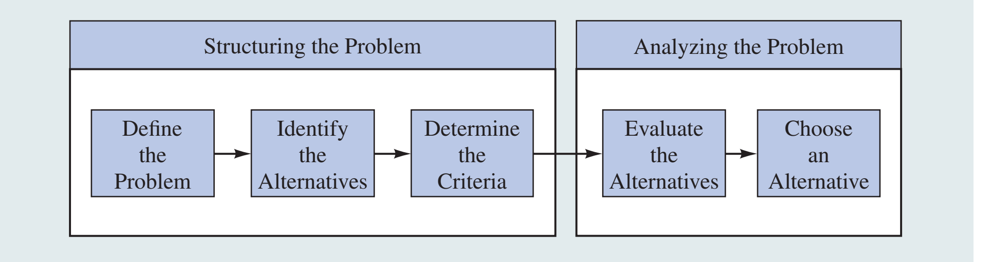
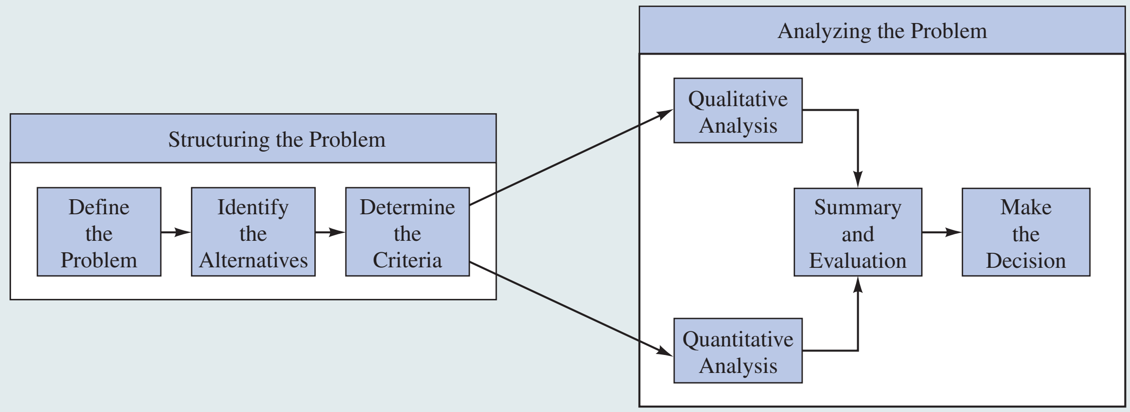


Figure 1.3 shows that the analysis phase of the decision-making process may take two basic forms: qualitative and quantitative. Qualitative analysis is based primarily on the manager's judgment and experience; it includes the manager's intuitive "feel" for the problem and is more an art than a science. If the manager has had experience with similar problems or if the problem is relatively simple, heavy emphasis may be placed on a qualitative analysis. However, if the manager has had little experience with similar problems, or if the problem is sufficiently complex, then a quantitative analysis of the problem can be an especially important consideration in the manager's final decision.

When using the quantitative approach, an analyst will concentrate on the quantitative facts or data associated with the problem and develop mathematical expressions that describe

Figure 1.3 The Role of Qualitative and Quantitative Analysis



the objectives, constraints, and other relationships that exist in the problem. Then, by using one or more quantitative methods, the analyst will make a recommendation based on the quantitative aspects of the problem.

Although skills in the qualitative approach are inherent in the manager and usually increase with experience, the skills of the quantitative approach can be learned only by studying the assumptions and methods of management science. A manager can increase decision-making effectiveness by learning more about quantitative methodology and by better understanding its contribution to the decision-making process. A manager who is knowledgeable in quantitative decision-making procedures is in a much better position to compare and evaluate the qualitative and quantitative sources of recommendations and ultimately to combine the two sources in order to make the best possible decision.

The box in Figure 1.3 titled "Quantitative Analysis" encompasses most of the subject matter of this text. We will consider a managerial problem, introduce the appropriate quantitative methodology, and then develop the recommended decision.

In closing this section, let us briefly state some of the reasons why a quantitative approach might be used in the decision-making process:

1. The problem is complex, and the manager cannot develop a good solution without the aid of quantitative analysis.
2. The problem is especially important (e.g., a great deal of money is involved), and the manager desires a thorough analysis before attempting to make a decision.
3. The problem is new, and the manager has no previous experience from which to draw.
4. The problem is repetitive, and the manager saves time and effort by relying on quantitative procedures to make routine decision recommendations.

1.3 Quantitative Analysis

From Figure 1.3, we see that quantitative analysis begins once the problem has been structured. It usually takes imagination, teamwork, and considerable effort to transform a rather general problem description into a well-defined problem that can be approached via quantitative analysis. The more the analyst is involved in the process of structuring the problem, the more likely the ensuing quantitative analysis will make an important contribution to the decision-making process.

To successfully apply quantitative analysis to decision making, the management scientist must work closely with the manager or user of the results. When both the management scientist and the manager agree that the problem has been adequately structured, work can begin on developing a model to represent the problem mathematically. Solution procedures can then be employed to find the best solution for the model. This best solution for the model then becomes a recommendation to the decision maker. The process of developing and solving models is the essence of the quantitative analysis process.

Model Development

Models are representations of real objects or situations and can be presented in various forms. For example, a scale model of an airplane is a representation of a real airplane. Similarly, a child's toy truck is a model of a real truck. The model airplane and toy truck are examples of models that are physical replicas of real objects. In modeling terminology, physical replicas are referred to as **iconic models**.

A second classification includes models that are physical in form but do not have the same physical appearance as the object being modeled. Such models are referred to as **analog models**. The speedometer of an automobile is an analog model; the position of the needle on the dial represents the speed of the automobile. A thermometer is another analog model representing temperature.

A third classification of models—the type we will primarily be studying—includes representations of a problem by a system of symbols and mathematical relationships or expressions. Such models are referred to as **mathematical models** and are a critical part of any quantitative approach to decision making. For example, the total profit from the sale of a product can be determined by multiplying the profit per unit by the quantity sold. If we let x represent the number of units sold and P the total profit, then, with a profit of \$10 per unit, the following mathematical model defines the total profit earned by selling x units:

$$P = 10x \qquad \textbf{(1.1)}$$

The purpose, or value, of any model is that it enables us to make inferences about the real situation by studying and analyzing the model. For example, an airplane designer might test an iconic model of a new airplane in a wind tunnel to learn about the potential flying characteristics of the full-size airplane. Similarly, a mathematical model may be used to make inferences about how much profit will be earned if a specified quantity of a particular product is sold. According to the mathematical model of equation (1.1), we would expect selling three units of the product ($x = 3$) would provide a profit of $P = 10(3) = \$30$.

In general, experimenting with models requires less time and is less expensive than experimenting with the real object or situation. A model airplane is certainly quicker and less expensive to build and study than the full-size airplane. Similarly, the mathematical model in equation (1.1) allows a quick identification of profit expectations without actually requiring the manager to produce and sell x units. Models also have the advantage of reducing the risk associated with experimenting with the real situation. In particular, bad designs or bad decisions that cause the model airplane to crash or a mathematical model to project a \$10,000 loss can be avoided in the real situation.

Herbert A. Simon, a Nobel Prize winner in economics and an expert in decision making, said that a mathematical model does not have to be exact; it just has to be close enough to provide better results than can be obtained by common sense.

The value of model-based conclusions and decisions is dependent on how well the model represents the real situation. The more closely the model airplane represents the real airplane, the more accurate the conclusions and predictions will be. Similarly, the more closely the mathematical model represents the company's true profit–volume relationship, the more accurate the profit projections will be.

Because this text deals with quantitative analysis based on mathematical models, let us look more closely at the mathematical modeling process. When initially considering a managerial problem, we usually find that the problem definition phase leads to a specific objective, such as maximization of profit or minimization of cost, and possibly a set of restrictions or **constraints**, such as production capacities. The success of the mathematical model and quantitative approach will depend heavily on how accurately the objective and constraints can be expressed in terms of mathematical equations or relationships.

A mathematical expression that describes the problem's objective is referred to as the **objective function**. For example, the profit equation $P = 10x$ would be an objective function for a firm attempting to maximize profit. A production capacity constraint would be necessary if, for instance, 5 hours are required to produce each unit and only 40 hours of production time are available per week. Let x indicate the number of units produced each week. The production time constraint is given by

$$5x \leq 40 \tag{1.2}$$

The value of $5x$ is the total time required to produce x units; the symbol $\leq$ indicates that the production time required must be less than or equal to the 40 hours available.

The decision problem or question is the following: How many units of the product should be scheduled each week to maximize profit? A complete mathematical model for this simple production problem is

$$\begin{array}{lll} \text{Maximize} & P = 10x & \text{objective function} \\ \text{subject to (s.t.)} & & \\ & \left.\begin{array}{r} 5x \leq 40 \\ x \geq 0 \end{array}\right\} & \text{constraints} \end{array}$$

The $x \geq 0$ constraint requires the production quantity x to be greater than or equal to zero, which simply recognizes the fact that it is not possible to manufacture a negative number of units. The optimal solution to this model can be easily calculated and is given by $x = 8$, with an associated profit of \$80. This model is an example of a linear programming model. In subsequent chapters we will discuss more complicated mathematical models and learn how to solve them in situations where the answers are not nearly so obvious.

In the preceding mathematical model, the profit per unit (\$10), the production time per unit (5 hours), and the production capacity (40 hours) are environmental factors that are not under the control of the manager or decision maker. Such environmental factors, which can affect both the objective function and the constraints, are referred to as **uncontrollable inputs** to the model. Inputs that are completely controlled or determined by the decision maker are referred to as **controllable inputs** to the model. In the example given, the production quantity x is the controllable input to the model. Controllable inputs are the decision alternatives specified by the manager and thus are also referred to as the **decision variables** of the model.

Once all controllable and uncontrollable inputs are specified, the objective function and constraints can be evaluated and the output of the model determined. In this sense, the output of the model is simply the projection of what would happen if those particular environmental factors and decisions occurred in the real situation. A flowchart of how controllable and uncontrollable inputs are transformed by the mathematical model into output is shown in Figure 1.4. A similar flowchart showing the specific details of the production model is shown in Figure 1.5.

As stated earlier, the uncontrollable inputs are those the decision maker cannot influence. The specific controllable and uncontrollable inputs of a model depend on the particular problem or decision-making situation. In the production problem, the production time available (40) is an uncontrollable input. However, if it were possible to hire more employees or use overtime, the number of hours of production time would become a controllable input and therefore a decision variable in the model.

Uncontrollable inputs can either be known exactly or be uncertain and subject to variation. If all uncontrollable inputs to a model are known and cannot vary, the model is referred to as a **deterministic model**. Corporate income tax rates are not under the influence of the manager and thus constitute an uncontrollable input in many decision

Figure 1.4 Flowchart of the Process of Transforming Model Inputs into Output

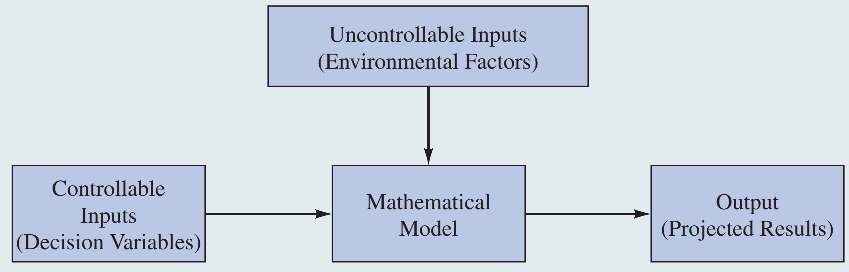
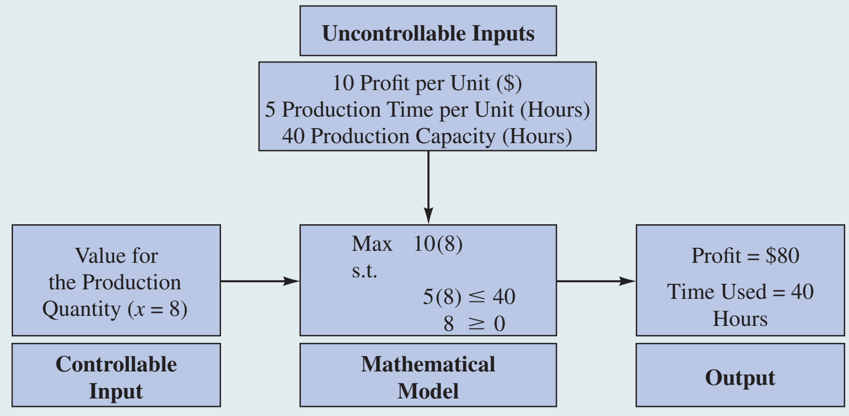


Figure 1.5 Flowchart for the Production Model



models. Because these rates are known and fixed (at least in the short run), a mathematical model with corporate income tax rates as the only uncontrollable input would be a deterministic model. The distinguishing feature of a deterministic model is that the uncontrollable input values are known in advance.

If any of the uncontrollable inputs are uncertain to the decision maker, the model is referred to as a **stochastic** or **probabilistic model**. An uncontrollable input to many production planning models is demand for the product. A mathematical model that treats future demand—which may be any of a range of values—with uncertainty would be called a stochastic model. In the production model, the number of hours of production time required per unit, the total hours available, and the unit profit were all uncontrollable inputs. Because the uncontrollable inputs were all known to take on fixed values, the model was deterministic. If, however, the number of hours of production time per unit could vary from 3 to 6 hours depending on the quality of the raw material, the model would be stochastic. The distinguishing feature of a stochastic model is that the value of the output cannot be determined even if the value of the controllable input is known because the specific values of the uncontrollable inputs are unknown. In this respect, stochastic models are often more difficult to analyze.

Data Preparation

Another step in the quantitative analysis of a problem is the preparation of the data required by the model. Data in this sense refer to the values of the uncontrollable inputs to the model. All uncontrollable inputs or data must be specified before we can analyze the model and recommend a decision or solution for the problem.

In the production model, the values of the uncontrollable inputs or data were \$10 per unit for profit, 5 hours per unit for production time, and 40 hours for production capacity. In the development of the model, these data values were known and incorporated into the model as it was being developed. If the model is relatively small and the uncontrollable input values or data required are few, the quantitative analyst will probably combine model development and data preparation into one step. In these situations the data values are inserted as the equations of the mathematical model are developed.

However, in many mathematical modeling situations, the data or uncontrollable input values are not readily available. In these situations the management scientist may know that the model will need profit per unit, production time, and production capacity data, but the values will not be known until the accounting, production, and engineering departments can be consulted. Rather than attempting to collect the required data as the model is being developed, the analyst will usually adopt a general notation for the model development step, and then a separate data preparation step will be performed to obtain the uncontrollable input values required by the model.

Using the general notation

$$c = \text{profit per unit}$$
$$a = \text{production time in hours per unit}$$
$$b = \text{production capacity in hours}$$

the model development step of the production problem would result in the following general model:

$$\begin{aligned} \text{Max} \quad & cx \\ \text{s.t.} \quad & \\ & ax \leq b \\ & x \geq 0 \end{aligned}$$

A separate data preparation step to identify the values for c, a, and b would then be necessary to complete the model.

Many inexperienced quantitative analysts assume that once the problem has been defined and a general model has been developed, the problem is essentially solved. These individuals tend to believe that data preparation is a trivial step in the process and can be easily handled by clerical staff. Actually, this assumption could not be further from the truth, especially with large-scale models that have numerous data input values. For example, a small linear programming model with 50 decision variables and 25 constraints could have more than 1300 data elements that must be identified in the data preparation step. The time required to prepare these data and the possibility of data collection errors will make the data preparation step a critical part of the quantitative analysis process. Often, a fairly large database is needed to support a mathematical model, and information systems specialists may become involved in the data preparation step.

Model Solution

Once the model development and data preparation steps are completed, we can proceed to the model solution step. In this step, the analyst will attempt to identify the values of the decision variables that provide the "best" output for the model. The specific decision-variable value or values providing the "best" output will be referred to as the **optimal solution** for the model. For the production problem, the model solution step involves finding the value of the production quantity decision variable x that maximizes profit while not causing a violation of the production capacity constraint.

One procedure that might be used in the model solution step involves a trial-and-error approach in which the model is used to test and evaluate various decision alternatives. In the production model, this procedure would mean testing and evaluating the model under various production quantities or values of x. Note, in Figure 1.5, that we could input trial values for x and check the corresponding output for projected profit and satisfaction of the production capacity constraint. If a particular decision alternative does not satisfy one or more of the model constraints, the decision alternative is rejected as being **infeasible**, regardless of the objective function value. If all constraints are satisfied, the decision alternative is **feasible** and a candidate for the "best" solution or recommended decision. Through this trial-and-error process of evaluating selected decision alternatives, a decision maker can identify a good—and possibly the best—feasible solution to the problem. This solution would then be the recommended decision for the problem.

Table 1.2 shows the results of a trial-and-error approach to solving the production model of Figure 1.5. The recommended decision is a production quantity of 8 because the feasible solution with the highest projected profit occurs at $x = 8$.

Although the trial-and-error solution process is often acceptable and can provide valuable information for the manager, it has the drawbacks of not necessarily providing the best solution and of being inefficient in terms of requiring numerous calculations if many decision alternatives are tried. Thus, quantitative analysts have developed special solution procedures for many models that are much more efficient than the trial-and-error approach. Throughout this text, you will be introduced to solution procedures that are applicable to the specific mathematical models that will be formulated. Some relatively small models or problems can be solved by hand computations, but most practical applications require the use of a computer.

Model development and model solution steps are not completely separable. An analyst will want both to develop an accurate model or representation of the actual problem situation and to be able to find a solution to the model. If we approach the model development step by attempting to find the most accurate and realistic mathematical model, we may find the model so large and complex that it is impossible to obtain a solution. In this case, a simpler and perhaps more easily understood model with a readily available solution procedure is preferred even if the recommended solution is only a rough approximation of the best decision. As you learn more about quantitative solution procedures, you will have a better idea of the types of mathematical models that can be developed and solved.

After a model solution is obtained, both the management scientist and the manager will be interested in determining how good the solution really is. Even though the analyst has undoubtedly taken many precautions to develop a realistic model, often the goodness or accuracy of the model cannot be assessed until model solutions are generated. Model testing and validation are frequently conducted with relatively small "test" problems that have known or at least expected solutions. If the model generates the expected solutions, and if other output information appears correct, the go-ahead may be given to use the model on the full-scale problem. However, if the model test and validation identify potential problems or inaccuracies inherent in the model, corrective action, such as model

Table 1.2 Trial-and-Error Solution for the Production Model of Figure 1.5

Decision Alternative (Production Quantity) x	Projected Profit	Total Hours of Production	Feasible Solution? (Hours Used ≤ 40)
0	0	0	Yes
2	20	10	Yes
4	40	20	Yes
6	60	30	Yes
8	80	40	Yes
10	100	50	No
12	120	60	No

modification and/or collection of more accurate input data, may be taken. Whatever the corrective action, the model solution will not be used in practice until the model has satisfactorily passed testing and validation.

Report Generation

An important part of the quantitative analysis process is the preparation of managerial reports based on the model's solution. In Figure 1.3, we see that the solution based on the quantitative analysis of a problem is one of the inputs the manager considers before making a final decision. Thus, the results of the model must appear in a managerial report that can be easily understood by the decision maker. The report includes the recommended decision and other pertinent information about the results that may be helpful to the decision maker.

A Note Regarding Implementation

As discussed in Section 1.2, the manager is responsible for integrating the quantitative solution with qualitative considerations in order to make the best possible decision. After completing the decision-making process, the manager must oversee the implementation and follow-up evaluation of the decision. The manager should continue to monitor the contribution of the model during the implementation and follow-up. At times this process may lead to requests for model expansion or refinement that will cause the management scientist to return to an earlier step of the quantitative analysis process.

Successful implementation of results is of critical importance to the management scientist as well as the manager. If the results of the quantitative analysis process are not correctly implemented, the entire effort may be of no value. It doesn't take too many unsuccessful implementations before the management scientist is out of work. Because implementation often requires people to do things differently, it often meets with resistance. People want to know, "What's wrong with the way we've been doing it?" and so on. One of the most effective ways to ensure successful implementation is to include users throughout the modeling process. A user who feels a part of identifying the problem and developing the solution is much more likely to enthusiastically implement the results. The success rate for implementing the results of a management science project is much greater for those projects characterized by extensive user involvement. The Management Science in Action, Quantitative Analysis and Supply Chain Management at the Heracles General Cement Company, discusses the use of management science techniques to optimize the operations of a supply chain.

Management Science in Action

Quantitative Analysis and Supply Chain Management at The Heracles General Cement Company*

Founded in 1911, the Heracles General Cement Company is the largest producer of cement in Greece. The company operates three cement plants in the prefecture of Evoia: one each in Volos, Halkis, and Milaki. Heracles' annual cement production capacity is 9.6 million tons, and the company manages 10 quarries that supply limestone, clay, and schist. Seven of these quarries are in the vicinity of the cement plants, and three are managed by Heracles' affiliate LAVA. The company also operates and maintains six distribution centers that are located across Greece; these distribution centers handle 2.5 million tons of domestic cement sales annually, which is over 40% of domestic sales in Greece.

Heracles faces a wide range of logistical challenges in transporting its products to customers. As a result, in 2005 the corporation decided to improve the efficiency of its supply chain by developing a platform for supply chain optimization and planning (SCOP) using mathematical programming. Heracles' objectives in creating SCOP were to (1) improve the efficiency of decision-making processes throughout its supply chain operations by synchronizing production plans, inventory control, and transportation policies; (2) integrate the core business processes of planning and budgeting with supply chain

*Based on G. Dikos and S. Spyropoulou, "SCOP in Heracles General Cement Company," *Interfaces* 43, no. 4 (July/August 2013): 297–312.

operations; and (3) achieve system-wide cost reductions through global optimization.

SCOP has been extremely successful. In addition to achieving its three initial goals through the development and implementation of SCOP, the platform provides Heracles with guidance for optimal revision of policies and responses to demand and cost fluctuations. The impact and success of SCOP translates into improved internal coordination, lower costs, greater operational efficiency, and increased customer satisfaction.

Notes + Comments

1. Developments in computer technology have increased the availability of management science techniques to decision makers. Many software packages are now available for personal computers. Microsoft Excel is widely used in management science courses and in industry.
2. Various chapter appendices provide step-by-step instructions for using Excel to solve problems in the text. Microsoft Excel has become the most used analytical modeling software in business and industry. We recommend that you read Appendix A, Building Spreadsheet Models, located in the back of this text.

1.4 Models of Cost, Revenue, and Profit

Some of the most basic quantitative models arising in business and economic applications are those involving the relationship between a volume variable—such as production volume or sales volume—and cost, revenue, and profit. Through the use of these models, a manager can determine the projected cost, revenue, and/or profit associated with an established production quantity or a forecasted sales volume. Financial planning, production planning, sales quotas, and other areas of decision making can benefit from such cost, revenue, and profit models.

Cost and Volume Models

The cost of manufacturing or producing a product is a function of the volume produced. This cost can usually be defined as a sum of two costs: fixed cost and variable cost. **Fixed cost** is the portion of the total cost that does not depend on the production volume; this cost remains the same no matter how much is produced. **Variable cost**, on the other hand, is the portion of the total cost that is dependent on and varies with the production volume. To illustrate how cost and volume models can be developed, we will consider a manufacturing problem faced by Nowlin Plastics.

Nowlin Plastics produces a line of cell phone covers. Nowlin's best-selling cover is its Viper model, a slim but very durable black and gray plastic cover. Several products are produced on the same manufacturing line, and a setup cost is incurred each time a changeover is made for a new product. Suppose that the setup cost for the Viper is \$3000. This setup cost is a fixed cost that is incurred regardless of the number of units eventually produced. In addition, suppose that variable labor and material costs are \$2 for each unit produced. The cost–volume model for producing x units of the Viper can be written as

$$C(x) = 3000 + 2x \tag{1.3}$$

where

$$x = \text{production volume in units}$$
$$C(x) = \text{total cost of producing } x \text{ units}$$

Once a production volume is established, the model in equation (1.3) can be used to compute the total production cost. For example, the decision to produce $x = 1200$ units would result in a total cost of $C(1200) = 3000 + 2(1200) = \$5400$.

Marginal cost is defined as the rate of change of the total cost with respect to production volume. That is, it is the cost increase associated with a one-unit increase in the production volume. In the cost model of equation (1.3), we see that the total cost $C(x)$ will increase by \$2 for each unit increase in the production volume. Thus, the marginal cost is \$2. With more complex total cost models, marginal cost may depend on the production volume. In such cases, we could have marginal cost increasing or decreasing with the production volume x.

Revenue and Volume Models

Management of Nowlin Plastics will also want information on the projected revenue associated with selling a specified number of units. Thus, a model of the relationship between revenue and volume is also needed. Suppose that each Viper cover sells for \$5. The model for total revenue can be written as

$$R(x) = 5x \tag{1.4}$$

where

$$\begin{aligned} x &= \text{sales volume in units} \\ R(x) &= \text{total revenue associated with selling } x \text{ units} \end{aligned}$$

Marginal revenue is defined as the rate of change of total revenue with respect to sales volume. That is, it is the increase in total revenue resulting from a one-unit increase in sales volume. In the model of equation (1.4), we see that the marginal revenue is \$5. In this case, marginal revenue is constant and does not vary with the sales volume. With more complex models, we may find that marginal revenue increases or decreases as the sales volume x increases.

Profit and Volume Models

One of the most important criteria for management decision making is profit. Managers need to be able to know the profit implications of their decisions. If we assume that we will only produce what can be sold, the production volume and sales volume will be equal. We can combine equations (1.3) and (1.4) to develop a profit–volume model that will determine the total profit associated with a specified production–sales volume. Total profit, denoted $P(x)$, is total revenue minus total cost; therefore, the following model provides the total profit associated with producing and selling x units:

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 5x - (3000 + 2x) = -3000 + 3x \end{aligned} \tag{1.5}$$

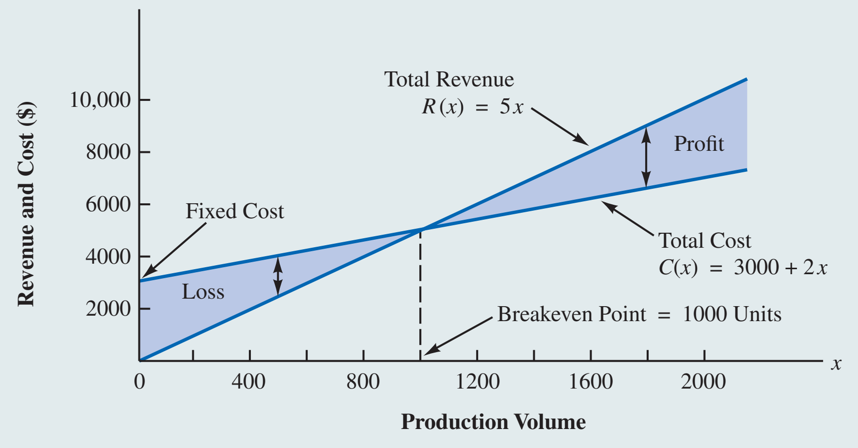
Thus, the profit–volume model can be derived from the revenue–volume and cost–volume models.

Breakeven Analysis

Using equation (1.5), we can now determine the total profit associated with any production volume x. For example, suppose that a demand forecast indicates that 500 units of the product can be sold. The decision to produce and sell the 500 units results in a projected profit of

$$P(500) = -3000 + 3(500) = -1500$$

Figure 1.6 Graph of the Breakeven Analysis for Nowlin Plastics



In other words, a loss of $1500 is predicted. If sales are expected to be 500 units, the manager may decide against producing the product. However, a demand forecast of 1800 units would show a projected profit of

$$P(1800) = -3000 + 3(1800) = 2400$$

This profit may be enough to justify proceeding with the production and sale of the product.

We see that a volume of 500 units will yield a loss, whereas a volume of 1800 provides a profit. The volume that results in total revenue equaling total cost (providing $0 profit) is called the **breakeven point**. If the breakeven point is known, a manager can quickly infer that a volume above the breakeven point will result in a profit, whereas a volume below the breakeven point will result in a loss. Thus, the breakeven point for a product provides valuable information for a manager who must make a yes/no decision concerning production of the product.

Let us now return to the Nowlin Plastics example and show how the total profit model in equation (1.5) can be used to compute the breakeven point. The breakeven point can be found by setting the total profit expression equal to zero and solving for the production volume. Using equation (1.5), we have

$$\begin{aligned} P(x) = -3000 + 3x &= 0 \\ 3x &= 3000 \\ x &= 1000 \end{aligned}$$

With this information, we know that production and sales of the product must be greater than 1000 units before a profit can be expected. The graphs of the total cost model, the total revenue model, and the location of the breakeven point are shown in Figure 1.6. In Appendix 1.1 we also show how Excel can be used to perform a breakeven analysis for the Nowlin Plastics production example.

1.5 Management Science Techniques

In this section we present a brief overview of the management science techniques covered in this text. Over the years, practitioners have found numerous applications for the following techniques:

Linear Programming Linear programming is a problem-solving approach developed for situations involving maximizing or minimizing a linear function subject to linear constraints that limit the degree to which the objective can be pursued. The production model developed in Section 1.3 (see Figure 1.5) is an example of a simple linear programming model.

Integer Linear Programming Integer linear programming is an approach used for problems that can be set up as linear programs, with the additional requirement that some or all of the decision variables be integer values.

Distribution and Network Models A network is a graphical description of a problem consisting of circles called nodes that are interconnected by lines called arcs. Specialized solution procedures exist for these types of problems, enabling us to quickly solve problems in such areas as supply chain design, information system design, and project scheduling.

Nonlinear Programming Many business processes behave in a nonlinear manner. For example, the price of a bond is a nonlinear function of interest rates; the quantity demanded for a product is usually a nonlinear function of the price. Nonlinear programming is a technique that allows for maximizing or minimizing a nonlinear function subject to nonlinear constraints.

Project Scheduling: PERT/CPM In many situations, managers are responsible for planning, scheduling, and controlling projects that consist of numerous separate jobs or tasks performed by a variety of departments, individuals, and so forth. The PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) techniques help managers carry out their project scheduling responsibilities.

Inventory Models Inventory models are used by managers faced with the dual problems of maintaining sufficient inventories to meet demand for goods and, at the same time, incurring the lowest possible inventory holding costs.

Waiting-Line or Queueing Models Waiting-line or queueing models have been developed to help managers understand and make better decisions concerning the operation of systems involving waiting lines.

Simulation Simulation is a technique used to model the operation of a system. This technique employs a computer program to model the operation and perform simulation computations.

Decision Analysis Decision analysis can be used to determine optimal strategies in situations involving several decision alternatives and an uncertain or risk-filled pattern of events.

Goal Programming Goal programming is a technique for solving multicriteria decision problems, usually within the framework of linear programming.

Analytic Hierarchy Process This multicriteria decision-making technique permits the inclusion of subjective factors in arriving at a recommended decision.

Forecasting Forecasting methods are techniques that can be used to predict future aspects of a business operation.

Markov Process Models Markov process models are useful in studying the evolution of certain systems over repeated trials. For example, Markov processes have been used to describe the probability that a machine, functioning in one period, will function or break down in another period.

Methods Used Most Frequently

Our experience as both practitioners and educators has been that the most frequently used management science techniques are linear programming, integer programming, network models (including supply chain models), and simulation. Depending upon the industry, the other methods in the preceding list are used more or less frequently.

Helping to bridge the gap between the manager and the management scientist is a major focus of the text. We believe that the barriers to the use of management science can best be removed by increasing the manager's understanding of how management science can be applied. The text will help you develop an understanding of which management science techniques are most useful, how they are used, and, most importantly, how they can assist managers in making better decisions.

The Management Science in Action, Impact of Operations Research on Everyday Living, describes some of the many ways quantitative analysis affects our everyday lives.

Management Science in Action

Impact of Operations Research on Everyday Living*

In an interview with Virginia Postrel of the *Boston Globe*, Mark Eisner, associate director of the School of Operations Research and Industrial Engineering at Cornell University, once said that operations research "is probably the most important field nobody's ever heard of." He further defines Operations Research as "the effective use of scarce resources under dynamic and uncertain conditions." As Professor Eisner's definition implies, the impact of operations research on everyday living is substantial.

Suppose you schedule a vacation to Florida and use Orbitz to book your flights. An algorithm developed by operations researchers will search among millions of options to find the cheapest fare. Another algorithm will schedule the flight crews and aircraft used by the airline, and yet another algorithm will determine the price you are charged for your ticket. If you rent a car in Florida, the price you pay for the car is determined by a mathematical model that seeks to maximize revenue for the car rental firm. If you do some shopping on your trip and decide to ship your purchases home using UPS, another algorithm tells UPS which truck to put the packages on, the route the truck should follow to avoid congested roads, and where the packages should be placed on the truck to minimize loading and unloading time. Do you subscribe to Netflix? The organization uses operations research, ratings you provide for movies, and your history of movie selections to recommend other movies that will likely appeal to you. Political campaigns even use operations research to decide where to campaign, where to advertise, and how to spend campaign funds in a manner that will maximize the candidate's chance of getting elected.

Operations Research is commonly used in the healthcare industry. Researchers from the Johns Hopkins Bloomberg School of Public Health, Pittsburgh Supercomputing Center (PSC), University of Pittsburgh, and University of California, Irvine use operations research algorithms in the Regional Healthcare Ecosystem Analyst (RHEA). RHEA is used to assess how increases or decreases in vancomycin-resistant enterococci (VRE) at one hospital ultimately change the incidence of VRE in neighboring hospitals. Because VRE is one of the most common bacteria that cause infections in healthcare facilities, RHEA could dramatically reduce the length of hospital stays and the cost of treatment by reducing the incidence of VRE.

"Our study demonstrates how extensive patient sharing among different hospitals in a single region substantially influences VRE burden in those hospitals," states Bruce Y. Lee, MD, MBA, lead author and associate professor of International Health and Director of Operations Research, International Vaccine Access Center, at the Johns Hopkins Bloomberg School of Public Health. "Lowering barriers to cooperation and collaboration among hospitals, for example, developing regional control programs, coordinating VRE control campaigns, and performing regional research studies could favorably influence regional VRE prevalence."

*Based on Virginia Postrel, "Operations Everything," *The Boston Globe*, June 27, 2004; "How Superbug Spreads Among Regional Hospitals: A Domino Effect," *Science News*, July 30, 2013; and Bruce Y. Lee, S. Levent Yilmaz, Kim F. Wong, Sarah M. Bartsch, Stephen Eubank, Yeohan Song, et al., "Modeling the Regional Spread and Control of Vancomycin-Resistant Enterococci," *American Journal of Infection Control*, 41, no. 8 (2013):668–673.

Notes + Comments

The Institute for Operations Research and the Management Sciences (INFORMS) and the Decision Sciences Institute (DSI) are two professional societies that publish journals and newsletters dealing with current research and applications of operations research and management science techniques.

Summary

This text is about how management science may be used to help managers make better decisions. The focus of this text is on the decision-making process and on the role of management science in that process. We discussed the problem orientation of this process and in an overview showed how mathematical models can be used in this type of analysis.

The difference between the model and the situation or managerial problem it represents is an important point. Mathematical models are abstractions of real-world situations and, as such, cannot capture all the aspects of the real situation. However, if a model can capture the major relevant aspects of the problem and can then provide a solution recommendation, it can be a valuable aid to decision making.

One of the characteristics of management science that will become increasingly apparent as we proceed through the text is the search for a best solution to the problem. In carrying out the quantitative analysis, we shall be attempting to develop procedures for finding the "best" or optimal solution.

Glossary

Analog model Although physical in form, an analog model does not have a physical appearance similar to the real object or situation it represents.
Breakeven point The volume at which total revenue equals total cost.
Constraints Restrictions or limitations imposed on a problem.
Controllable inputs The inputs that are controlled or determined by the decision maker.
Decision The alternative selected.
Decision making The process of defining the problem, identifying the alternatives, determining the criteria, evaluating the alternatives, and choosing an alternative.
Decision variable Another term for controllable input.
Deterministic model A model in which all uncontrollable inputs are known and cannot vary.
Feasible solution A decision alternative or solution that satisfies all constraints.
Fixed cost The portion of the total cost that does not depend on the volume; this cost remains the same no matter how much is produced.
Iconic model A physical replica, or representation, of a real object.
Infeasible solution A decision alternative or solution that does not satisfy one or more constraints.
Marginal cost The rate of change of the total cost with respect to volume.
Marginal revenue The rate of change of total revenue with respect to volume.
Mathematical model Mathematical symbols and expressions used to represent a real situation.
Model A representation of a real object or situation.
Multicriteria decision problem A problem that involves more than one criterion; the objective is to find the "best" solution, taking into account all the criteria.
Objective function A mathematical expression that describes the problem's objective.
Optimal solution The specific decision-variable value or values that provide the "best" output for the model.
Problem solving The process of identifying a difference between the actual and the desired state of affairs and then taking action to resolve the difference.
Single-criterion decision problem A problem in which the objective is to find the "best" solution with respect to just one criterion.
Stochastic (probabilistic) model A model in which at least one uncontrollable input is uncertain and subject to variation; stochastic models are also referred to as probabilistic models.
Uncontrollable inputs The environmental factors or inputs that cannot be controlled by the decision maker.
Variable cost The portion of the total cost that is dependent on and varies with the volume.

Problems

1. **Vocabulary.** Define the terms *management science* and *operations research*. **LO 1**
2. **The Decision-Making Process.** List and discuss the steps of the decision-making process. **LO 2**
3. **Roles of Qualitative and Quantitative Approaches to Decision Making.** Discuss the different roles played by the qualitative and quantitative approaches to managerial decision making. Why is it important for a manager or decision maker to have a good understanding of both of these approaches to decision making? **LO 2**
4. **Production Scheduling.** A firm just completed a new plant that will produce more than 500 different products, using more than 50 different production lines and machines. The production scheduling decisions are critical in that sales will be lost if customer demands are not met on time. If no individual in the firm has experience with this production operation and if new production schedules must be generated each week, why should the firm consider a quantitative approach to the production scheduling problem? **LO 2**
5. **Advantages of Modeling.** List the advantages of analyzing and experimenting with a model as opposed to a real object or situation. **LO 3**
6. **Choosing Between Models.** Suppose that a manager has a choice between the following two mathematical models of a given situation: (a) a relatively simple model that is a reasonable approximation of the real situation, and (b) a thorough and complex model that is the most accurate mathematical representation of the real situation possible. Why might the model described in part (a) be preferred by the manager? **LO 3**
7. **Modeling Fuel Cost.** Suppose you are going on a weekend trip to a city that is d miles away. Develop a model that determines your round-trip gasoline costs. What assumptions or approximations are necessary to treat this model as a deterministic model? Are these assumptions or approximations acceptable to you? **LO 3**
8. **Adding a Second Product to a Production Model.** Recall the production model from Section 1.3:

$$\begin{aligned} &\text{Max} \quad 10x \\ &\text{s.t.} \\ &\qquad 5x \le 40 \\ &\qquad\ \ x \ge 0 \end{aligned}$$

   Suppose the firm in this example considers a second product that has a unit profit of \$5 and requires 2 hours of production time for each unit produced. Use y as the number of units of product 2 produced. **LO 4**

   a. Show the mathematical model when both products are considered simultaneously.
   b. Identify the controllable and uncontrollable inputs for this model.
   c. Draw the flowchart of the input–output process for this model (see Figure 1.5).
   d. What are the optimal solution values of x and y?
   e. Is the model developed in part (a) a deterministic or a stochastic model? Explain.

9. **Stochastic Production Models.** Suppose we modify the production model in Section 1.3 to obtain the following mathematical model:

$$\begin{aligned} &\text{Max} \quad 10x \\ &\text{s.t.} \\ &\qquad ax \le 40 \\ &\qquad\ \ x \ge 0 \end{aligned}$$

where a is the number of hours of production time required for each unit produced. With $a = 5$, the optimal solution is $x = 8$. If we have a stochastic model with $a = 3$, $a = 4$, $a = 5$, or $a = 6$ as the possible values for the number of hours required per unit, what is the optimal value for x? What problems does this stochastic model cause? **LO 4**

10. **Modeling Shipments.** A retail store in Des Moines, Iowa, receives shipments of a particular product from Kansas City and Minneapolis. Define x and y as follows. **LO 4**

$$x = \text{number of units of the product received from Kansas City}$$
$$y = \text{number of units of the product received from Minneapolis}$$

   a. Write an expression for the total number of units of the product received by the retail store in Des Moines.
   b. Shipments from Kansas City cost \$0.20 per unit, and shipments from Minneapolis cost \$0.25 per unit. Develop an objective function representing the total cost of shipments to Des Moines.
   c. Assuming the monthly demand at the retail store is 5000 units, develop a constraint that requires 5000 units to be shipped to Des Moines.
   d. No more than 4000 units can be shipped from Kansas City, and no more than 3000 units can be shipped from Minneapolis in a month. Develop constraints to model this situation.
   e. Of course, negative amounts cannot be shipped. Combine the objective function and constraints developed to state a mathematical model for satisfying the demand at the Des Moines retail store at minimum cost.

11. **Modeling Demand as a Function of Price.** For most products, higher prices result in a decreased demand, whereas lower prices result in an increased demand. Let

$$d = \text{annual demand for a product in units}$$
$$p = \text{price per unit}$$

Assume that a firm accepts the following price–demand relationship as being realistic:

$$d = 800 - 10p$$

where p must be between \$20 and \$70. **LO 4**

   a. How many units can the firm sell at the \$20 per-unit price? At the \$70 per-unit price?
   b. What happens to annual units demanded for the product if the firm increases the per-unit price from \$26 to \$27? From \$42 to \$43? From \$68 to \$69? What is the suggested relationship between the per-unit price and annual demand for the product in units?
   c. Show the mathematical model for the total revenue (TR), which is the annual demand multiplied by the unit price.
   d. Based on other considerations, the firm's management will only consider price alternatives of \$30, \$40, and \$50. Use your model from part (b) to determine the price alternative that will maximize the total revenue.
   e. What are the expected annual demand and the total revenue corresponding to your recommended price?

12. **Modeling a Special Order Decision.** The O'Neill Shoe Manufacturing Company will produce a special-style shoe if the order size is large enough to provide a reasonable profit. For each special-style order, the company incurs a fixed cost of \$2000 for the production setup. The variable cost is \$60 per pair, and each pair sells for \$80. **LO 4**
   a. Let x indicate the number of pairs of shoes produced. Develop a mathematical model for the total cost of producing x pairs of shoes.
   b. Let P indicate the total profit. Develop a mathematical model for the total profit realized from an order for x pairs of shoes.
   c. How large must the shoe order be before O'Neill will break even?

13. **Breakeven Point for a Training Seminar.** Micromedia offers computer training seminars on a variety of topics. In the seminars each student works at a personal computer, practicing the particular activity that the instructor is presenting. Micromedia is currently planning a two-day seminar on the use of Microsoft Excel in statistical analysis. The projected fee for the seminar is \$600 per student. The cost for the conference room, instructor compensation, lab assistants, and promotion is \$9600. Micromedia rents computers for its seminars at a cost of \$120 per computer per day. **LO 4**
    a. Develop a model for the total cost to put on the seminar. Let x represent the number of students who enroll in the seminar.
    b. Develop a model for the total profit if x students enroll in the seminar.
    c. Micromedia has forecasted an enrollment of 30 students for the seminar. How much profit will be earned if their forecast is accurate?
    d. Compute the breakeven point.
14. **Breakeven Point for a Textbook.** Eastman Publishing Company is considering publishing a paperback textbook on spreadsheet applications for business. The fixed cost of manuscript preparation, textbook design, and production setup is estimated to be \$160,000. Variable production and material costs are estimated to be \$6 per book. The publisher plans to sell the text to college and university bookstores for \$46 each. **LO 4**
    a. What is the breakeven point?
    b. What profit or loss can be anticipated with a demand of 3800 copies?
    c. With a demand of 3800 copies, what is the minimum price per copy that the publisher must charge to break even?
    d. If the publisher believes that the price per copy could be increased to \$50.95 and not affect the anticipated demand of 3800 copies, what action would you recommend? What profit or loss can be anticipated?
15. **Breakeven Point for Stadium Luxury Boxes.** Preliminary plans are under way for the construction of a new stadium for a major league baseball team. City officials have questioned the number and profitability of the luxury corporate boxes planned for the upper deck of the stadium. Corporations and selected individuals may buy the boxes for \$300,000 each. The fixed construction cost for the upper-deck area is estimated to be \$4,500,000, with a variable cost of \$150,000 for each box constructed. **LO 4**
    a. What is the breakeven point for the number of luxury boxes in the new stadium?
    b. Preliminary drawings for the stadium show that space is available for the construction of up to 50 luxury boxes. Promoters indicate that buyers are available and that all 50 could be sold if constructed. What is your recommendation concerning the construction of luxury boxes? What profit is anticipated?
16. **Modeling Investment Strategies.** Financial Analysts, Inc., is an investment firm that manages stock portfolios for a number of clients. A new client is requesting that the firm handle an \$800,000 portfolio. As an initial investment strategy, the client would like to restrict the portfolio to a mix of the following two stocks:

Stock	Price/ Share	Estimated Annual Return/Share	Maximum Possible Investment
Oil Alaska	\$50	\$6	\$500,000
Southwest Petroleum	\$30	\$4	\$450,000

Define x and y as follows. **LO 4**

x = number of shares of Oil Alaska
y = number of shares of Southwest Petroleum

a. Develop the objective function, assuming that the client desires to maximize the total annual return.
b. Show the mathematical expression for each of the following three constraints:
(1) Total investment funds available are \$800,000.
(2) Maximum Oil Alaska investment is \$500,000.
(3) Maximum Southwest Petroleum investment is \$450,000.

Note: Adding the $x \geq 0$ and $y \geq 0$ constraints provides a linear programming model for the investment problem. A solution procedure for this model will be discussed in Chapter 2.

17. **Modeling an Inventory System.** Models of inventory systems frequently consider the relationships among a beginning inventory, a production quantity, a demand or sales, and an ending inventory. For a given production period j, define s, x, and d as follows. **LO 4**

s_{j-1} = ending inventory from the previous period (beginning inventory for period j)
x_j = production quantity in period j
d_j = demand in period j
s_j = ending inventory for period j

a. Write the mathematical relationship or model that describes how these four variables are related.
b. What constraint should be added if production capacity for period j is given by C_j?
c. What constraint should be added if inventory requirements for period j mandate an ending inventory of at least I_j?

18. **Blending Fuels.** Esiason Oil makes two blends of fuel by mixing oil from three wells, one each in Texas, Oklahoma, and California. The costs and daily availability of the oils are provided in the following table.

Source of Oil	Cost per Gallon	Daily Gallons Available
Texas well	0.30	12,000
Oklahoma well	0.40	20,000
California well	0.48	24,000

Because these three wells yield oils with different chemical compositions, Esiason's two blends of fuel are composed of different proportions of oil from its three wells. Blend A must be composed of at least 35% of oil from the Texas well, no more than 50% of oil from the Oklahoma well, and at least 15% of oil from the California well. Blend B must be composed of at least 20% of oil from the Texas well, at least 30% of oil from the Oklahoma well, and no more than 40% of oil from the California well.

Each gallon of Blend A can be sold for \$3.10 and each gallon of Blend B can be sold for \$3.20. Long-term contracts require at least 20,000 gallons of each blend to be produced.

Define x, y, and i as follows. **LO 4**

x_i = number of gallons of oil from well i used in production of Blend A
y_i = number of gallons of oil from well i used in production of Blend B
i = 1 for the Texas well, 2 for the Oklahoma well, 3 for the California well

a. Develop the objective function, assuming that the client desires to maximize the total daily profit.
b. Show the mathematical expression for each of the following three constraints:
(1) Total daily gallons of oil available from the Texas well is 12,000.
(2) Total daily gallons of oil available from the Oklahoma well is 20,000.
(3) Total daily gallons of oil available from the California well is 24,000.
c. Should this problem include any other constraints? If so, express them mathematically in terms of the decision variables.